Border Lecture

Anderson and van Wincoop: Gravity with Gravitas: A Solution to the Border Puzzle

- Start with McCallum (1995). Shipment data from Canadian provinces to other provinces and to U.S. states

$$\begin{aligned} \ln x_{ij} &= \alpha_1 + \alpha_2 \ln h_i + \alpha_3 \ln y_j \\ &\quad + \alpha_4 \ln d_{ij} + a_5 \delta_{ij} + \varepsilon_{ij} \end{aligned}$$

where

$$\begin{aligned} x_{ij} &: \text{ exports from region } i \text{ to } j \\ y_i \text{ and } y_j &: \text{ gross domestic production at } i \text{ and } j \\ d_{ij} &: \text{ distance } i \text{ to } j \\ \delta_{ij} &: \text{ dummy}=1\text{, province/province, }=0\text{, state/province} \end{aligned}$$

- McCallum adds atheoretic “remoteness” variable (that will try to capture ideas in this paper)

$$REM_i = \sum_{m \neq j} \frac{d_{im}}{y_m}$$

- Results
  - Notice large border coefficient for Canada. 16.4! (similar to McCallum of 22)
  - Notice that get something very different when do same exercise with US orginations
  - Notice the slick why the paper is transitioned into unitary income elasticity.
  - Notice adding REM doesn't change anything.

Table 1—McCallum Regressions

Data	McCallum regressions			Unitary income elasticities		
	(i) CA–CA CA–US	(ii) US–US CA–US	(iii) US–US CA–CA CA–US	(iv) CA–CA CA–US	(v) US–US CA–US	(vi) US–US CA–CA CA–US
Independent variable						
ln y_i	1.22	1.13	1.13	1	1	1
	(0.04)	(0.03)	(0.03)			
ln y_j	0.98	0.98	0.97	1	1	1
	(0.03)	(0.02)	(0.02)			
ln d_{ij}	−1.35	−1.08	−1.11	−1.35	−1.09	−1.12
	(0.07)	(0.04)	(0.04)	(0.07)	(0.04)	(0.03)
Dummy–Canada	2.80		2.75	2.63		2.66
	(0.12)		(0.12)	(0.11)		(0.12)
Dummy–U.S.		0.41	0.40		0.49	0.48
		(0.05)	(0.05)		(0.06)	(0.06)
Border–Canada	16.4		15.7	13.8		14.2
	(2.0)		(1.9)	(1.6)		(1.6)
Border–U.S.		1.50	1.49		1.63	1.62
		(0.08)	(0.08)		(0.09)	(0.09)
$\bar{R}^2$	0.76	0.85	0.85	0.53	0.47	0.55
Remoteness variables added						
Border–Canada	16.3		15.6	14.7		15.0
	(2.0)		(1.9)	(1.7)		(1.8)
Border–U.S.		1.38	1.38		1.42	1.42
		(0.07)	(0.07)		(0.08)	(0.08)
$\bar{R}^2$	0.77	0.86	0.86	0.55	0.50	0.57

Figure 1:

Model (Armington Model from Last Class)

- Utility of j is CES over location i goods,

$$U_j = \left(\sum_i \beta_i^{(1-\sigma)/\sigma} c_{ij}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

  maximize subject to

$$\sum_i p_{ij} c_{ij} = y_j$$

- Endowment of good i at location iTrade costs $p_{ij} = p_i t_{ij}$. Total income

$$y_i = \sum_j x_{ij}.$$

- To get total revenues from sales from location i to j, of course

$$Q_j = \frac{y_j}{P_j}$$

and demand per unit from i at j is $\beta_{ij}\left(\beta_{ij}p_{ij}\right)^{-\sigma}P_j^{\sigma}$, so spending by j at i is

$$\begin{aligned} x_{ij} &= p_{ij}\beta_{ij}\left(\beta_{ij}p_{ij}\right)^{-\sigma}P_j^{\sigma}\frac{y_j}{P_j} \\ &= \left(\frac{\beta_i p_i t_{ij}}{P_j}\right)^{(1-\sigma)} y_j \end{aligned}$$

where we use $p_{ij} = p_i t_{ij}$. And as usual,

$$P_j = \left[\sum_i \left(\beta_i p_i t_{ij}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

- General equilibrium

$$
\begin{aligned}
y_i &= \sum_j x_{ij} \\
&= \sum_j \left(\frac{\beta_i p_i t_{ij}}{P_j} \right)^{(1-\sigma)} y_j \\
&= (\beta_i p_i)^{1-\sigma} \sum_j \left(\frac{t_{ij}}{P_j} \right)^{(1-\sigma)} y_j
\end{aligned}
$$

So solve out for $\beta_i p_i$

$$
\left(\frac{y_i}{\sum_j \left(\frac{t_{ij}}{P_j} \right)^{(1-\sigma)} y_j} \right)^{\frac{1}{1-\sigma}} = \beta_i p_i
$$

Then get the **gravity equation**

$$
\begin{aligned}
x_{ij} &= \left(\frac{\beta_i p_i t_{ij}}{P_j}\right)^{(1-\sigma)} y_j = (\beta_i p_i)^{1-\sigma} \left(\frac{t_{ij}}{P_j}\right)^{(1-\sigma)} y_j \\
&= \frac{y_i}{\sum_j \left(\frac{t_{ij}}{P_j}\right)^{(1-\sigma)} y_j} \left(\frac{t_{ij}}{P_j}\right)^{(1-\sigma)} y_j \\
&= \frac{y_i y_j}{y^W} \left(\frac{t_{ij}}{\left[\sum_j \left(\frac{t_{ij}}{P_j}\right)^{(1-\sigma)} \frac{y_j}{y^W}\right]^{\frac{1}{1-\sigma}} P_j}\right)^{(1-\sigma)} \\
&= \frac{y_i y_j}{y^W} \left(\frac{t_{ij}}{\Pi_i P_j}\right)^{(1-\sigma)}
\end{aligned}
$$

for

$$\Pi_i \equiv \left(\sum_j \left(\frac{t_{ij}}{P_j} \right)^{1-\sigma} \theta_j \right)^{1/(1-\sigma)}$$

We can substitute into the equilibrium scaled prices to get

$$\begin{aligned} P_j &= \left[\sum_i \left(\beta_i p_i t_{ij}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ &= \left[\sum_i (\beta_i p_i)^{1-\sigma} \left(t_{ij}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ &= \left[\sum_i \frac{y_i}{\sum_j \left(\frac{t_{ij}}{P_j}\right)^{(1-\sigma)} y_j} \left(t_{ij}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ &= \left[\sum_i \frac{\theta_i}{\sum_j \left(\frac{t_{ij}}{P_j}\right)^{(1-\sigma)} \theta_j} \left(t_{ij}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ & \left[\sum_i \theta_i \left(\frac{t_{ij}}{\Pi_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \end{aligned}$$

So now have:

$$\Pi_i \equiv \left(\sum_j \left(\frac{t_{ij}}{P_j} \right)^{1-\sigma} \theta_j \right)^{1/(1-\sigma)}$$

$$P_j = \left[\sum_i \theta_i \left(\frac{t_{ij}}{\Pi_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

- Can solve for Π_i and P_i in terms of income shares θ_i and $\left\{ t_{ij} \right\}$ and σ.

- Suppose symmetry: $t_{ij} = t_{ji}$. Then get a solution where

$P_j = \Pi_i$,

$$\begin{aligned} P_j^{1-\sigma} &= \sum_i \theta_i \left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} \\ &= \sum_i P_i^{\sigma-1}\theta_i t_{ij}^{1-\sigma} \end{aligned}$$

Now the gravity equation can be written

$$\begin{aligned} x_{ij} &= \frac{y_i y_j}{y^W}\left(\frac{t_{ij}}{\Pi_i P_j}\right)^{(1-\sigma)} \\ &= \frac{y_i y_j}{y^W}\left(\frac{t_{ij}}{P_i P_j}\right)^{(1-\sigma)} \end{aligned}$$

- Call $\{P_i\}$ multilateral resitance, as they depend upon bilateral resitance t_{ij}.

- A rise in trade barriers with all trading partners will raise the index.

- If $t_{ij} = 1$, then $P_i = 1$

- Intuition for why trade depends on multilateral resitance of both the importer j and the exporter i.

- What happens then t_{ij} all increase proportionately (including t_{ii}). Homeogeneous of degree zero, while P_i are homogeneous of degree $1/2$.

Implications

- Implication 1: Trade Barriers reduce size adjusted trade between the large countries more than between small countries

On one basis can such a claim be made? Paper considers the following. Start at $t_{ij} = 1$. Then set $dt_{ij} = dt$, $i \neq j$, and $dt_{ii} = 0$. Start with

$$P_j^{1-\sigma} = \sum_i P_i^{\sigma-1} \theta_i t_{ij}^{1-\sigma}$$

Totally differentiate

$$\begin{aligned}(1-\sigma) P_j^{-\sigma} dP_j &= \sum_i (\sigma-1) P_i^{\sigma} dP_i \theta_i t_{ij}^{1-\sigma} \\ &+ \sum_i P_i^{\sigma-1} d\theta_i t_{ij}^{1-\sigma} \\ &+ (1-\sigma) \sum_i P_i^{\sigma-1} \theta_i t_{ij}^{-\sigma} dt_{ij}\end{aligned}$$

set $P_j = 1 = t_{ij}$,

$$(1-\sigma) dP_j = \sum_i (\sigma-1) dP_i \theta_i + \sum_i d\theta_i + (1-\sigma) \sum_i \theta_i dt_{ij}$$

or

$$dP_j = -\sum_i dP_i \theta_i + \sum_i \theta_i dt_{ij}$$

Multiply by θ_j gives

$$\theta_j dP_j = -\theta_j \sum_i dP_i \theta_i + \sum_i \theta_j \theta_i dt_{ij}$$

Sum over j?

$$\sum_j \theta_j dP_j = -\sum_j \theta_j \sum_i dP_i \theta_i + \sum_j \sum_i \theta_j \theta_i dt_{ij}$$

using $dt_{ij} = 1$, for $i \neq j$ and $dt_{ii} = 0$, somehow gets to

$$dP_i = \left(\frac{1}{2} - \theta_i + \frac{1}{2} \sum_k \theta_k^2 \right) dt$$

Form this conclude that a uniform increase in trade barriers raises multilaterial resistance more for a small country than a large country.

Next

$$d\left(x_{ij} \frac{y^W}{y_i y_j} \right) = -(\sigma - 1) \left[\theta_i + \theta_j - \sum_k \theta_k^2 \right] dt$$

and this gets us implication 1.

- skip some other stuff

- get to a comparison of theoretical gravity equation and McCallum

- Assume this specification:

$$t_{ij} = b_{ij} d_{ij}^{\rho}$$

$$\begin{aligned} \ln x_{ij} &= k + \ln y_i + \ln y_j + (1-\sigma)\,\rho \ln d_{ij} \\ &\quad + (1-\sigma)\ln b_{ij} - (1-\sigma)\ln P_i \\ &\quad - (1-\sigma)\ln P_j \end{aligned}$$

- Now how estimate? Given model parameters $t_{ij} = b_{ij} d_{ij}^{\rho}$ and data θ_i

$$\begin{aligned} P_j^{1-\sigma} &= \sum_i P_i^{\sigma-1}\theta_i t_{ij}^{1-\sigma} \\ &= \sum_i P_i^{\sigma-1}\theta_i b_{ij}^{(1-\sigma)} d_{ij}^{\rho(1-\sigma)} \\ &= \sum_i P_i^{\sigma-1}\theta_i e^{a_1 \ln d_{ij} + a_2(1-\delta_{ij})} \end{aligned}$$

for

$$b_{ij} = b^{1-\delta_{ij}}$$

where $\delta_{ij} = 1$ if in same country

$$\begin{aligned} a_1 &= (1-\sigma)\rho \\ a_2 &= (1-\sigma)\ln b \end{aligned}$$

So have nonlinear equations to be solved in $P_i^{\sigma-1}$. So solve equation for this (note

- Note allow locations to have own internal distance $d_{ii} > 1$.

- Get it down to

$$\begin{aligned}\ln z_{ij} &= \ln\left(\frac{x_{ij}}{y_i y_j}\right) = k + a_1 \ln d_{ij} + a_2(1-\delta_{ij}) \\ &\quad - \ln P_i^{1-\sigma} - \ln P_j^{1-\sigma} + \varepsilon_{ij}\end{aligned}$$

  take ε_{ij} to be measurement error. Take as given $\sigma = 5$, then given k, a_1, a_2, have nonlinear least squares

$$\ln z = h(d, \delta, \theta; k, a_1, a_2) + \varepsilon$$

- Estimates

TABLE 2—ESTIMATION RESULTS

		Two-country model	Multicountry model
Parameters	$(1-\sigma)\rho$	−0.79	−0.82
		(0.03)	(0.03)
	$(1-\sigma)\ln b_{US,CA}$	−1.65	−1.59
		(0.08)	(0.08)
	$(1-\sigma)\ln b_{US,ROW}$		−1.68
			(0.07)
	$(1-\sigma)\ln b_{CA,ROW}$		−2.31
			(0.08)
	$(1-\sigma)\ln b_{ROW,ROW}$		−1.66
			(0.06)
Average error terms:	US–US	0.06	0.06
	CA–CA	−0.17	−0.02
	US–CA	−0.05	−0.04

Figure 2:

TABLE 3—AVERAGE OF $P^{1-\sigma}$

	US	Canada
Two-country model		
With border barrier (BB)	0.77	2.45
	(0.03)	(0.12)
Borderless trade (NB)	0.75	1.18
	(0.03)	(0.01)
Ratio (BB/NB)	1.02	2.08
	(0.00)	(0.08)

Figure 3:

- Estimates: Resulting average of $P^{1-\sigma}$