Tradeoff between Scale Economies and Tranportation Costs

- Scale Economies

—Many types, but usually boils down to some kind of fixed cost (or nonrivalous input)

- Transportation costs

—If consolidate production, average transportation cost must increase

—Also some other forces that offset scale economies: congestion costs, managerial diseconomies,...

A Simple Model of Retail (Hotelling Model)

- Geographic space is the real line.
- Consumers are uniformly distributed on the line with density of m per unit distance
- F is fixed cost of opening a store at a particular location
- c is constant marginal cost
- t is transportation cost per mile.
- D is distance between stores

Social Planner's Problem

- Choose D to minimize average total cost (ATC)

- $S = Dm$ is store size given D

- Average Production Cost

$$APC = \frac{cS + F}{S} = c + \frac{F}{S}$$

- Average Transportation Cost (see figure!)

$$ATrC = \frac{Dt}{4}$$

- ATC

$$\begin{aligned} ATC &= APC + ATrC \\ &= c + \frac{F}{S} + \frac{Dt}{4} \\ &= c + \frac{F}{Dm} + \frac{Dt}{4} \end{aligned}$$

- Minimize ATC. First order necessary condition (differentiate w.r.t. D)

$$0 = -\frac{F}{mD^2} + \frac{t}{4}$$

- Note sufficient second order condition holds

- Solving for D ...

$$\frac{F}{mD^2} = \frac{t}{4}$$

$$
\begin{aligned}
D^2 &= \frac{4F}{mt} \\
D^* &= \frac{4F}{mt}
\end{aligned}
$$

- Store Size

$$
\begin{aligned}
S^* &= mD^* \\
&= \frac{4mF}{t}
\end{aligned}
$$

- Important comparative statics

—D^* increases in F, decreases in m and t

—S^* increases in F and m, decreases in t

Market Equilibrium Problem

(Hotelling/Salop Monopolistic Competition model)

- Each store a separate firm

- Firms set price taking as given prices of neighboring firms (Bertrand competition)

- An equilibrium is a (p^e, D^e) so that

(1) p^e is profit maximizing given other firms set p^e and distance is D^e

(2) firms make zero profit

Problem of the Firm

- Consider firm located at point 0 on the line

- Suppose firm sets price p and neighbors set p°.

- Let x be location of consumer on the right indifferent between firm at 0 and next firm on the right

$$\begin{aligned} p + tx &= p^\circ + (D - x)t \\ 2tx &= p^\circ - p + Dt \\ x &= \frac{p^\circ - p}{2t} + \frac{D}{2} \end{aligned}$$

- Demand of firm at 0 is (adding up demand on both sides)

$$\begin{aligned} Q^d &= 2mx \\ &= 2m\left[\frac{p^\circ - p}{2t} + \frac{D}{2}\right] \end{aligned}$$

- Profit is

$$\begin{aligned} \pi &= (p-c)\,Q^d - F \\ &= 2m\,(p-c)\left[\frac{p^\circ - p}{2t} + \frac{D}{2}\right] - F \end{aligned}$$

- FONC

$$2m\left[\frac{p^\circ - p}{2t} + \frac{D}{2}\right] + 2m\,(p-c)\left[-\frac{1}{2t}\right] = 0$$

- In a symmetric equilibrium, $p^{\circ} - p = 0$. So we get

$$mD - m\frac{(p-c)}{t} = 0$$

or

$$\begin{aligned} p - c &= tD \\ p &= c + tD \end{aligned}$$

- Profit

$$\begin{aligned} \pi &= (p-c)\,S - F \\ &= (p-c)\,Dm - F \\ &= tD^2 m - F \end{aligned}$$

Setting equal to zero yields

$$\begin{aligned} D^e &= \frac{F}{tm} = \frac{1}{2}\frac{4F}{tm} = \frac{1}{2}D^* \\ S^e &= mD^e = \frac{mF}{t} = \frac{1}{2}S^* \end{aligned}$$

So stores are too close with monopolistic competition. Excess Entry.

- Direction of the comparative statics the same

Application: School Sizes (Meg Ledyard's work)

Decile (10th Grade Density)	Cutoff Density	Mean School Size
1	0.65	552
2	1.43	859
3	2.72	1,185
4	5.14	1,351
5	9.48	1,547
6	16.11	1,736
7	26.09	1,867
8	38.56	1,841
9	68.93	1,878
10	606.02	2,355

: