Lecture—Pakes, Ostrovsky, and Berry

Dynamic *and* Stochastic Model of Industry

- Let π_n be flow profit of incumbant firms when n firms are in the industry.
  - $\pi_1 > 0$,
  - $0 \leq \pi_2 < \pi_1$
  - $\pi_n = -\infty$, for $n \geq 3$.
- Incumbent firm draw an exit value ϕ each period from the standard exponential distribution, so the density and c.d.f. is

$$f(\phi) \;=\; \sigma e^{-\frac{\phi}{\sigma}}$$

$$F(\phi) \;=\; 1 - e^{-\frac{\phi}{\sigma}}.$$

where the expected value of ϕ is σ. If leave get this tomorrow.

- One possible entrant in each period. It draws an entry cost of $\kappa = 0$ with probability γ and cost $\kappa = \infty$ with probability $1 - \gamma$.

- β discount factor.

Markov-perfect Equilibrium

- Let VC_n be the continuation value (i.e. return starting next period in next period dollars) when remain in the industry and there are n firms.

- Let $V_n(\phi)$ be the discounted value given n firms today and given exit value of ϕ,

$$V_n(\phi) = \max\{\pi_n + \beta\phi, \pi_n + \beta VC_n\}.$$

Let $\hat{\phi}_n$ be a cutoff rule such that an incumbent exits if $\phi > \hat{\phi}_n$ when there are n firms.

- Since $\pi_1 > 0$, $0 \leq \pi_2 < \pi_1$ and $\pi_n = -\infty$, for $n \geq 3$, it is immediate that there will never be entry if $n = 2$.

- Since the entry cost draws an entry cost of $\kappa = 0$ with probability γ and cost $\kappa = \infty$ with probability $1 - \gamma$, it is clear that the entrant comes if $\kappa = 0$ and $n \leq 1$ and otherwise doesn't enter.

- Taking this entry behavior as given, a MPE is a list

$$\{\hat{\phi}_1, \hat{\phi}_2, VC_1, VC_2\}$$

such that $\hat{\phi}_n$ is the optimal policy rule at state n taking as given that other firms obey $(\hat{\phi}_1, \hat{\phi}_2)$ and (VC_1, VC_2) are the continuation values given behavior according to these rules.

Derivation of MPE

- Let $F_2 = F(\hat{\phi}_2)$, the probability an incumbent stays in when there are two firms.

- Then

$$
\begin{aligned}
VC_2 &= (1 - F_2)\, EV_1 + F_2 EV_2 \\
VC_1 &= (1 - \gamma)\, EV_1 + \gamma EV_2
\end{aligned}
$$

- Next

$$
\begin{aligned}
V_n(\phi) &= \pi_n + \beta VC_n, \text{ if } \phi < VC_n \\
&= \pi_n + \beta\phi, \text{ if } \phi \geq VC_n
\end{aligned}
$$

- Observe that given the exponential assumption on ϕ, (has mean 1).

$$E\{\phi|\phi > VC_n\} = \sigma + VC_n.$$

- Hence

$$\begin{aligned} EV_n &= \pi_n + \beta F_n VC_n + \beta(1 - F_n)(\sigma + VC_n) \\ &= \pi_n + \beta VC_n + \beta(1 - F_n)\sigma \end{aligned}$$

- Results in two equations:

$$\begin{aligned} VC_1 &= (1-\gamma)\left[\pi_1 + \beta VC_1 + \beta(1 - F_1)\sigma\right] \\ &\quad +\gamma\left[\pi_2 + \beta VC_2 + \beta(1 - F_2)\sigma\right] \\ VC_2 &= (1-F_2)\left[\pi_1 + \beta VC_1 + \beta(1 - F_1)\sigma\right] \\ &\quad +F_2\left[\pi_2 + \beta VC_2 + \beta(1 - F_2)\sigma\right] \end{aligned}$$

- Rewrite as

$$\begin{aligned} VC_1 &= (1-\gamma)(\pi_1 + \beta(1-F_1)\sigma) \\ &\quad + \gamma(\pi_2 + \beta(1-F_2)\sigma) + (1-\gamma)\beta VC_1 + \gamma\beta VC_2 \\ VC_2 &= (1-F_2)(\pi_1 + \beta(1-F_1)\sigma) \\ &\quad + F_2(\pi_2 + \beta(1-F_2)\sigma) + (1-F_2)\beta VC_1 + F_2\beta VC_2 \end{aligned}$$

or

$$\begin{aligned} & \begin{pmatrix} 1-(1-\gamma)\beta & -\gamma\beta \\ -(1-F_2)\beta & 1-F_2\beta \end{pmatrix} \begin{pmatrix} VC_1 \\ VC_2 \end{pmatrix} \\ = & \begin{pmatrix} (1-\gamma)(\pi_1 + \beta(1-F_1)\sigma) + \gamma(\pi_2 + \beta(1-F_2)\sigma) \\ (1-F_2)(\pi_1 + \beta(1-F_1)\sigma) + F_2(\pi_2 + \beta(1-F_2)\sigma) \end{pmatrix} \end{aligned}$$

or

$$\begin{aligned} & \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} VC_1 \\ VC_2 \end{pmatrix} \\ = & \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \end{aligned}$$

- Use the fact that $\hat{\phi}_i = VC_i$ and substitute out for VC_i above and add the two equations

$$F_i = 1 - e^{-\hat{\phi}_i},\ i = 1, 2 \tag{1}$$

and solve the four equations in four unknowns $\{\hat{\phi}_1, \hat{\phi}_2, F_1, F_2\}$.

Estimation—Overview

- Data: history of industry.

- Suppose know π_0, π_1 and π_2, and β and want to estimate σ.

- Standard "nested fixed point" approach (e.g. Rust). Take a set of parameters, $\theta = (\sigma, \pi_0, \pi_1, \pi_2, \beta)$. Solve for equilibrium. Then write down the likelihood function. Here easy, but usually hard. Pick σ to maximize likelihood. Note need to recalculate equilibirum at every iteration.

- Two-Stage Approach (POB, Hotz-Miller, Bajari-Benkard-Levin.) Stage 1. Use data to estimate reduced-form policy functions.

Use realizations to estimate VC_1 and VC_2. (since can see π_0) and $\hat{F}_1$ and $\hat{F}_2$. Note given knowledge of π_0 see everything that the firm sees. State 2. Now find parameters consistent with these policies.

- No nest. Estimate VC_1 and VC_2 once and for all.

Estimation—Implementation for the Monopoly Case

Assume $\pi_2 = -\infty$ (so $n \leq 1$)

$$V_1(\phi) = \max\left\{\pi_1 + \beta\phi, \pi_1 + \beta VC_1\right\}.$$

Then

$$VC_1 = EV_1$$

but (remember trick from above)

$$\begin{aligned} EV_1 &= \pi_1 + \beta F_1 VC_1 + \beta\left(1 - F_1\right)\left(\sigma + VC_n\right) \\ &= \pi_1 + \beta VC_1 + \beta\left(1 - F_1\right)\sigma \end{aligned}$$

so

$$VC_1 = \pi_1 + \beta VC_1 + \beta (1 - F_1) \sigma$$

or

$$VC_1 = \frac{\pi_1}{1-\beta} + \frac{\beta}{1-\beta} (1 - F_1) \sigma$$

Now solution is obtained by solving

$$\begin{aligned} F_1 &= F(\hat{\phi}_1, \sigma) \\ \hat{\phi}_1 &= VC_1 \end{aligned}$$

- Estimation. Recall parameters
  - π_1, γ (entry), β, and σ where distribution of ϕ is

$$F(\phi, \sigma) = 1 - e^{-\frac{\phi}{\sigma}}.$$

  - Say that π_1, β, and γ are known. Want to estimate σ.

- Nested Fixed point approach
  - Take a given value of σ and solve the dynamic programming problem.
  - Pins down $\hat{\phi}_1(\sigma)$ and $F_1(\hat{\phi}_1(\sigma), \sigma)$. Now take data. Suppose have n periods of data and let n_x be the number of periods where firm exits and n_s be number where firm stays,

$$n = n_x + n_s$$

  - The likelihood is

$$\begin{aligned} L &= k(n_x, n_s) * F_1(\hat{\phi}_1(\sigma), \sigma)^{n_s} * \left[1 - F_1(\hat{\phi}_1(\sigma), \sigma\right]^{n_x} \\ \ln(L) &= n_s \ln F_1(\hat{\phi}_1(\sigma), \sigma) + n_x \ln\left[1 - F_1(\hat{\phi}_1(\sigma), \sigma\right] \end{aligned}$$

- "Simple" Alternative

  – Let

$$\tilde{F}_1 = \frac{n_s}{n}$$

  – For a given value of σ, let

$$V\tilde{C}_1(\sigma) = \frac{\pi_1}{1-\beta} + \frac{\beta}{1-\beta}\left(1 - \tilde{F}_1\right)\sigma$$

  Note there is no nested fixed point here. Directly estimating from observed payoffs (in the nest we know σ)

  – Can use a moment condition

$$V\tilde{C}_1(\sigma) = \hat{\phi}_1$$

$$\tilde{F}_1 = F(V\tilde{C}_1(\sigma), \sigma)$$

– Straightforward to see how this generalizes to the duopoly case from last class. Now as add more to the model, the "simple" alternative gets no more complicated. But the nested fixed point case? Have to solve the fixed point. Have to worry about perhaps multiple equilibria.

But full power of this approach is really with the duopoly case...