Go from Single Agent Problem to Entry Game

- Like before: time $t = 1, 2, ...$
- Two kinds of agents: Incumbents in industry already. Numbered $i = 1$ to ω_t, where $\omega_t < \bar{n}$, maximum number of agents.
  - $a^I_{i,t} = 1$ stay in
  - $a^I_{i,t} = 0$, exit.
- Suppose one potential entrant each period. Entry feasible only if $\omega < \bar{n}$ (simplifies the state space)

- $a_t^E = 1$, enter
- $a_t^E = 0$, not enter.

- State variables at time t
  - State ω_t =number of firms in industry at beginning of period.(since symmetric, can keep track just number of firms).
  - A utility shock to each choice of incuments $i = 1, ...n_t$ and potential entrant
    * $\varepsilon_{i,t,0}$ utility shock to $a_{i,t}^I = 0$
    * $\varepsilon_{i,t,1}$ utility shock to $a_{it}^I = 1$
    * analogous shocks for potential entrant

- Play of game.
  - Start with ω_t firms.
  - Random utility shocks are private information!
  - Simultaneously incumbents make exit decision and potential entrant makes entry decision. Let

$$n_t = \sum_{i=1}^{\omega_t} \mathbf{1}_{[a_{it}^I=1]} + \mathbf{1}_{[a_t^E=1]}$$

    this is the number of firms at end of period

- Payoffs (other than utility shock mentioned above
  - π_n profit to each firm in the industry (including entrant, if it comes in)

    * Expect $\pi_n > \pi_{n+1}$.
    * Maybe $\pi_1 > 2\pi_2$.
  - Entry cost ϕ
  - Exit value ξ

- Transition $\omega_{t+1} = n_t$

Equilibrium

- Will define a symmetric equilibrium where firms in the same state do the same thing

- Policy function of incumbent $\tilde{a}^I(\omega, \varepsilon_0^I, \varepsilon_1^I)$ choice given state and two draws

- Policy function of entrant is similar $\tilde{a}^E(\omega, \varepsilon_0^E, \varepsilon_1^E)$

- Value functions $V^I(\omega, \varepsilon_0^I, \varepsilon_1^I)$ and $V^E(\omega, \varepsilon_0^E, \varepsilon_1^E)$

- Equilibrium: Values solve the Bellman equations, given the policies. Policies are optimal given the Value functions and that other firms following the policy rules

- Look at choice specific value functions

$$
\begin{aligned}
V^I(\omega, 0, \varepsilon_0) &= \xi + \varepsilon_0 \\
V^I(\omega, 1, \varepsilon_1) &= E_n[\pi_n + \beta E_\varepsilon[V^I(n, \varepsilon)]|\omega, a_1^I = 1] + \varepsilon_1 \\
V^E(\omega, 0, \varepsilon_0) &= \varepsilon_0 \\
V^E(\omega, 1, \varepsilon_1) &= -\phi + E_n[\pi_n + \beta E_\varepsilon[V^I(n, \varepsilon)]|\omega, a_1^E = 1] + \varepsilon_1
\end{aligned}
$$

- Then

$$
\begin{aligned}
V^I(\omega, \boldsymbol{\varepsilon}) &= \max\left\{V^I(\omega, 0, \varepsilon_0), V^I(\omega, 1, \varepsilon_1)\right\} \\
V^E(\omega, \boldsymbol{\varepsilon}) &= \max\left\{V^E(\omega, 0, \varepsilon_0), V^E(\omega, 1, \varepsilon_1)\right\}
\end{aligned}
$$

- Policy can be summarized by cutoff rules

$$
\begin{aligned}
\tilde{x}_\omega^I &< \varepsilon_{1,t}^I - \varepsilon_{0,t}^I \text{ then } \tilde{a}^I(\omega, \varepsilon_0^I, \varepsilon_1^I) = 1, \text{ otherwise} = 0, \\
\tilde{x}_\omega^E &< \varepsilon_{1,t}^E - \varepsilon_{0,t}^E, \text{ then } \tilde{a}^E(\omega, \varepsilon_0^I, \varepsilon_1^I) = 1, \text{ otherwise} = 0.
\end{aligned}
$$

then incumbent (entrant) stays, where

$$\begin{aligned}
\tilde{x}^I_\omega &= \xi - E_n[\pi_n + \beta E_\varepsilon[V^I(n,\varepsilon)]|\omega, a^I_1 = 1] \\
\tilde{x}^E_\omega &= -\phi + E_n[\pi_n + \beta E_\varepsilon[V^E(n,\varepsilon)]|\omega, a^E_1 = 1]
\end{aligned}$$

- Let $\tilde{\mathbf{x}} = \left(\tilde{x}^I_1, \tilde{x}^I_2, ...\tilde{x}^I_{\bar{n}}, \tilde{x}^E_1, \tilde{x}^E_2, ...\tilde{x}^E_{\bar{n}}\right)$ be a vector of cutoff rule.

- Can use recursive methods for solve for an equilibrium.

Data

- See state of industry each period ω_t and n_t

- Let $exit_t$ the the number of firms exiting each period, and $entry_t$ the number of entrants

$$n_t = \omega_t - exit_t + entry_t$$

- So have data set $(\omega_t, exit_t, entry_t) t = 1, ..., T$

- Parameter vector $\theta = (\pi_1, \pi_2, ...\pi_{\bar{n}}, \phi, \xi)$

- What next?

- How implement a nested-fixed point approach?

- How implement a two-step approach? (Assumptions? Advantages?)

Related Game that is Simpler (throws out the dynamics)

- Incumbent firms simultaneously decide whether or not to produce in a period. Production not related to whether produce today or not. Finally, sometimes only one gets to make the decision. So the state is $\omega = \{1, 2\}$ where ω is the count of firms that can choice to be in . ω is public information.

- Payoffs (n is number of producers)

$$\begin{aligned} \text{stay out} &: \varepsilon_{i,0} \\ \text{produce} &: \pi_n + \varepsilon_{i,1} \end{aligned}$$

  Where $\varepsilon_{i,a}$ extreme value

- Taking as given agent 2 uses rule produce if

$$\varepsilon_{2,1} - \varepsilon_{2,0} = x_2 \geq \tilde{x}_2(2).$$

- Let $V_1(a, \varepsilon_{1,a}|\omega)$ be the choice specific value function for agent 1.

$$\begin{aligned} V_1(0, \varepsilon_{1,0}|\omega &= 1,2) = \varepsilon_{1,0} \\ V_1(1, \varepsilon_{1,1}|1) &= \pi_1 + \varepsilon_{1,1} \\ V_1(1, \varepsilon_{1,1}|2) &= F(\tilde{x}_2)\pi_1 + (1 - F(\tilde{x}_2))\pi_2 + \varepsilon_{1,1} \end{aligned}$$

Enter if

$$V_1(1, \varepsilon_{1,1}|\omega) \geq V_1(0, \varepsilon_{1,0}|\omega)$$

If $\omega = 2$, enter if

$$\varepsilon_{1,1} - \varepsilon_{1,0} \geq \tilde{X}_1^*(\tilde{x}_2|2) \equiv -F(\tilde{x}_2)\pi_1 - (1 - F(\tilde{x}_2))\,\pi_2$$

Define

$$\delta_1(2) = -\tilde{X}_1^*(\tilde{x}_2|2)$$

If $\omega = 1$, enter if

$$\varepsilon_{1,1} - \varepsilon_{1,0} \geq \tilde{X}_1^*(\tilde{x}_2|1) \equiv -\pi_1$$

Let's plot $\tilde{X}_1^*(\tilde{x}_2|2)$: the optimal cutoff of firm 1 given behavior of firm 2.

Assume for starters $\pi_2 < \pi_1$.

What if $\pi_2 > \pi_1$?. The cut-off solves:

$$\tilde{X}_1^*(\tilde{x}_2|2) \equiv -F(\tilde{x}_2)\pi_1 - (1 - F(\tilde{x}_2))\,\pi_2$$

$$\frac{d\tilde{x}_1}{d\tilde{x}_2} = f\,(\tilde{x}_2)\,(\pi_2 - \pi_1)$$

- When have multiple equilibria?

- Let's say we have multiple equilibria. With two-stage approach (also called a partial solution approach) can remain agnostic about which equilibrium the agents settle on. (Assume they play the same equilibrium across the data points. Otherwise, unobserved heterogeneity that we come back to below)

- Let's estimate the CCP on a first stage.

- Using the expressions for the CCP, can invert to get utilities:

$$\begin{aligned}\text{Pr}(a|i,\omega) &= \frac{\exp(\delta_a(i,\omega))}{\exp(\delta_0(i,\omega)) + \exp(\delta_1(i,\omega))} \\ &= \frac{\exp(\delta_a(i,\omega))}{1 + \exp(\delta_1(i,\omega))}\end{aligned}$$

- $$\ln(\Pr(1|i,\omega)) - \ln(\Pr(0|i,\omega)) = \delta_1(i,\omega)$$

- $$\begin{aligned} \delta_1(i,1) &= \pi_1 \\ \delta_2(i,2) &= \Pr(0|-i,2)\pi_1 + \Pr(1|-i,2)\pi_2 \end{aligned}$$

- $$V(i,\omega) = \gamma + \log\left(\exp(\delta_0(i,\omega)) + \exp(\delta_1(i,\omega)\right)$$

- With estimates of π_1 and π_2 in hand, can ask whether there is multiple equilibria, and if so, whether they are coordinating on the good equilibrium or the bad one.

Big Picture

- Entry model above, can estimate vector π_1, π_2,...$\pi_{\bar{n}}$
- What is potentially interesting economics?
- Bring in market size. Think of a broader data set. Write $\pi(n, pop)$ for n firms and population.
- $(\omega_{j,t}, exit_{j,t}, entry_{j,t}, pop_j)$ for markets j, and time periods $t = 1, ...T$
- Breshahan and Reiss idea

– This literature has been influential. Use of revealed preference to back out parameters related to entry. Later class see an application to Wal-Mart

Now Generalize the Setup (Quick Overview of Ericson-Pakes Full Model)

Give incumbents more to do. Not just staying in or out. But get better somehow. But let's discretize it. Make a continuous decision x of investment. Let's put in some aggregate state that moves around (demand, aggregate costs)

- Each firm is in discrete state ω_i
- Have some stage game with reduced form profits $\pi_i(\omega_i, \omega_{-i})$.
  - e.g. if have differentiated products oligopoly with demand $q_i(p_i, p_{-i}, \omega_i, \omega_{-i})$ cost $c_i(\omega_i, \omega_{-i})$, and some reduced from equilibrium price function $p_i^e(\omega_i, \omega_{i-1})$, then

$$\begin{aligned}\pi_i(\omega_i, \omega_{-i}) &= [p_i^e(\omega_i, \omega_{-i}) - c_i(\omega_i, \omega_{-i})] \\ &\quad \times q_i(p_i(\omega_i, \omega_{-i}), p_{-i}(\omega_i, \omega_{-i}), \omega_i, \omega_{-i})\end{aligned}$$

- Make decision to exit and take ξ (let this be random now) or invest at x and move state for next period.

- Ericson Pakes particular investment model:

$$\Pr(\nu|x_i) = \begin{array}{l} \frac{\alpha x_i}{1+\alpha x_i} \text{if } \nu = 1 \\ = \frac{1}{1+\alpha x_i} \text{ if } \nu = 0 \end{array}$$

and

$$\omega_i' = \omega_i + \nu - \eta \text{ (where } \eta \text{ market)}$$

- Incumbent's problem, takes a given that other firms k are obeying Markov policy rules $x = \sigma_k(\omega)$, and let $\sigma = (\sigma_1, \sigma_2, ...\sigma_N)$ (if N firms)

$$V(\omega_i, \omega_{-i}, \phi) = \pi(\omega_i, \omega_{-i}) + \max\left\{\xi, \max_{x_i} H(x_i, \omega_i, \omega_{-i}, \xi)\right\}$$

where

$$H(x_i, \omega_i, \omega_{-i}, \xi) \equiv -x_i + \beta E[V(\omega_i', \omega_{-i}', \xi') | \omega_i, \omega_{-i}, x_i, \sigma_{-i}(\omega)]$$

and the solution to the above is $x_i = \sigma_i(\omega)$.

- Entrant (suppose for similicity):

$$V^e(\omega, \phi) = \max \left\{ 0, \max_{x_i^e} -\phi - x_i^e + ... \right\}$$

- A *Symmetric* Markov-Perfect Equilibrum: Investment Policy functions $\tilde{x}_i(\omega_i, \omega_{i-1})$, Entry rules, $Entry(\omega, \phi^e)$, Exit rules $Exit(\omega_i, \omega_{-i}, \xi)$, and value functions $V(\omega_i, \omega_{-i}, \xi)$, satisfying the Bellman equations and optimality conditions above.

- Symmetry has bite, all differences in behavior coming through differences in states.

Illustrate Multiplicity with Example of Cournot Model with Fixed Cost

- Look at Cournot example, $P = 12 - Q$ and suppose have a fixed cost equal to F (paid if $q > 0$, otherwise if $q = 0$ is avoided). Given output q_2, firm 1 solves the problem:

$$\begin{aligned} \text{Either } q_1 &> 0 \text{ and } q_1 = \arg\max_{q_1} \left[12 - q_1 - q_2\right] q_1 - F \\ \text{of } q_1 &= 0 \text{ and } \pi = 0. \end{aligned}$$

So if $q_1 > 0$, then reaction is

$$\text{max (given positive)} = 6 - \frac{q_2}{2}$$

So compare

$$pq_1 = [12 - q_1 - q_2]\left[6 - \frac{q_2}{2}\right] = \left[6 - \frac{q_2}{2}\right]^2$$

with fixed cost

$$\left[6 - \frac{q_2}{2}\right]^2 = F$$

Let

$$\begin{aligned} 6 - \frac{q_2}{2} &= F^{\frac{1}{2}} \\ \hat{q}_2 &= 12 - 2F^{\frac{1}{2}} \end{aligned}$$

So

$$\begin{aligned} \tilde{R}_1(q_2) &= 6 - \frac{q_2}{2},\ q_2 < 12 - 2F^{\frac{1}{2}} \\ &= 0,\ q_2 < 12 - 2F^{\frac{1}{2}}, \end{aligned}$$

- If $F = 0$, then unique equilibrium is $q_1 = q_2 = 4$. Both firms earn 16.

- Suppose $F = 15$: Then have three pure strategy equilibria, one symmetric, two asymmetric

- Suppose $F = 17$?