Lecture 3: More on Entry Games

An Aside: Analogy Between Entry Game Literature and Auction Literature

- Consider Symmetric First-Price Auction Models

- Assume independing private values.

  - n bidders, x_i private valuation of i drawn i.i.d. from c.d.f. F_X with support $[\underline{x}, \bar{x}]$

  - Let y be maximum of $(n-1)$ draws, let F_Y be c.d.f.

$$
\begin{aligned}
F_Y(y) &= F_X(y)^{(n-1)} \\
f_Y(y) &= (n-1)\, F_X(y)^{(n-2)} f_X(y)
\end{aligned}
$$

– Suppose have symmetric equilibrium with bid function $\beta(x)$ and inverse η.

– Suppose a reserve price $r > \underline{x}$.

- Then bidder 1's profits from bidding b given value x are

$$\pi(b, x) = (x - b)\, F_Y(\eta(b))$$

Differentiating w.r.t. b

$$[x - b] f_{Y(\eta(b))} \frac{d\eta}{db} - F_Y(\eta(b)) = 0$$

and imposing symmetry and $x = \eta(b)$ and $\frac{d\eta}{d\beta} = 1/\beta'(x)$ yields

$$[x - \beta(x)] f_Y(x) - \beta'(x) F_Y(x) = 0$$

The equilibrium bid solves this differential equation, subject to the boundary condition $\beta(r) = r$

$$\beta(x) = x - \frac{\int_r^x F_X(s)^{n-1} ds}{F_X(x)^{n-1}}$$

the mark down $x - \beta(x)$ is decreasing in the number of bidders n and increasing in the dispersion of the value distribution.

Estimation

- Early work (e.g. Paarsch). Full solution (or gain nexted fixed point approarch. Parameterize the distribution function. Take given parameters, solve for the equilibrium, construct a likelihood function. (Then after estimation can calculate optimal reserve price, for example)

- Let's instead go over a nonparametric approach (Elyakime, Laffont, Loisel, Vuong (1994)+Gueere, Perrigne, and Vuong (2000))

- Data on bids $\left\{\{b_{it}\}_{i=1}^{m_t}, n_t, r_t\right\}_{t=1}^{T}$

- Define $M = \beta(Y)$ as the maximum bid of bidder 1's rivals and let the distribution be denoted $G_M(\cdot)$ and density given by $g_M(\cdot)$. Montonicity of η implies that for any $m \in (r, \beta(\bar{x}))$,

$$G_M(m) = F_Y(\eta(m))$$

The associated density function is given by

$$g_M(m) = f_Y(\eta(m))\eta'(m)$$

Recall the FOC

$$[x-b]f_{Y(\eta(b))}\frac{d\eta}{db} - F_Y(\eta(b)) = 0$$

Substituting the above into the FOC yields

$$(\eta(b) - b)g_M(b) - G_M(b)$$

or

$$\eta(b) = b + \frac{G_M(b)}{g_M(b)} = b + \frac{G(b)^{n-1}}{(n-1)\,G(b)^{(n-2)}g(b)} = b + \frac{G(b)}{(n-1)\,g(b)}$$

- Estimation proceeds in two steps
  - Step 1: Estimate G or G_M and g or g_M either parametrically or nonparametrically Then obtain

    $$\hat{x}_{it} = b_{it} + \frac{\hat{G}_{M_i}(b_{it})}{\hat{g}_{M_i}(b_{it})}$$

  - Step 2: Estimate $f_X(\hat{x}_{it})$ and $F_X(\hat{x}_{it})$. Then can run counterfactuals (i.e. change in reserve price, change in number of bidders, of course we have kept number of bidders as exogenous here. If bidders endogeneous, need to do more):

Embellish Entry Game with Permanent Unobserved Heterogeneity

- Incumbent firms simultaneously decide whether or not to produce in a period. Production not related to whether produce today or not. Finally, sometimes only one gets to make the decision. So the state is $\omega = \{1, 2\}$ where ω is the count of firms that have the choice of whether to be in or out.

- Assume ω is public information.

- Also a fixed cost $\xi \in \{\xi_1, \xi_2\}$ with λ_ℓ probability of drawing ξ_ℓ (*finite mixture model*)

  – Take ξ as known, parameter to estimate is λ

- Let $\Pr(n) = \gamma_n$ (assume known, otherwise, trivial to estimate)

- Payoffs (n is number of producers)

$$\begin{aligned} \text{stay out} &: \varepsilon_{i,0} \\ \text{produce} &: \pi_n - \xi_\ell + \varepsilon_{i,1} \end{aligned}$$

  Where $\varepsilon_{i,a}$ extreme value

- Suppose have a cross section of locations playing this game. Any luck identifying this model?

  - What do we see? $\Pr(a = 1|n)$....

- Suppose have a panel, say two observations on each market.

- Let markets be indexed by m, time $t = 1, 2$.

- Let n_{mt} be publicly observed state at m at time t

- Let a_{mt} be actions (e.g. if $n = 1$ whether the one firm enters,

- Let $\{x_{mt} = (n_{mt}, a_{mt}), m = 1....M, t = 1, 2\}$ be the data

- Full solution MLE approach to estimating (θ, λ), where $\theta = (\pi_1, \pi_2)$
  - For a given θ, solve for $\Pr(x_{mt}|\xi = \xi_\ell, \theta, n)$
  - Use Bayes Rule to calculate $\Pr(\xi_m = \xi_\ell|(x_{m1}, x_{m2}), \theta, \lambda)$
  - Likelihood of outcome in market i can be written

$$
\begin{aligned}
& \Pr(x_{m1}, x_{m2}|\theta, \lambda) \\
= \; & \Pr(\xi_m = \xi_1|(x_{m1}, x_{m2}), \theta, \lambda) \Pr(x_{m1}, x_{m2}|\theta, \xi_m = \xi_1) + \\
\Pr(\xi_m = \; & \xi_2|(x_{m1}, x_{m2}), \theta, \lambda) \Pr(x_{m1}, x_{m2}|\theta, \xi_m = \xi_2)
\end{aligned}
$$

- Not going to turn this into a two step approach because we are using states from multiple periods

- Trick in the literature is to get everything Markov, and get information about the unobservable state contained in the observable state.

- Let's tweak the model. Add another parameter where there is another fixed cost that depends on whether the firm produced in the previous period.

  - The state $\omega_{i,t} = \{0, 1, 2\}$

    * 0 (can't produce)

    * 1, didn't produce last period

* $*$ 2 produced last period

- Returns

$$\begin{aligned} \text{stay out} &: \varepsilon_{i,0} \\ \text{produce} &: \pi_n - \phi_\omega - \xi_\ell + \varepsilon_{i,1} \end{aligned}$$

where $\phi_2 < \phi_1$.

- To estimate: λ and $\theta = (\pi_1, \pi_2, \phi_1, \phi_2, \gamma)$ (assume discount factor β known)

- Consider estimation strategies that do not exploit the panel aspect of the data.

- Full solution

  - Given ℓ, and θ solve for equilibrium

  - Then take a draw from the stationary distribution, Let $\omega_{m,t} = (\omega_{1mt}, \omega_{2mt})$. Calculate $\Pr(\xi = \xi_\ell | \omega, \theta)$ then calculate likelihood as above

Partial Solution Approach

- Let's say we have estimates of CCP, conditional on the latent variable ℓ. Call it $\hat{P}_\ell^\circ$ (the "$\circ$" is there because we are going to update it later)
  - Can start with this by just assuming constant across ℓ, then plug in the CCP off of the data
- Run the CCP and get the steady state distribution of ω for each ℓ., also $\Pr(\xi = \xi_\ell|\omega)$
- Obtain a pseudo-maximum likelihood of θ (figure the likelihood of the data given θ, $\Pr(\xi = \xi_\ell|\omega)$, and $\hat{P}_\ell^\circ$)
  - e.g., likelihood firm 1 at $\omega_1 = 2$ enters, given $\omega_2 = 1$.

- Use the $\hat{P}_\ell^\circ$ to plug in firm 2's likelihood of entry.
- if $\beta = 0$ done.
- if $\beta > 0$ well you have the parameters, and how firm 1 is behaving in the future, so done to.

- Update the CCP using the best response. Note we are not solving a fixed point here, not running through and solving the equilibrium of the model.

- Rinse and repeat...

- EM algorithm, can look it up.

- Good reference is part 2 of Aguirregabiria and Nevo handbook chapter on syllabus.