Incentives and the Theory of the Firm

- Tradeoff between incentives and insurance (Holmstrom)

- Multi-Tasking: Get What You Pay For (Holmstrom and Milgrom)

- Holdup Problem (Grossman and Hart, Williamson)

- Tradeoff between centralization and decentralization (Alonso, Dessein, Matoushek, 2006)

Classic Moral Hazard

—a: agent effort, $c(a)$ cost of effort

—ε: events beyond agent's control

—$y = a + \varepsilon$ total output, publicly observed

—$w(y)$ compenation scheme

—In classic case, cost of effort additively separable (cost in utils) so agent solves

$$\max_a Eu(w(y)) - c(y)$$

—Here consider effort cost in dollars,

$$\max_a E\left[u(w(y)) - c(y)\right]$$

Suppose

- 1. Restrict attention to linear compensation, $w(y) = s + by$

  2. Assume CARA, $u(x) = -\exp(-rx)$

  3. Suppose ε is $N(0, \sigma^2)$

Agent's problem

$$\max_a -e^{-r(s+ba-c(a)} \int_\varepsilon e^{-rb\varepsilon}\phi(\varepsilon)d\varepsilon$$

So $a^*(b)$ solves $c'(a) = b$.

- Agent's certainty equivalent

$$CE(s,b) = s + ba^*(b) - c(a^*(b)) - \frac{1}{2}rb^2\sigma^2$$

- Principal expected profit

$$E\Pi(s,b) = (1-b)a^*(b) - s$$

- Total Surplus

$$CE(s,b) + E\Pi(s,b) = a^*(b) - c(a^*(b)) - \frac{1}{2}rb^2\sigma^2$$

- Optimal slope b

$$b^* = \frac{1}{1 + r\sigma^2 c''}$$

Linearity?

- In problem described above can do better with some step function contract

$$
\begin{aligned}
w_H, \text{if } y &\geq y_0, \\
w_L, \text{if } y &< y_0
\end{aligned}
$$

  for some $w_L < w_H$ and some y_0

- In general optimal incentive contracts not even monotonic

- Holmstrom and Milgrom rescue linear contracts in reinterpretation.

- In more recent thinking goes beyond tradeoff between incentives and insurance...

You Get What You Pay For

- Suppose

—$y = a + \varepsilon$

—$p = a + \phi$

—$w = s + bp$

—$a = a_1 + a_2$

- Ex 1. $y = a_1 + a_2$, $p = a_1$.
- Ex 2. $y = a_1$, $p = a_1 + a_2$
- Ex 3. $y = a_1$, $p = a_2$

Ex.4

- $y = f_1 a_1 + f_2 a_2 + \varepsilon$

- $p = g_1 a_1 + g_2 a_2 + \phi$

- $w = s + bp$

- $y - w$

- Payoff to risk neutral agent $w - c(a_1, a_2)$ where

$$c(a_1, a_2) = \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2$$

- Timing

1—Contract ($w = s + bp$) determined

2—Agent picks a_1 and a_2

3—ε and ϕ occur

4—Agent paid

- Agent solves

$$\max_{(a_1,a_2)} s + E\left[bg_1a_1 + bg_2a_2 + b\phi\right] - \frac{1}{2}a_1^2 - \frac{1}{2}a_2^2$$

So $a_1^*(b) = g_1b$ and $a_2^*(b) = g_2b$

- Principal expected payoff

$$E(y-w) = f_1 a_1^*(b) + f_2 a_2^*(b) - s - b\left[g_1 a_1^*(b) + g_2 a_2^*(b)\right]$$

- Agent expected payoff

$$E(w) - c(a_1, a_2) = s + b\left[g_1 a_1^*(b) + g_2 a_2^*(b)\right] - \frac{1}{2} a_1^*(b)^2 - \frac{1}{2} a_2^*(b)^2$$

- Total Surplus

$$E[y] - c(a_1, a_2) = f_1 a_1^*(b) + f_2 a_2^*(b) - \frac{1}{2} a_1^*(b)^2 - \frac{1}{2} a_2^*(b)^2$$

- FONC

$$\begin{aligned} f_1 a_1^{*\prime}(b) + f_2 a_2^{*\prime}(b) - a_1^*(b) a_1^{*\prime}(b) - a_2^*(b) a_2^{*\prime}(b) &= 0 \\ f_1 g_1 + f_2 g_2 - g_1^2 b - g_2^2 b &= 0 \end{aligned}$$

So

$$b^* = \frac{f_1 g_1 + f_2 g_2}{g_1^2 + g_2^2}$$

- Scaling and alignment

- Special cases.

Holmstrom and Milgrom

- Multi-tasking and multidimensional contracts

- Observe p_1, p_2,,,p_n

- Payment is $s + b_1 p_2 + b_2 p_2 + ... + b_n p_n$

- Cases where go to corner .

—Extreme case 1: $b_1 = 0$, $b_2 = 0....b_n = 0$ (employee)

—Extreme case 2: $b_1 = 1$, $b_2 = 1....b_n = 1$ (separate firm)

The Holdup Problem

Property Rights and the Nature of the Firm

- Two agents, supplier and buyer and one asset.

- Supplier makes investment x in the asset.

—$f(x)$ is benefit if continue with relationship, $f' > 0$ $f'' < 0$, $f(0) = 0$.

—$f_r(x)$ if walk away and can reuse it with another party $f_r(0) = 0$, $f_r'(x) < f'(x)$, $x > 0$.

—0 if buyer takes asset away.

- Buyer makes investmet y

—$g(y)$ if continue with the relationship

—$g_r(y)$ is walk away and find it new supplier, $g_r(0) = 0$, $g_r'(y) < g'(y)$, $y > 0$

—0 if supplier take asset away.

- Incomplete contracts

1—Sign contract (assign residual rights to control). Agree to lump sum transfer and who gets to walk away with the asset.

2—Supplier and buyer pick x and y

3—Whoever is assigned residual rights of control can exercise this right. Nash Bargaining with parameter α on the supplier.

Case 1—assign supplier residual rights of control

- Stage 3: Outside option

—Supplier has value $f_r(x)$

—Buyer has value 0

—Total value when agree is $f(x) + g(y)$

—Distribution is

$$\begin{aligned} v_S &= f_r(x) + \alpha\left[f(x) + g(y) - f_r(x)\right] \\ v_B &= 0 + (1-\alpha)\left[f(x) + g(y) - f_r(x)\right] \end{aligned}$$

- Stage 2:

—Supplier problem

$$\max_x -x + f_r(x) + \alpha\left[f(x) + g(y) - f_r(x)\right]$$
$$FONC \;:\; \alpha f'(x) + (1-\alpha)\, f_r'(x) - 1 = 0$$

Let x_S^{**} solve above. Let x^* solve $f'(x) = 1$. Note $x_S^{**} < x^*$ if $\alpha < 1$.

—Buyer problem

$$\max_y -y + 0 + (1-\alpha)\left[f(x) + g(y) - f_r(x)\right]$$
$$FONC \;:\; (1-\alpha)g'(y) - 1 = 0$$

Let y_S^{**} solve above. Let y^* solve $g'(y) = 1$, $y_S^{**} < y^*$.

Case 2—assign buyer residual rights of control

- Stage 3: Outside option

—Supplier has value 0

—Buyer has value $g_r(y)$

—Total value when agree is $f(x) + g(y)$

—Distribution is

$$\begin{aligned} v_S &= 0 + \alpha\,[f(x) + g(y) - f_r(x)] \\ v_B &= g_r(y) + (1-\alpha)\,[f(x) + g(y) - f_r(x)] \end{aligned}$$

- Stage 2:

—Supplier problem

$$\begin{aligned} & \max_x -x + \alpha \left[f(x) + g(y) - f_r(x) \right] \\ FONC \quad : \quad & \alpha f'(x) - 1 = 0 \end{aligned}$$

Let x_B^{**} solve above. Note $x_B^{**} < x_S^{**} < x^*$.

—Buyer problem

$$\begin{aligned} & \max_y -y + g_r(y) + (1 - \alpha) \left[f(x) + g(y) - f_r(x) \right] \\ FONC \quad : \quad & \alpha g_r'(y) + (1 - \alpha) g'(y) - 1 = 0 \end{aligned}$$

Let y_B^{**} solve above. $y_S^{**} < y_B^{**} \leq y^*$ ($y_B^{**} < y^*$ if $\alpha > 0$)

Stage 1

- Supplier ownership. Total surplus is

$$\begin{aligned} TS_S &= f(x_S^{**}) + g(y_S^{**}) - x_S^{**} - y_S^{**} \\ &< f(x^*) + g(y^*) - x^* - y^* \end{aligned}$$

- Buyer ownership

$$\begin{aligned} TS_B &= f(x_B^{**}) + g(y_B^{**}) - x_B^{**} - y_B^{**} \\ &< f(x^*) + g(y^*) - x^* - y^* \end{aligned}$$

- Pick ownership structure to solve

$$\max \{TS_S, TS_B\}$$

- Divide ex ante surplus somehow.

Generalization to Multiple Assets

- Suppose A_1, $A_2 ... A_n$

- Can have general functions $f(x, y)$, $g(x, y)$ where $x = (x_1, ..., x_n)$.

- Can specify walkaway returns for various partitions of the assets

- Have vertical integration if one party has all resisual rights of control.

- Williamson vs. Hart and Moore

Williamson Hold-up Model

- Two kinds of individuals, type A and type B.

—N_i measure of type i, $N_A < .5N_B$.

- $t = \{0, 1\}$. $\beta = 1$.

- Each individual has a single labor unit in each period

- Technology 1: Regular

—type j produces q_j per unit of time, $q_A > q_B$

- Technology 2: Special

—period 0, type A builds a factory of quality i with i labor units

—period 0 factor has no output

—period 1: output is $f(i) + q_S$, where $f(0) = 0$, $f'(0) > 0$, and $f''(0) < 0$ when managed with one unit of time (of any type)

—factory must be customized in period 0. If another person manages it, output is q_S instead of $f(i)$

Alonso, Dessein, Matoushek

When Does Coordination Require Centralization?

- Two divisions. Profits depend upon decisions d_1 and d_2 and local conditions θ_1 and θ_2

$$\begin{aligned} \pi_1 &= -(d_1 - \theta_1)^2 - \delta (d_1 - d_2)^2 \\ \pi_2 &= -(d_1 - \theta_2)^2 - \delta (d_1 - d_2)^2 \end{aligned}$$

- Information: Manager 1 sees θ_1. Common knowledge uniform draw from $[-s_1, s_1]$. Analogous for manager 2. θ_1 and θ_2 independent

- Manager 1 maximizes $\lambda\pi_1 + (1 - \lambda)\pi_2$, $\lambda > \frac{1}{2}$. Analogous for manager 2

- Headquarters manager maximizes $\pi_1 + \pi_2$.

Incomplete contracts.

- Centralization. Managers 1 and 2 communicate with HQs, cheap talk message m_1 and $m_2$then HQ manage picks d_1 and d_2 to solve

$$\max_{d_1, d_2} E[\pi_1 + \pi_2 | m_1, m_2]$$

- Decentralization. Managers 1 and 2 communicate with each other. Then manager 1 picks d_1 to solve

$$\max_{d_1} E[\lambda \pi_1 + (1 - \lambda)\pi_2 | \theta_1, m_2]$$

Centralization problem

$$\begin{aligned}
& E[\pi_1 + \pi_2 | m_1, m_2] \\
= \; & E\left[-(d_1 - \theta_1)^2 - (d_2 - \theta_2)^2 - 2\delta (d_1 - d_2)^2\right] \\
= \; & E\left[\begin{array}{c} -d_1^2 - \theta_1^2 + 2d_1\theta_1 - d_2^2 - \theta_2^2 + 2d_2\theta_2 \\ -2\delta d_1^2 - 2\delta d_2^2 + 4\delta d_1 d_2 \end{array}\right] \\
= \; & -d_1^2 - \theta_1^2 + 2d_1 E\theta_1 - d_2^2 - \theta_2^2 + 2d_2 E\theta_2 \\
& -2\delta d_1^2 - 2\delta d_2^2 + 4\delta d_1 d_2
\end{aligned}$$

So rule solves:

$$\begin{aligned}
-2d_1 + 2E\theta_1 - 2\delta d_1 + 4\delta d_2 &= 0 \\
-2d_2 + 2E\theta_2 - 2\delta d_1 + 4\delta d_1 &= 0
\end{aligned}$$