Wages

Helpman, Itskhoki, and Redding

- In the end, very interested in how trade impacts the distribution of the pie.

- Naturally, can see an angle here where the literature on firm heterogeneity gets linked up to worker heterogeneity.

- Natually, can see that can start plugging in models where different types of firms tend to demand different types of workers, and we are off to the races.

- This paper mixes a bunch of different models into the soup, making some clever assumptions along the way as is often typical of a paper with Helpman in it.

- Perhaps the stucture is useful....

Model

- Usual CES

$$Q = \left[\int_{j \in J} q(j)^{\beta} dj \right]^{1/\beta}, \ 0 < \beta < 1$$

as usual,

$$\begin{aligned} q(j) &= A^{\frac{1}{1-\beta}} p(j)^{-1/(1-\beta)}, \text{ for } A = E^{1-\beta} P^{\beta} \\ & \quad E, \text{ expenditure, } P \text{ price index} \end{aligned}$$

for today, useful to invert and get price

$$p(j) = A q(j)^{-(1-\beta)}$$

and then revenue

$$r(j) = p(j) q(j) = A q(j)^{\beta}$$

For imports, if set price equal to $p(j)\tau$, then letting $q^*(j)$ be the quantity shipped,

$$
\begin{aligned}
q^*(j) &= \tau\left[A^{*\frac{1}{1-\beta}} p(j)^{-1/(1-\beta)} \tau^{-1/(1-\beta)}\right] \\
p^*(j) &= \tau^{1-\beta} A^* q^*(j)^{-(1-\beta)} \tau^{-1} \\
&= \tau^{-\beta} A^* q^*(j)^{-(1-\beta)} \\
r^*(j) &= p^*(j) q^*(j) = \tau^{-\beta} A^* q^*(j)^{\beta}
\end{aligned}
$$

- Usual Melitz stuff
  - Pay sunk cost f_e, production involves a fixed cost f_d, exporting involves a fixed cost of f_x (and iceberg cost factor $\tau > 1$)
  - Observe θ drawn from $G(\theta) = 1 - (\theta_{\text{min}}/\theta)^z$, for $\theta \geq \theta_{\text{min}} > 0$, and shape parameter $z > 1$.

- As aside, the right tail is $(\theta_{\text{min}}/\theta)^z$, so z is the Zipf coefficient measuring firm size by θ, i.e., ln of right tail is $z \ln \theta_{\text{min}} - z \ln \theta$, so slope is θ. The closer z to one, the fatter the tail.

- Worker Ability
  - Pareto, $G_a(a) = 1 - (a_{\text{min}}/a)^k$, for $a \geq a_{\text{min}} > 1$

- Some diminishing returns (span of control issue)
  - $y = \theta h^\gamma \bar{a}$, $0 < \gamma < 1$
  - for h measure of workers hired
  - $\bar{a}$ average ability

- Labor Market Search and Matching Frictions
  - Diamond, Mortenson, Pissaredes approach (prescient to include Diamand and anticipate the Nobel Prize!)
  - A firm that pays a search cost of bn units of numeraire can randomly match with a measure n workers.
  - Search cost b is endogenous as explained below.
  - worker ability cannot be costless observed when workers and firms are matched.
  - pay a screening cost of ca_c^{δ}/δ units of numeraire, a firm can identify workers with an ability below a_c.
  - structure will generate the result that more productive firms will employ more workers, screen more, pay higher wages.

– bargaining occurs under conditions of symmetric information, because workers don't know any more about their ability than firms.

Working it Out

- Suppose a firm chooses a screening threshold a_c and matches with a measure n workers.
  - The number of hires is $h = (a_{\text{min}}/a_c)^k \, n$
  - The average quality? The mean of Pareto with scale parameter a_c and shape parameter k is

$$\bar{a} = \frac{k}{k-1} a_c$$

  - So can rewrite production technology as

$$\begin{aligned} y &= \theta h^{\gamma} \bar{a}, \\ &= \theta \left((a_{\text{min}}/a_c)^k \, n \right)^{\gamma} \frac{k}{k-1} a_c \\ &= \kappa_y \theta n^{\gamma} a_c^{1-k\gamma} \end{aligned}$$

- Assume that $k\gamma < 1$ (or $\gamma < \frac{1}{k}$), the tail of the ability distribution is big and there is a lot of curvature (so really want assortative matching)

- Otherwise don't sort. (Assuming opportunity cost of the labor is zero)

- With the curvature in costs, we lose the property that the output decisions in each market are independent. Going to allocate output between the two markets to equate marginal revenue. Recall

$$\begin{aligned} r(j) &= Aq(j)^{\beta} \\ r^{*}(j) &= \tau^{-\beta} A^{*} q^{*}(j)^{\beta} \end{aligned}$$

Need to allocate $y(\theta)$ between $y_d(\theta)$ and $y_x(\theta)$. We require

$$\begin{aligned}
\beta A y_d^{\beta-1} &= \beta \tau^{-\beta} A^* y_x^{\beta-1} \\
\left(\frac{y_x}{y_d}\right)^{1-\beta} &= \tau^{-\beta}\frac{A^*}{A} \\
\frac{y_x}{y_d} &= \tau^{-\frac{\beta}{1-\beta}}\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}
\end{aligned}$$

$$\begin{aligned}
y &= y_d + y_x \\
&= y_d + y_d \tau^{-\frac{\beta}{1-\beta}}\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \\
y_d &= \frac{1}{1+\tau^{-\frac{\beta}{1-\beta}}\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}} y \\
y_x &= \frac{\tau^{-\frac{\beta}{1-\beta}}\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}}{1+\tau^{-\frac{\beta}{1-\beta}}\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}} y
\end{aligned}$$

- So total revenue is

$$
\begin{aligned}
r &= r_d + r_x = A{y_d}^{\beta} + \tau^{-\beta} A^* {y_x}^{\beta} \\
&= A\left[\frac{1}{1+\tau^{-\frac{\beta}{1-\beta}}\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}} y\right]^{\beta} \\
&\quad + A\tau^{-\beta}\frac{A^*}{A}\left[\frac{\tau^{-\frac{\beta}{1-\beta}}\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}}{1+\tau^{-\frac{\beta}{1-\beta}}\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}} y\right]^{\beta} \\
&= \Upsilon(\theta)^{1-\beta} A y(\theta)^{\beta}
\end{aligned}
$$

enough (not getting it perfectly here...)

$$
\Upsilon(\theta) = 1 + \tau^{-\beta/(1-\beta)}\left(\frac{A^*}{A}\right)^{1/(1-\beta)}
$$

And if don't export then set $\Upsilon(\theta) = 1$. Since if export $\Upsilon(\theta) > 0$, can see that the gains from exporting captured in a clean way.

- Firm revenue is using $y = \theta h^\gamma \bar{a}$

$$\begin{aligned} r &= \Upsilon(\theta)^{1-\beta} A y(\theta)^\beta \\ &= \Upsilon(\theta)^{1-\beta} A \theta^\beta h^{\beta\gamma} \bar{a}^\beta \end{aligned}$$

- Suppose bargain with all workers as a union, assume firm's nash bargaining share is $\frac{1}{1+\beta\gamma}$. Then know how to work this out.

- Instead bargain individually with each worker who has ability $\bar{a}$, internalize impact on wages of rest if lose a guy, get these shares (Stole and Zwiebel). I don't know this paper, looks like something cool to have figured out. In any case, can see what the firm gets

$$
\begin{aligned}
\pi(\theta) &= \max_{\substack{n \geq 0 \\ a_c \geq a_{\min} \\ I_x \in \{0,1\}}} \left\{ \frac{1}{1+\beta\gamma} \Upsilon(\theta)^{1-\beta} A y(\theta)^{\beta} - bn - \frac{c}{\delta} a_c^{\delta} - f_d - I_x f_x \right\} \\
&= \max_{\substack{n \geq 0 \\ a_c \geq a_{\min} \\ I_x \in \{0,1\}}} \left\{ \frac{1}{1+\beta\gamma} \Upsilon(\theta)^{1-\beta} A \left[\kappa_y \theta n^{\gamma} a_c^{1-k\gamma} \right]^{\beta} - bn - \frac{c}{\delta} a_c^{\delta} - f_d - I_x f_x
\end{aligned}
$$

- Usual cutoff rule property with θ (for export and staying in business

- FONC for n and a_c,

$$
\begin{aligned}
\frac{\beta\gamma}{1+\beta\gamma} r(\theta) \frac{1}{n(\theta)} &= b \\
\frac{\beta\gamma}{1+\beta\gamma} r(\theta) &= bn(\theta)
\end{aligned}
$$

$$\begin{aligned}\frac{1}{1+\beta\gamma}\left(1-k\gamma\right)\beta\frac{r}{a_c} &= ca_c^{\delta-1} \\ \frac{1}{1+\beta\gamma}\left(1-k\gamma\right)\beta r &= ca_c^{\delta}\end{aligned}$$

- Results:

  – Firms with larger revenue sample more workers

  – Measure of hires $h = n\left(\frac{a_{\text{min}}}{a_c}\right)^k$ increasing in n, decreasing in a_c. Look at

$$\begin{aligned}\frac{\gamma}{(1-k\gamma)}\frac{1}{1+\beta\gamma}\left(1-k\gamma\right)\beta r &= \frac{\gamma}{(1-k\gamma)}ca_c^{\delta} \\ \frac{\beta\gamma}{1+\beta\gamma}r &= \frac{\gamma}{(1-k\gamma)}ca_c^{\delta}\end{aligned}$$

  – Under assumption $\delta > k$, firms not only sample more workers, but also hire more workers (where it this, don't quite see it.)

– From division of revenue, total wage bill is a constant share of revenue, so

$$w = \frac{\beta\gamma}{1+\beta\gamma}\frac{r}{h} = b\frac{n}{h} = b\frac{n}{n\left(\frac{a_{\text{min}}}{a_c}\right)^k} = b\left(\frac{a_c}{a_{\text{min}}}\right)^k$$

so higher θ firms pay higher wages.

– Expected wage conditional on being sample is

$$\frac{w(\theta)h(\theta)}{n(\theta)} = b$$

so firms have no incentive to direct their search

– Now take $h = n\,(a_{\text{min}}/a_C)^k$, or $(a_C/a_{\text{min}})^k = n/h$

$$\begin{aligned} w &= b\left(\frac{a_C}{a_{\text{min}}}\right)^k \\ &= b\frac{n}{h} \end{aligned}$$

not getting this

$$\ln w(\theta) = \text{constant} + \frac{k}{\delta - k}\ln h(\theta)$$

so $\delta > k$, get employer size wage effect.

Labor Market Tightness

- Search cost b

$$b = \alpha_0 x^{\alpha_1}$$
$$x = \frac{N}{L}$$

where x is probability of being sampled (market tightness),

- Outside option for worker has value ω

- Expected wage conditional on being sampled $w(\theta)\, h(\theta)/n(\theta) = b$

- So

$$\begin{aligned} \omega &= xb \\ &= x\alpha_0 x^{\alpha_1} \\ &= \alpha_0 x^{1+\alpha_1} \end{aligned}$$

so

$$\begin{aligned} x &= \left(\frac{\omega}{\alpha_0}\right)^{\frac{1}{1+\alpha_1}} \\ b &= \alpha_0^{1/(1+\alpha_1)}\omega^{\alpha_1/(1+\alpha_1)} \end{aligned}$$

- Can fix ω and x and plug away, cutffs θ_d and θ_x

- Two measures of openness, first $\rho \equiv \frac{\theta_d}{\theta_x}$, determines ratio of exporting firms to surviving firms

$$\frac{1 - G_\theta(\theta_x)}{1 - G_\theta(\theta_d)} = \rho^z$$

$$(17) \quad r(\theta) = \Upsilon(\theta)^{(1-\beta)/\Gamma} \cdot r_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\beta/\Gamma}, \quad r_d \equiv \frac{1+\beta\gamma}{\Gamma} f_d,$$

$$h(\theta) = \Upsilon(\theta)^{(1-\beta)(1-k/\delta)/\Gamma} \cdot h_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\beta(1-k/\delta)/\Gamma},$$

$$h_d \equiv \frac{\beta\gamma}{\Gamma} \frac{f_d}{b} \left[\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{c a_{\min}^{\delta}}\right]^{-k/\delta},$$

$$w(\theta) = \Upsilon(\theta)^{k(1-\beta)/(\delta\Gamma)} \cdot w_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\beta k/(\delta\Gamma)},$$

$$w_d \equiv b \left[\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{c a_{\min}^{\delta}}\right]^{k/\delta},$$

Figure 1:

- Sector Wage Inequality

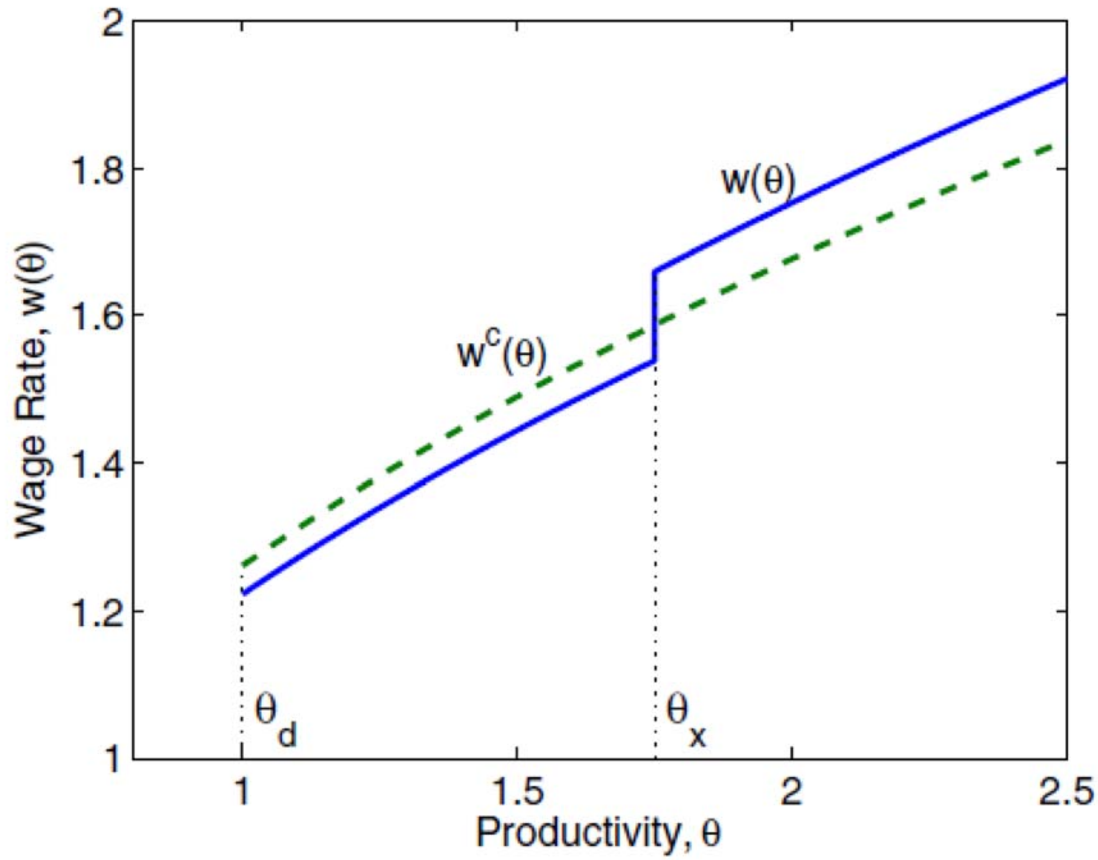

FIGURE 1.—Wages as a function of firm productivity.

Figure 2:

$$
(18) \qquad G_w(w) = \begin{cases} S_{h,d} G_{w,d}(w), \\ \qquad \text{for } w_d \leq w \leq w_d / \rho^{\beta k/(\delta \Gamma)}, \\ S_{h,d}, \quad \text{for } w_d / \rho^{\beta k/(\delta \Gamma)} \leq w \leq w_d {Y_x}^{k(1-\beta)/(\delta \Gamma)} / \rho^{\beta k/(\delta \Gamma)}, \\ S_{h,d} + (1 - S_{h,d}) G_{w,x}(w), \\ \qquad \text{for } w \geq w_d {Y_x}^{k(1-\beta)/(\delta \Gamma)} / \rho^{\beta k/(\delta \Gamma)}, \end{cases}
$$

Figure 3:

- Next compare with open economy, create counter factual distribution $G_w^c(w)$ that is Pareto with same shape parameter as closed economy, but same mean as open economy.

$$S_{h,d} = \frac{1 - \rho^{z-\beta(1-k/\delta)/\Gamma}}{1 + \rho^{z-\beta(1-k/\delta)/\Gamma}[\Upsilon_x^{(1-\beta)(1-k/\delta)/\Gamma} - 1]},$$

which depends on the extensive and intensive margins of trade openness.

The distributions of wages across workers employed by domestic and exporting firms can also be derived from the solutions for firm-specific variables (17). Given that productivity is Pareto distributed, and both wages and employment are power functions of productivity, the distribution of wages across workers employed by domestic firms is a truncated Pareto distribution

$$G_{w,d}(w) = \frac{1 - \left(\dfrac{w_d}{w}\right)^{1+1/\mu}}{1 - \rho^{z-\beta(1-k/\delta)/\Gamma}} \quad \text{for} \quad w_d \le w \le w_d/\rho^{k\beta/(\delta\Gamma)}.$$

Similarly, the distribution of wages across workers employed by exporters, $G_{w,x}(w)$, is an untruncated Pareto distribution

$$G_{w,x}(w) = 1 - \left[\frac{w_d}{w}\Upsilon_x^{k(1-\beta)/(\delta\Gamma)}\rho^{-k\beta/(\delta\Gamma)}\right]^{1+1/\mu}$$

$$\text{for} \quad w \ge w_d \Upsilon_x^{k(1-\beta)/(\delta\Gamma)}/\rho^{k\beta/(\delta\Gamma)}.$$

The wage distributions for workers employed by domestic firms and by exporters have the same shape parameter, $1 + 1/\mu$, where μ is defined as

$$\mu \equiv \frac{\beta k/\delta}{z\Gamma - \beta}, \quad \text{where} \quad \Gamma \equiv 1 - \beta\gamma - \frac{\beta}{\delta}(1 - \gamma k).$$

Figure 4:

PROPOSITION 1: *In the closed economy,* μ *is a sufficient statistic for sectoral wage inequality. In particular,* (i) *the coefficient of variation of wages is* $\mu/\sqrt{1-\mu^2}$; (ii) *the Lorenz curve is represented by* $s_w = 1-(1-s_h)^{1/(1+\mu)}$, *where* s_h *is the fraction of workers and* s_w *is the fraction of their wages when workers are ordered from low- to high-wage earners*; (iii) *the Gini coefficient is* $\mu/(2+\mu)$; *and* (iv) *the Theil index is* $\mu - \ln(1+\mu)$.

Figure 5:

PROPOSITION 2: *In the closed economy, inequality in the sectoral distribution of wages is increasing in firm productivity dispersion (lower* z*) and increasing in worker ability dispersion (lower* k*) if and only if* $z^{-1}+\delta^{-1}+\gamma > \beta^{-1}$.

Since more productive firms pay higher wages, greater dispersion in firm productivity (lower z) implies greater sectoral wage inequality. In contrast, greater dispersion in worker ability (lower k) has an ambiguous effect on sectoral wage inequality because of two counteracting forces. On the one hand, a reduction in k increases relative employment in more productive firms (from (17)) that pay higher wages, which increases wage inequality. On the other hand, a reduction in k decreases relative wages paid by more productive firms (from (17)), which reduces wage inequality. When the parameter inequality in the proposition is satisfied, the change in relative employment dominates the change in relative wages, and greater dispersion in worker ability implies greater sectoral wage inequality.

Figure 6:

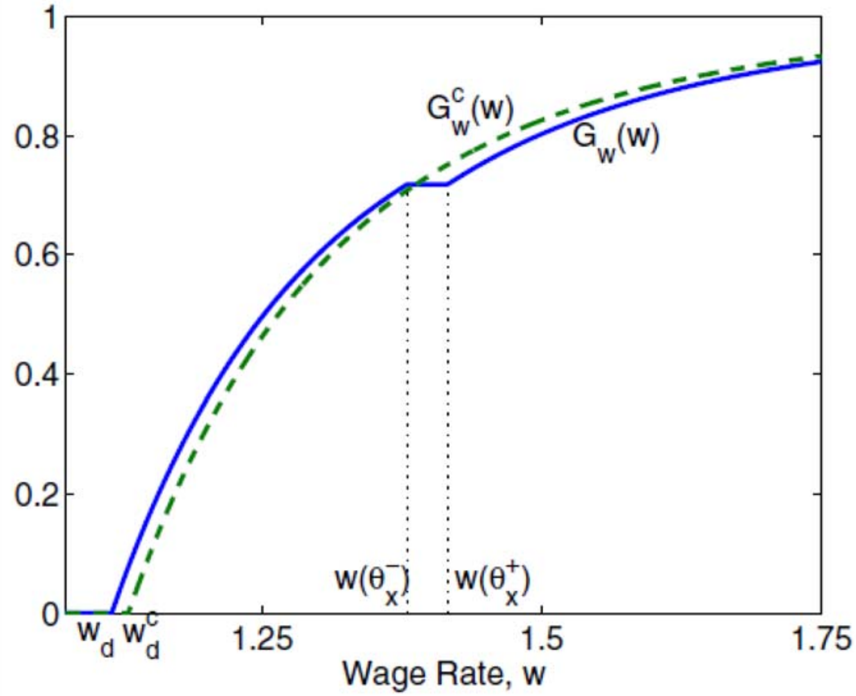

FIGURE 2.—Cumulative distribution function of wages.

Figure 7:

Proposition 3: (i) *Sectoral wage inequality in the open economy when some but not all firms export is strictly greater than in the closed economy and* (ii) *sectoral wage inequality in the open economy when all firms export is the same as in the closed economy.*

Figure 8:

Corollary to Proposition 3: *An increase in the fraction of exporting firms raises sectoral wage inequality when the fraction of exporting firms is sufficiently small and reduces sectoral wage inequality when the fraction of exporting firms is sufficiently large.*

Figure 9:

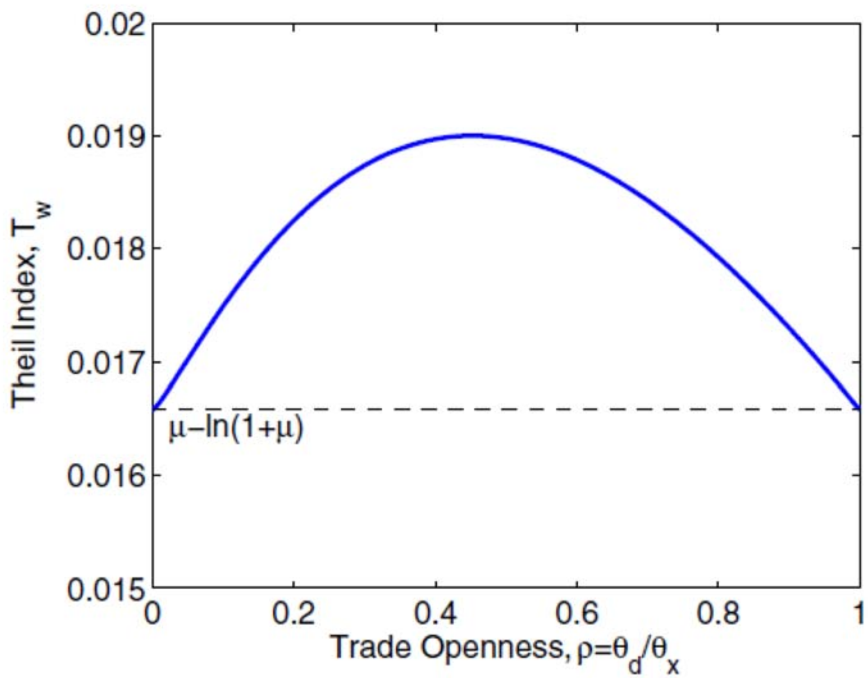

FIGURE 3.—Trade openness and sectoral wage inequality.

Figure 10:

[The Theil index can be interpreted] as the expected information content of the indirect message which transforms the population shares as prior probabilities into the income shares as posterior probabilities.

Henri Theil (1967:125-126)

But the fact remains that [the Theil index] is an arbitrary formula, and the average of the logarithms of the reciprocals of income shares weighted by income is not a measure that is exactly overflowing with intuitive sense.

Amartya Sen (1997:36)

Figure 11: