Econ 8601—Fall 2017—Take-Home Final

Please work alone. You can choose when to work on it, but please work only one day on it.

Question 1. Perfect Price Discrimination. Consider the Logit Model of Product Differentiation from homework 1. There are n firms indexed by j and an "outside good" labeled by 0. Each firm $j \geq 1$ has constant marginal cost equal to c_j. There is a unit measure consumers. Let i index an individual consumer and suppose the utility of consumer i from purchasing good j and paying price $p_{i,j}$ has utility

$$\begin{aligned} U_{i,j} &= \xi_j - \alpha p_{i,j} + \varepsilon_{i,j} \text{ for } j = 1, 2, ...n \\ &= \varepsilon_{i,0} \text{ for good 0.} \end{aligned}$$

A consumer is therefore summarized by his or her vector of draws $\boldsymbol{\varepsilon}_i = (\varepsilon_{i,0}, \varepsilon_{i,1}, ...\varepsilon_{i,n})$ since consumers are otherwise the same. Assume the $\varepsilon_{i,j}$ are drawn type 1 extreme value which delivers the logit choice probabilities.

We allow for perfect price discrimination. Each firm observes the entire vector of draws $\boldsymbol{\varepsilon}_i$ for each consumer i and can set prices contingent on $\boldsymbol{\varepsilon}_i$ (i.e. can set a price $p_{i,j}$ specific to individual i.). The n firms compete in a Bertrand fashion for each individual consumer.

(a) Take as given the n firms in the industry and calculate the equilibrium of price competition when perfect price discrimination is feasible. Derive formulas for the market shares of each firm.

(b) A potential entrant is considering entry into this industry. If it comes in, it will be firm $j = n + 1$, and consumers will all get a new logit draw $\varepsilon_{i,n+1}$ for this firm. The firm has given values of ξ_{n+1} and c_{n+1}. If the firm enters, it pays a fixed cost ϕ_{n+1}. Derive a condition determining whether the firm enters. Discuss the connection between the private incentive for entry and the social incentive. That is, how does the total of producers and consumers surplus change if the firm enters?

Question 2

Consider the following competitive industry with three stages. In stage 1, a measure of firms N enters the industry. Each entering firm pays a fixed cost ϕ and draws a productivity parameter $\theta \in [0, \overline{\theta}]$ from a continuous distribution $F(\theta)$ with positive density $f(\theta)$ on the support $[0, \overline{\theta}]$. There is an unlimited supply of potential entrants to this industry, all *ex ante* identical.

In stage 2, after observing its own initial productivity draw θ°, each firm chooses whether to pay $\kappa > 0$ dollars to increase productivity to $\theta = \lambda\theta^{\circ}$, where $\lambda > 1$, or to not incur this cost in which case productivity remains at the initial level θ°.

In stage 3, there is a completive market for the industry. The production function of a firm with productivity θ is $q = \theta g(x)$, where x is the amount of labor employed by the firm and q is the output and $g(x)$ increasing and strictly concave. The demand curve for the industry is $Q^D = D(p)$, where p is the industry price. Assume that $D(p) > 0$, $D'(p) < 0$ for all p, and $\lim_{p\to\infty} D(p) = 0$. Assume the supply of labor to this industry is perfectly elastic at wage w. Let y denote the fraction of firms entering in stage 1 that invest to increase productivity in stage 2.

(a) Assume the cost κ is small enough that in equilibrium, the fraction y investing to increase productivity is strictly positive. Characterize the competitive equilibrium in this economy.

(b) Suppose demand doubles to $\tilde{D}(p) = 2D(p)$. How does this change the competitive equilibrium?

(c) Suppose you interested in estimating the parameters of this industry. Suppose you directly observe: p and w. Suppose $g(x)$ has the form $g(x) = x^{\alpha}$ for $\alpha < 1$. Suppose for each firm in the industry you observe employment x and output q. How would you estimate α, λ, κ, and the distribution of initial productivity draws $F(\cdot)$?

Question 3

Go to the notes on shares with CES and Frechet at Lecture 11 on the list of overheads for the class. Do the exercise at the end and calculate the given share.

Suppose we observe the observed shares for different industries. Discuss identification of parameters as well as additional data you might want to procure.