Econ 8601, Fall 2016

Homework 1

Due Tues. Sept. 19

Question 1

Consider the following duopoly model. There are two firms, 1 and 2, and time is discrete, $t \in \{0, 1, 2, ...\}$. Let $q_{1,t}$ and $q_{2,t}$ be the output of each firm in period t and let $Q_t = q_{1,t} + q_{2,t}$ be total output. Suppose the inverse demand is constant over time and linear, $P = A - Q$.

The average production cost in period t is constant in current production but depends upon *industry* production in the previous period. Let $X_t = Q_{t-1}$ denote total industry output from the previous period. Specifically, the marginal cost (and average cost) in period t of each firm is $c(X_t)$, where $c(0) \equiv \overline{c} < A$, $c' < 0$, $c'' > 0$, and $\lim_{X\to\infty} c(X) \equiv \underline{c} > 0$. So given last period industry output X, if firm i produces q_i units in the current period, its total cost in the period is $q_i c(X)$. Note the learning by doing here occurs at the industry level. There is a *knowledge spillover* here since firm 2's production cost next period are smaller when firm 1 produces more in the current period.

Suppose the discount factor is $\beta < 1$.

Assume in each period t the two firms simultaneously choose output levels $q_{1,t}$ and $q_{t,t}$ in a Cournot fashion.

(a) Define a Markov-perfect equilibrium in this model. Define a stationary equilibrium.

(b) Suppose $\beta = 0$. Determine the transition equation mapping last period's industry output X to this period's industry output Q. Under what condition does there exist a unique stationary equilibrium?

(c) Consider a two-period version of the model, $t \in \{1, 2\}$ and let $X_1 = 0$ be the initial state. If is possible that an equilibrium path for $\beta > 0$ would be the same as the equilibrium path when $\beta = 0$?

Question 2

Take the dynamic industry model discussed in class. Assume the parameterization

$$\begin{aligned} c(q) &= \frac{q^2}{2} \\ D(p) &= p^{-\varepsilon_D} = p^{-2} \\ P(Q) &= Q^{-\frac{1}{2}} \end{aligned}$$

where ε_D is the elasticity of demand. Assume $\beta = .5$ and $\sigma = 1 - \delta = .5$. Following the class notes:

$$\begin{aligned} q^* &= \frac{1}{\sigma} = 2 \\ p^*_C &= (1-\beta)c'(q^*) + \beta\sigma c(q^*) \\ &= .5q^* + .25\frac{q^{*2}}{2} = 1.5 \\ p^*_M &= \frac{\varepsilon_D}{\varepsilon_D - 1} p^*_C = 3 \\ Q^*_C &= p_C^{*-2} \\ Q^*_M &= p_M^{*-2} \\ K^*_C &= \sigma Q^*_C \\ K^*_M &= \sigma Q^*_M \end{aligned}$$

(a) Use value function iteration and Chebyshev approximation (page 223 in Judd) to calculate the equilibrium value function for the monopoly problem.

Use $n = 5$ (the order of the polynomials) and $m = 10$ (the number of grid points). Let $a = .5K^*_M$ and $b = 1.5K^*_M$; these are the endpoints of the grid using Judd's notation.

Iterate on the vector $(a_0, a_1, ..., a_n)$ which is the vector determining the approximation of $w(K)$ (Sorry for the awkward notation where a denotes two things; this is Judd's fault). Start with $a_i = 0$ for all i and stop when

$$\max_{i \in \{0,n\}} |a_i^{t+1} - a_i^t| < .000001$$

where t denotes a particular iteration.

After the value function converges approximate the policy function $q(K)$. Let the initial capital level be $K_0 = a = .5K^*_M$ and calculate for periods 1-25 the following variables: K_t,

q_t, P_t and $w_t(K_t)$. Make a table with this information. Compare with K_M^*, q^*, P_M^* and w_M^*, the stationary monopoly levels.

(b) Let $(a_0, ...a_n)$ be the coefficient vector for the value function $v_1(K_1, K_2)$ approximation and $(b_0, ..., b_n)$ the coefficient vector for the policy function $q_1(K_1, K_2)$ approximation. Use Judd's techniques for approximation in R^2 (page 238) to approximate the Markov perfect equilibrium. Note you need to iterate on q_1 as well as v_1 since firm 1 takes firm 2's action as given in the problem (and $q_2(x, y) = q_1(y, x)$).

Let $a = .25K_M^*$ and $b = K_C^*$ be the end points of the grid.

Solve for the equilibrium path for the first 25 periods starting at $K_{1,0} = b$ and $K_{2,0} = a$. Print out $q_{1,t}, q_{2,t}, K_{1,t}$, $K_{2,t}$, and P_t. Again, put this information in a table. What happens to market share over time?