Urban Economics: Economics 4621 (Holmes)

Dept. of Economics, University of Minnesota

Homework 3

(Due Tuesday, April 20)

Following the notation from class, suppose the average time cost function is

$$\begin{aligned} A(Q,X) &= 1, Q \leq X \\ &= 1 + \left(\frac{Q}{X} - 1\right)^2, Q > X, \end{aligned}$$

when highway capacity is X and there are Q drivers.

1. Suppose that $X = 1$, so that the average cost function simplifies to

$$A(Q) = 1 + (Q-1)^2.$$

Suppose that the (inverse) demand function for driving is $D(Q) = 8 - Q$.

(a) Derive the total driving cost function $T(Q)$ and the marginal cost function $M(Q)$.

(b) What is the equilibrium level of driving Q^e with no congestion tax? What is the average time cost at the equilibrium?

(c) What is the value of the externality term $A'(Q^e)Q^e$ at the equilibrium from (b)

(d) What is the socially optimal level of driving Q^* and what is the average time cost at the optimum?

(e) What congestion tax implements the socially optimal level of driving

(f) Graph your answer and illustrate the deadweight loss from the externality when there is no congestion tax.

(g) Suppose that the cost of capacity equals c per unit of capacity (in units of time). Suppose that the social planner picks Q and X to maximize total surplus and

sets the socially optimal congestion tax. What would the cost of capacity c have to be if $X^* = 1$ were the solution to the social planner's problem?

2. Suppose for this problem that there are no congestion taxes. Consider a different demand function. Assume that

$$\begin{aligned} D(Q) &= 5,\ Q \in [0, 10] \\ &= 0,\ Q > 10. \end{aligned}$$

This demand is perfectly elastic at a price of 5 time units up to 10 drivers. An interpretation of this demand is that there are 10 drivers each with the same willingness to pay of 5 units of time to use the road.

(a) Suppose that initially capacity $X = 1$. What is the equilibrium number of drivers and the equilibrium time cost. Graph your answer.

(b) Suppose that capacity is raised to $X = 2$. What is the new equilibrium number of drivers and the new equilibrium time cost? Suppose it is costly to raise capacity to $X = 2$ compared with $X = 1$. What can you say about the welfare effects of this capacity expansion?

(c) You are in charge of the department of transportation. Assume you cannot impose a congestion tax. You get to pick capacity X and the cost per unit of capacity is $c = 25$ in units of time. Your choice of capacity can be any level $X \geq 0$ (and not just the levels $X = 1$ and $X = 2$ considered so far). What is the optimal level of capacity in this case?

(d) Suppose instead that the cost of capacity is

$$c = \frac{25}{32}.$$

Show that the optimal capacity is $X^* = 8$. What is the equilibrium time spent in traffic? What is total surplus net of highway construction costs?