Lecture 1

Evolution of Market Concentration

- Take a look at : Doraszelski and Pakes, "A Framework for Applied Dynamic Analysis in IO," Handbook of I.O. Chapter. (see link at syllabus).

- Matt Shum's notes are also pretty helpful

- This lecture will examine concentration in a structure with long-run constant returns to scale

- Static Cournot Duopoly (do this quickly!)

- Dynamic Duopoly (Today use a deterministic structure. Next lecture consider a stochastic structure)

Technology

- K_i capital of firm i
- Q_i output of firm i
- $q = \frac{Q}{K}$ output per unit of capital
- $c(q)$ cost per unit of capital when output intensity is q. $c' > 0$, $c'' > 0$.
- $C(q) = Kc(q)$ is total cost

Example:

- Cobb-Douglas $Q = L^{\alpha}K^{1-\alpha}$.

- Suppose L is \$1 per unit.

$$
\begin{aligned}
C(Q) &= \left[\frac{Q}{K^{1-\alpha}}\right]^{\frac{1}{\alpha}} \\
c(q) &= q^{\frac{1}{\alpha}}
\end{aligned}
$$

Static Cournot

- K_i fixed

- $P(Q)$ industry demand where $P'(Q) < 0$.

- Cournot problem. Firm 1 takes q_2 as fixed. Maximize profits per unit of capital

$$\max_{q_1} P(K_1 q_1 + K_2 q_2) q_1 - c(q_1)$$

- FONC

$$P(K_1 q_1 + K_2 q_2) + P'(K_1 q_1 + K_2 q_2) K_1 q_1 - c'(q_1) = 0$$

- SOC

  $$2P'(K_1q_1 + K_2q_2)K_1 + P''(K_1q_1 + K_2q_2)K_1^2q_1 - c''(q_1) < 0$$

- Reaction function $q_1 = R(q_2)$ solves above.

- If $K_1 = K_2$, then weak conditions get existence of symmetric equilibrium (if reaction function continuous. ($P'' \leq 0$ is sufficient)

- Let q^c solve $q^c = R(q^c)$.

Infintely Repeated Game (supergame)

- $K_1 = K_2 = 1$ fixed over time.
- β discount factor
- Can collusion be supported?
- $$\max_{q_1,q_2} P(q_1+q_2)(q_1+q_2) - c(q_1) - c(q_2)$$
$$FONC \; : \; P + P' - c'(q_i) = 0$$
- Let q^m solve the above

$$\begin{aligned} \pi^c &= P(q^c)q^c - c(q^c) \\ \pi^m &= P(q^m)q^m - c(q^m) \end{aligned}$$

- Can show $\pi^c < \pi^m$. So have standard prisoner's dilemma.

- Can collusive solution be supported?

Trigger Strategies

- If deviate play Cournot forever, otherwise q^m

- Return to cooperation

$$\frac{1}{1-\beta}\pi^m$$

- Return to deviating

$$\begin{aligned} & \max_{q_1} P(q_1 + q^m)q_1 - c(q_1) + \frac{\beta}{1-\beta}\pi^c \\ = \quad & \pi^{dev} + \frac{\beta}{1-\beta}\pi^c \end{aligned}$$

- Won't deviate iff

$$\pi^{dev} - \pi^m \leq \frac{\beta}{1-\beta}\left(\pi^m - \pi^c\right)$$

so get cooperation for sufficiently high β.

- More complicated solutions if there is uncertainty, imperfect monitoring, etc. (Abreu, Pearce, and Staccetti).

Markov Perfect Equilibria (Maskin and Tirole)

- Equilibrium policy functions depend only on *payoff relevant* states. Let s be a vector of such states.

- $\pi_i(a_1, a_2, s)$ current period payoff to player i given actions a_1 and a_2 in the current period and state s. π_1

- $s' = f(a_1, a_2, s)$ be transition function

- Let $\tilde{a}_i(s)$ be policy function and suppose $\tilde{v}_i(s)$ satisfies

$$\tilde{v}_1(s) = \max_{a_1} \pi\left(a_1, \tilde{a}_2(s), s\right) + \beta \tilde{v}_1(f(a_1, \tilde{a}_2(s), s))$$

  and let $\tilde{a}_1$ be the solution Suppose $\tilde{v}_2(s)$ and $\tilde{a}_2(s)$ satisfy the analogous relationships. Then $(\tilde{a}_1, \tilde{a}_2, \tilde{v}_1, \tilde{v}_2)$ is a Markov-perfect equilibrium.

Cournot Duopoly

- Suppose

$$K_1 = K_2 = 1$$

fixed over time.

—What is the set of Markov-perfect equilibria?

—What is the set of payoff-relevant states?

- Suppose

$$K_{i,t} = Q_{i,t-1}(1-\delta)$$

—Intepretation: use capital to make new capital.

—Adjustment costs (Lucas 1967, Prescott and Visscher (1980))

- Can separate output and investment. Add an output stage after the investment state. Assume Q_i is capital and Y_i is output. Suppose $Y_i \leq Q_i$ and zero marginal cost up to capacity. Suppose demand is elastic. Then firms always produce up to capacity.

- Define a Markov-perfect equilibrium

- What is a steady state?

Dynamics with $\beta = 0$

- Given (K_1, K_2), solve the (asymmetric) Cournot duopoly problem

- Claim: if $K_1 > K_2$ then $q_1 < q_2$, but $q_1 K_1 > q_2 K_2$.

—FONC for two firms

$$\begin{aligned} P + P' q_1 K_1 - c'(q_1) &= 0 \\ P + P' q_2 K_2 - c'(q_2) &= 0 \end{aligned}$$

Suppose instead that $q_1 \geq q_2$.

$\Rightarrow c'(q_1) \geq c'(q_2)$

$\Rightarrow P'q_1K_1 \geq P'q_2K_2$

$\Rightarrow K_1 \leq K_2$, a contradiction.

- Claim market shares converge to equality.

- $$\begin{aligned}\frac{K_1'}{K_2'} &= \frac{q_1K_1(1-\delta)}{q_2K_2(1-\delta)} \\ &= \frac{q_1K_1}{q_2K_2} \\ &< \frac{K_1}{K_2}\end{aligned}$$

  But

  $$1 < \frac{K_1'}{K_2'}$$

- So converge to 50-50 monotonically.

—Kydland, Dominant firm literature

- Intuition?

- Suppose $\beta > 0$

—analytic results difficult

—will go to computer and work this out

—Suppose commit to sequence of outputs. Does this matter? Look at $T = 2$ case.

Comment About the Role of Commitment

- MPE equilibrium very differernt from outcome of simultaneous move game where firm one and two pick vectors $(q_{11}, q_{12}, q_{13}, ...)$ and $(q_{21}, q_{22}, q_{23}, ...)$

Benchmark Case of Perfect Competition Steady State

- Suppose agents take as given a constant price p. .

- Let v be the discounted value of owning one unit of capital at the beginning of a period

$$v = \max_q pq - c(q) + \beta\sigma qv$$

where

$$\sigma = 1 - \delta$$

- FONC

$$p - c'(q) + \beta\sigma v = 0 \tag{1}$$

- In a stationary equilibrium,

$$\begin{aligned} \sigma q &= 1 \\ q^* &= \frac{1}{\sigma} \end{aligned}$$

- v^* solves

$$\begin{aligned} v^* &= pq^* - c(q^*) + \beta\sigma q^* v^* \\ &= pq^* - c(q^*) + \beta v^* \end{aligned}$$

  so

$$v^* = \frac{pq^* - c(q^*)}{1-\beta}$$

- From the FONC

$$p = c'(q^*) - \beta\sigma v^*$$

- Plugging in the formula for v^* yields

$$p = c'(q^*) - \beta\sigma\frac{pq^* - c(q^*)}{1-\beta}$$

  Solving for p yields the stationary competitive price

$$p_C^* = (1-\beta)c'(q^*) + \beta\sigma c(q^*).$$

- Q_C^* be the stationary competitive output

- $x_C^* = \sigma Q_C^*$ be the stationary competitive capital level.

Pure Monopoly.

- The state variable is K at the beginning of period capital. Let $w(K)$ be discounted maximized monopoly profit. This solves

$$w(K) = \max_q P\left(Kq\right)Kq - Kc(q) + \beta w\left(\sigma Kq\right)$$

- The FONC is

$$PK + P'K^2 q - Kc' + \beta\sigma K \frac{dw}{dK} = 0$$

- Dividing by x,

$$P + P'Kq - c' + \beta\sigma \frac{dw}{dK} = 0$$

- Use the envelope theorem to verify that

$$\frac{dw}{dK} = qc'(q) - c(q)$$

(Think of Q as the choice variable....).

- Plugging this into the first-order condition and evaluating at the steady state output level $q^* = \frac{1}{\sigma}$ yields

$$p + P'qK - c' + \beta\sigma \left[qc' - c\right] = 0$$

or

$$\begin{aligned} p + P'q^*K &= (1-\beta)\, c' + \beta\sigma c \\ &= P_C^*. \end{aligned}$$

- Let K solving the above be denoted K_M^*. .

- Now calculate the equilbrium off the steady state

A Technical Aside

Numerical Solutions of Dynamic Programming Problems

Monopoly Problem

- Statement of problem. $w(K)$ value function and $q(K)$ is policy function. Contraction mapping: Let w_0 be value function beginning next period. Then

$$w_1(K) = \max_q P(Kq)\,Kq - Kc(q) + \beta w_0(\sigma Kq)\,.$$

A solution is where $w_1(K) = w_0(K)$ for all K.

- Iterate

- How do numerically? Need an approximation for w_0.

- Discretize? Works well with single agent decision theory. For duopoly problem though continuity is useful.

- Polynomial approximation.

Example with Linear Approximation

1. Start with approximation

$$\hat{w}_0(K) = \alpha_0 + \beta_0 K$$

2. Take a set of m evaluation points $\tilde{K} = \{\tilde{K}_1, \tilde{K}_2, ..., \tilde{K}_m\}$

3. Solve problem at each of this points with $\hat{w}_0(K)$ instead of $w_0(K)$.

$$\tilde{w}_{1,i} = \max_q P\left(\tilde{K}_i q\right)\tilde{K}_i q - \tilde{K}_i c(q) + \beta \hat{w}_0\left(\sigma \tilde{K}_i q\right).$$

4. Yields a vector $\tilde{W}_1 = (\tilde{w}_{1,1}, \tilde{w}_{1,2}, ...\tilde{w}_{1,m})$

5. Use OLS to determine a new approximation

$$\begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} = \left(X'X\right)^{-1} X'\tilde{W}_1$$

$$X = 1\tilde{\ }\hat{K}$$

6. Iterate until obtain convergence in (α_t, β_t)

General Polynomial Approximation

- Chebyshev polynomials (in class of orthogonal polynomials)

- Defined on range $x \in [-1, 1]$

$$T_n(x) = \cos(n \cos^{-1} x)$$

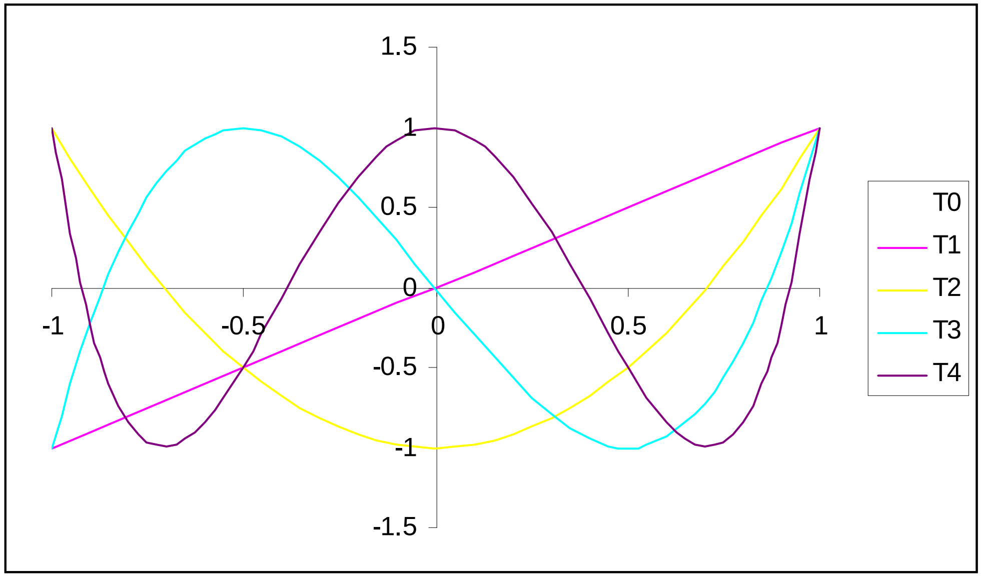

Figure 1:

Recipe in Judd

- Step 1: Evaluation points

$$z_k = -\cos\left(\frac{2k-1}{2m}\pi\right), k = 1, ..., m$$

- Step 2: Adjust the notes to the [a,b] interval (here $a = .5K_M^*, b = 1.5K_M^*$)

$$x_k = (z_k + 1)\left(\frac{b-a}{2}\right) + a,\ k = 1, ..., m$$

- Step 3: Evaluate $w(x)$ at the approximation nodes

$$\tilde{w}_k = w(x_k),\ k = 1, ..., m$$

- Step 4: Compute the Chebyshev coefficients (remember T_i orthogonal)

$$a_i = \frac{\sum_{k=1}^{m} \tilde{w}_k T_i(z_k)}{\sum_{k=1}^{m} T_i(z_k)^2}$$

- To arrive at the approximation

$$\hat{w}(x) = \sum_{i=0}^{n} a_i T_i(2\frac{x-a}{b-a} - 1)$$

Hints for Duopoly Problem

- $(a_0, ...a_n)$ coefficient vector for the value function $v_1(K_1, K_2)$ approximation

- $(b_0, ..., b_n)$ coefficient vector for the policy function $q_1(K_1, K_2)$ approximation.

- Use Judd's techniques for approximation in R^2 (page 238)

- You need to iterate on q_1 as well as v_1 since firm 1 takes firm 2's action as given in the problem (and $q_2(x, y) = q_1(y, x)$).

Contrast this Discretization Approach from First-Year Macro:

- Given k_0, solve problem

$$\max_{c_0, c_1, c_2, \cdots} \sum_{t=0}^{\infty} u(c_t)$$

subject to

$$\begin{aligned} c_t &= y_t - c_t \\ y_t &= f(k_t) \\ k_{t+1} &= (1-\delta)(y_t - c_t) \end{aligned}$$

- Set up as a dynamic programming problem

$$v(k) = \max_{k'} u(f(k) - \frac{k'}{1-\delta}) + \beta v(k')$$

Can discretize the state space $k_1, k_2, ...k_m$ evaluation points. Then write it as

$$v(k_t) = \max_{k_{t+1} \in \{k_1, k_2, ...k_m\}} u(f(k_t) - \frac{k_{t+1}}{1-\delta}) + \beta v(k_{t+1})$$

So solve for a vector $(v_1, v_2....v_m)$, and iterate on this. Do exhausive search (or can take into account convexity of the problem.

- What we are doing instead is to iterate on parameters pinning down a continuous function that we can differentiate. Take for iteration j $(\alpha_0^j, \alpha_1^j, ...\alpha_m^j)$,

$$v^j(k) = \sum_{i=0}^{m} \alpha_i^j T^j(k)$$

Then can take a first-order approach for solving the maximization problem

$$u'(c)\left[-\frac{1}{1-\delta}\right]+\beta v'(k')=0$$