Lecture Notes 9/7/05

1A: The Cost of Monopoly in General Equilibrium

- Set of goods $[0,1]$, $x \in [0,1]$ a particular good.

- Utility function of representative consumer

$$
\begin{aligned}
U &= \left(\int_0^1 q(x)^{\frac{1}{\mu}} dx \right)^{\mu} \\
\sigma &= \frac{\mu}{\mu - 1}
\end{aligned}
$$

  for $\mu > 1$.

- Unit time endowment.

- Technology: one unit of labor per unit of good.

- Let labor be numeraire, $w = 1$

  Goods $x \in [0, \lambda]$ are controlled by a monopolist

- Goods $x \in (\lambda, 1]$ are perfectly competitive.

- The representative consumer owns shares in all the firms.

Solution

- Let π_M be the equilibrium monopoly profit of a representative monopolist.

- Income of the representative consumer

$$I = 1 + \lambda \pi_M.$$

- $p_C = 1$.

- Constant elasticity of demand $\Rightarrow$the price in monopoly industries is $p_M = \mu$.

- Let q_M and q_C be quantities in the equilibrium of this economy.

- Consumer MRS implies:

$$\frac{q_M}{q_C} = \left(\frac{p_M}{p_C}\right)^{-\sigma}$$

But $p_C = 1$ and $p_M = \mu$, so

$$q_M = q_C \mu^{-\sigma}$$

- Resource constraint for labor,

$$\begin{aligned} \lambda q_M + (1-\lambda) q_C &= 1 \\ \lambda q_C \mu^{-\sigma} + (1-\lambda) q_C &= \\ q_C &= \frac{1}{(1-\lambda+\lambda\mu^{-\sigma})} \end{aligned}$$

Welfare gains from antitrust

- v be the compensating variation (the change in income at the new prices so the representative consumer is indifferent to old system).

- New prices, $p = 1$ everywhere.

- $Y = 1 - v$ be income.

- $U = Y$ and

$$1 - v \;=\; \left(\lambda q_M^{\frac{1}{\mu}} + (1-\lambda)\, q_C^{\frac{1}{\mu}}\right)^{\mu}$$

$$
\begin{aligned}
&= \left(\lambda \left(q_C \mu^{-\sigma}\right)^{\frac{1}{\mu}} + (1-\lambda)\, q_C^{\frac{1}{\mu}}\right)^{\mu} \\
&= q_C \left(\lambda \mu^{-\frac{\sigma}{\mu}} + 1 - \lambda\right)^{\mu} \\
&= \frac{\left(\lambda \mu^{-\frac{\sigma}{\mu}} + 1 - \lambda\right)^{\mu}}{(1 - \lambda + \lambda \mu^{-\sigma})}
\end{aligned}
$$

or

$$
v = 1 - \frac{\left(1 - \lambda + \lambda \mu^{-\frac{1}{\mu-1}}\right)^{\mu}}{\left(1 - \lambda + \lambda \mu^{-\frac{\mu}{\mu-1}}\right)}
$$

$$
v = 1 - \frac{\left(1 - \lambda + \lambda \mu^{-\frac{1}{\mu-1}}\right)^{\mu}}{\left(1 - \lambda + \lambda \mu^{-\frac{\mu}{\mu-1}}\right)}
$$

Observe that

$$\lim_{\mu \to 1} \mu^{-\frac{1}{\mu-1}} = \lim_{\mu \to 1} \mu^{-\frac{\mu}{\mu-1}} = .3679$$

So

$$\lim_{\mu \to 1} v = 0$$

Other parameters:

μ	λ						
	0	.2	.4	.6	.8	.9	1.0
1.1	.000	.005	.009	.012	.011	.007	.000
1.5	.000	.025	.045	.058	.053	.036	.000
2.0	.000	.047	.086	.109	.100	.069	.000
5.0	.000	.141	.248	.312	.301	.227	.000

Discussion

- Harberger .1 percent of GDP. Credible?
  - Not the fashion in IO today to produce aggegate estimates
  - Much work today produces analytical tools to give to the Justice Dept

- Other costs of monopoly
  - Rent Seeking (could dissipate the profit)
    - Posner, Hsieh and Moretti example of real estate agents.
  - Effect of competition on productivity
    - x-inefficiency? Leibenstein (1966). Not much there besides an an ugly term.
    - Recent treatments: Parente and Prescott, Holmes and Schmitz

Lec 1B—Dixit Stiglitz

- Unbounded set of possible goods, $x \in [0, \infty]$

- Utility function of representative consumer

$$
\begin{aligned}
U &= \left(\int_0^\infty q(x)^{\frac{1}{\mu}} dx \right)^{\mu} \\
\sigma &= \frac{\mu}{\mu - 1}
\end{aligned}
$$

  for $\mu > 1$.

- L time endowment for economy

- Technology: ϕ fixed cost (labor) to setup a product. Constant marginal cost of β (labor)

- Let labor be numeraire, $w = 1$
- Let $[0, N]$ be interval of goods produced in the market. Let $p(x)$ be price of good x.

Definition of Equilibrium

$\{N$, $p(x)$, $q(x)$, $x \in [0, N]\}$ such that

1. Consumer demands $q(x)$ maximize utility given the budget constraint

2. $p(x)$ is the profit maximizing price of firm x, taking as given the prices of all other firms

3. Firms that enter make nonnegative profit

4. No incentive for further entry

(Note (3+4)$\Rightarrow$ zero profit).

- Problem of Consumer:

$$\max_{q(\cdot)} \left[\int_0^N q(x)^{\frac{1}{\mu}} dx \right]^{\mu} \tag{1}$$

subject to

$$\int_0^N p(x)q(x)dx = L$$

- MRS condition: goods x_1 and x_0.

$$
\begin{aligned}
\frac{\mu\,[]^{\mu-1}\left(\frac{1}{\mu}\right) q_1^{\frac{1}{\mu}-1}}{\mu\,[]^{\mu-1}\left(\frac{1}{\mu}\right) q_0^{\frac{1}{\mu}-1}} &= \frac{p_1}{p_0} \\
\left(\frac{q_1}{q_0}\right)^{-\frac{1}{\sigma}} &= \frac{p_1}{p_0} \\
q_1 &= p_1^{-\sigma}\left(p_0^{\sigma} q_0\right) \\
&= p_1^{-\sigma} k
\end{aligned}
$$

The FONC of firm 1

$$\begin{aligned} p_1^{-\sigma} k - \sigma \left(p_1 - \beta \right) p_1^{-\sigma - 1} k &= 0 \\ p_1 &= \sigma \left(p_1 - \beta \right) \\ \frac{p_1 - \beta}{p_1} &= \frac{1}{\sigma} \\ p_1 &= \mu \beta \end{aligned}$$

Constant markup over cost.

- Zero-profit condition

$$\mu\beta q - \beta q - \phi = 0, \tag{2}$$

$$\beta\,(\mu - 1)\,q = \phi$$

So

$$q^* = \frac{\phi}{\beta(\mu - 1)}$$

- Use resource constraint to determine number of products:

$$\begin{aligned} N\left(\beta q^{*}+\phi\right) &= L \\ N &= \frac{L}{\beta q^{*}+\phi}=\frac{L}{\frac{\phi}{(\mu-1)}+\phi} \\ &= L\left(\frac{\mu-1}{\mu}\right)\frac{1}{\phi}=\frac{L}{\sigma\phi} \end{aligned}$$

- Consumer Welfare (per capita)

$$
\begin{aligned}
\text{utility per capita} &= \frac{\left(\int_0^\infty q(x)^{\frac{1}{\mu}} dx\right)^\mu}{L} \\
&= \frac{\left(N {q^*}^{\frac{1}{\mu}}\right)^\mu}{L} \\
&= \frac{N^\mu q^*}{L} = \frac{\left(\frac{L}{\sigma\phi}\right)^\mu q^*}{L} \\
&= \frac{\phi^{1-\mu}\sigma^{-\mu}}{\beta(\mu-1)} L^{\mu-1}
\end{aligned}
$$

Increasing in L(love of variety).

Lec 1C

Basic Address Model of Product Differentiation (Hotelling, Salop)

- Geographic space is the real line.

- Consumers are uniformly distributed on the line with density of L per unit distance

- ϕ is fixed cost of opening a store at a particular location

- β is constant marginal cost

- t is transportation cost per mile.
- D is distance between stores

Social Planner's Problem

- Choose D to minimize average total cost (ATC)
- $S = DL$ is store size given D
- Average Production Cost

$$APC = \frac{\beta S + \phi}{S} = \beta + \frac{\phi}{S}$$

- Average Transportation Cost (see figure!)

$$ATrC = \frac{Dt}{4}$$

- ATC

$$\begin{aligned} ATC &= APC + ATrC \\ &= \beta + \frac{\phi}{S} + \frac{Dt}{4} \\ &= \beta + \frac{\phi}{DL} + \frac{Dt}{4} \end{aligned}$$

- Minimize ATC. First order necessary condition (differentiate w.r.t. D)

$$0 = -\frac{\phi}{LD^2} + \frac{t}{4}$$

- Note sufficient second order condition holds

- Solving for D ...

$$\frac{\phi}{LD^2} = \frac{t}{4}$$

$$
\begin{aligned}
D^2 &= \frac{4\phi}{Lt} \\
D^* &= \left(\frac{4\phi}{Lt}\right)^{\frac{1}{2}}
\end{aligned}
$$

- Store Size

$$
\begin{aligned}
S^* &= LD^* \\
&= \left(\frac{4L\phi}{t}\right)^{\frac{1}{2}}
\end{aligned}
$$

- Important comparative statics

—D^* increases in ϕ, decreases in L and t

—S^* increases in ϕ and L, decreases in t

Market Equilibrium Problem

(Hotelling/Salop Monopolistic Competition model)

Note: model has "issues" that we will discuss at the end

- Each store a separate firm
- Firms set price taking as given prices of neighboring firms (Bertrand competition)
- An equilibrium is a (p^e, D^e) so that

(1) p^e is profit maximizing given other firms set p^e and distance is D^e

(2) firms make zero profit

Problem of the Firm

- Consider firm located at point 0 on the line
- Suppose firm sets price p and neighbors set p°.
- Let x be location of consumer on the right indifferent between firm at 0 and next firm on the right

$$\begin{aligned} p + tx &= p^\circ + (D - x)t \\ 2tx &= p^\circ - p + Dt \\ x &= \frac{p^\circ - p}{2t} + \frac{D}{2} \end{aligned}$$

- Demand of firm at 0 is (adding up demand on both sides)

$$\begin{aligned} Q^d &= 2Lx \\ &= 2L\left[\frac{p^\circ - p}{2t} + \frac{D}{2}\right] \end{aligned}$$

- Profit is

$$\begin{aligned} \pi &= (p-\beta)\, Q^d - \phi \\ &= 2L\,(p-\beta)\left[\frac{p^\circ - p}{2t} + \frac{D}{2}\right] - \phi \end{aligned}$$

- FONC

$$2L\left[\frac{p^\circ - p}{2t} + \frac{D}{2}\right] + 2L\,(p-\beta)\left[-\frac{1}{2t}\right] = 0$$

- In a symmetric equilibrium, $p^{\circ} - p = 0$. So we get

$$LD - L\frac{(p-\beta)}{t} = 0$$

or

$$\begin{aligned} p - \beta &= tD \\ p &= \beta + tD \end{aligned}$$

- Profit

$$\begin{aligned} \pi &= (p-\beta)\,S - \phi \\ &= (p-\beta)\,DL - \phi \\ &= tD^2 L - \phi \end{aligned}$$

Setting equal to zero yields

$$D^e = \left(\frac{\phi}{tL}\right)^{\frac{1}{2}} = \frac{1}{2}\left(\frac{4\phi}{tL}\right)^{\frac{1}{2}} = \frac{1}{2}D^*$$

$$S^e = LD^e = \left(\frac{L\phi}{t}\right)^{\frac{1}{2}} = \frac{1}{2}S^*$$

So stores are too close with monopolistic competition. Excess Entry.

- Direction of the comparative statics the same

Issues

- Didn't specify this as a two stage game.

- Suppose try. Say a circle.
  - Stage 1 N firms enter
  - Stage 2. firms are equally spaced around the circle
  - Stage 3. Price game.
  - Straightforward to generalize the above analysis. Given discreteness, will in general have positive profits. (But next guy in makes profits negative).

- Problem: model assumes firms are equally spaced. If firms are free to pick locations that aren't equally spaced, a pure strategy equilibrium may not exist in the price subgame. There is discontinuities in demand. (If assume transportation costs is quadratic rather than linear, no discontinuity.)

Part 1D: Mankiw and Whinston

- Homogenous product market demand $P(Q)$, Q total output. $P'(Q) < 0$

- Fixed cost ϕ

- Variable costs $c(q)$, $c(0) = 0$, $c'(q) \geq 0$, $c''(q) \geq 0$.

- Second stage, output per entrant is determined. Let q_N be equilibrium output per firm, given N entrants (you pick model of competition). But assume (easy to check this is satisfied with Cournot and $P''(Q) \leq 0$):

- $Nq_N > \hat{N}\hat{q}_N$, $N > \hat{N}$ and $\lim_{N\to\infty} Nq_N = M < \infty$
- $q_N < q_{\hat{N}}$, for $N > \hat{N}$.
- $P(Nq_N) - c'(q_N) > 0$ for all N.

- First stage entry: N^e, then $\pi_{N^e} \geq 0$, and $\pi_{N^e+1} < 0$.

Social Planner

- Planner controls entry but not pricing given entry.

- Maximizes total surplus. So problem is

$$\max_N W(N) = \int_0^{Nq_N} P(s)ds - Nc(q_N) - N\phi$$

- Ignore integer constraint, for now. The Planner's FONC is

$$\begin{aligned} W'(N^*) &= P(Nq_N)\left[N\frac{\partial q_N}{\partial N} + q_N\right] - c(q_N) - Nc'(q_N)\frac{\partial q_N}{\partial N} - \phi \\ &= [Pq_N - c - \phi] + N\left[P - c'\right]\frac{\partial q_N}{\partial N} \\ &= \pi_N + N\left[P - c'\right]\frac{\partial q_N}{\partial N} \\ &= 0 \end{aligned}$$

- Evaluate at N^e, observe that $\pi_{N^e} = 0$, so $W'(N^e) < 0$, (since $P > c'$, and $\frac{\partial q_N}{\partial N} < 0$. Excessive entry.

- Intuition

- If impose the integer constraint then $N^e \geq N^* - 1$.