Econ 8601—Fall 2016—Take-Home Final

Please work alone. You can choose when to work on it, but please work only one day on it.

Question 1. Perfect Price Discrimination. Consider the Logit Model of Product Differentiation from homework 1. There are n firms indexed by j and an "outside good" labeled by 0. Each firm $j \geq 1$ has constant marginal cost equal to c_j. There is a unit measure consumers. Let i index an individual consumer and suppose the utility of consumer i from purchasing good j and paying price $p_{i,j}$ has utility

$$\begin{aligned} U_{i,j} &= \xi_j - \alpha p_{i,j} + \varepsilon_{i,j} \text{ for } j = 1, 2, ...n \\ &= \varepsilon_{i,0} \text{ for good 0.} \end{aligned}$$

A consumer is therefore summarized by his or her vector of draws $\boldsymbol{\varepsilon}_i = (\varepsilon_{i,0}, \varepsilon_{i,1}, ...\varepsilon_{i,n})$ since consumers are otherwise the same. Assume the $\varepsilon_{i,j}$ are drawn type 1 extreme value which delivers the logit choice probabilities.

We allow for perfect price discrimination. Each firm observes the entire vector of draws $\boldsymbol{\varepsilon}_i$ for each consumer i and can set prices contingent on $\boldsymbol{\varepsilon}_i$ (i.e. can set a price $p_{i,j}$ specific to individual i.). The n firms compete in a Bertrand fashion for each individual consumer.

(a) Take as given the n firms in the industry and calculate the equilibrium of price competition when perfect price discrimination is feasible. Derive formulas for the market shares of each firm.

(b) A potential entrant is considering entry into this industry. If it comes in, it will be firm $j = n + 1$, and consumers will all get a new logit draw $\varepsilon_{i,n+1}$ for this firm. The firm has given values of ξ_{n+1} and c_{n+1}. If the firm enters, it pays a fixed cost ϕ_{n+1}. Derive a condition determining whether the firm enters. Discuss the connection between the private incentive for entry and the social incentive. That is, how does the total of producers and consumers surplus change if the firm enters?

Question 2

Consider the following competitive industry with three stages. In stage 1, a measure of firms N enters the industry. Each entering firm pays a fixed cost ϕ and draws a productivity parameter $\theta \in [0, \overline{\theta}]$ from a continuous distribution $F(\theta)$ with positive density $f(\theta)$ on the support $[0, \overline{\theta}]$. There is an unlimited supply of potential entrants to this industry, all *ex ante* identical.

In stage 2, after observing its own initial productivity draw θ°, each firm chooses whether to pay $\kappa > 0$ dollars to increase productivity to $\theta = \lambda\theta^{\circ}$, where $\lambda > 1$, or to not incur this cost in which case productivity remains at the initial level θ°.

In stage 3, there is a completive market for the industry. The production function of a firm with productivity θ is $q = \theta g(x)$, where x is the amount of labor employed by the firm and q is the output and $g(x)$ increasing and strictly concave. The demand curve for the industry is $Q^D = D(p)$, where p is the industry price. Assume that $D(p) > 0$, $D'(p) < 0$ for all p, and $\lim_{p \to \infty} D(p) = 0$. Assume the supply of labor to this industry is perfectly elastic at wage w. Let y denote the fraction of firms entering in stage 1 that invest to increase productivity in stage 2.

(a) Assume the cost κ is small enough that in equilibrium, the fraction y investing to increase productivity is strictly positive. Characterize the competitive equilibrium in this economy.

(b) Suppose demand doubles to $\tilde{D}(p) = 2D(p)$. How does this change the competitive equilibrium?

(c) Suppose you interested in estimating the parameters of this industry. Suppose you directly observe: p and w. Suppose $g(x)$ has the form $g(x) = x^{\alpha}$ for $\alpha < 1$. Suppose for each firm in the industry you observe employment x and output q. How would you estimate α, λ, κ, and the distribution of initial productivity draws $F(\cdot)$?

Question 3

Consider a homogeneous product industry with N firms. Firm i has productivity z_i, meaning the firm can produce z_i units of industry product per unit of labor employed. The productivity of each firm is an i.i.d. draw from the Pareto distribution,

$$F(z) = 1 - z^{-\theta},\ z \geq 1.$$

Define z^* to be the maximum across the N draws and z^{**} to be the second highest. Let $F^{**}(z, N)$ be the c.d.f. of the second highest draw z^{**}. The details of the particular functional firm are not important here, but two properties are worth noting. First, for any $z > 1$, $F^{**}(z, N)$ strictly decreases in N, and second

$$\lim_{N \to \infty} F^{**}(z, N) = 0, \text{ for } z > 1.$$

Conditional on the second highest draw z^{**}, the distribution of the highest draw is

$$F^*(z^*|z^{**}) = 1 - \left(\frac{z^*}{z^{**}}\right)^{-\theta}.$$

Note this is independent of the number of draws.

Assume demand is unit elastic in the industry, $D(p) = Ap^{-1}$. Let w be the wage. Assume firms compete in price in a Bertrand fashion.

(a) For a particular industry outcome (i.e., a realization of z^* and z^{**}, the remaining productivities are irrelevant), determine the equilibrium price and quantity in the industry, as well as the share of labor costs in total revenue.

(b) Suppose we measure productivity as the ratio of revenue to labor costs. How does the distribution of this measure of productivity vary as N, the level of competition, is varied?

(c) Suppose instead we measure productivity by z^*. How does the distribution vary with N?

(d) Suppose instead of Pareto, firms draw from the uniform distribution

$$F^{uniform}(z) = 1 - \frac{z-1}{2}, \; z \in [1, 2]$$

Compare the case of $N = 2$ with an alternative case where N is large. How are the qualitative results in this comparison different than your findings in part (b) and (c) for the Pareto?