Efficiency Of Free Entry: Some Example Models
Model 1:Mankiw and Whinston

- Homogenous product market demand $P(Q)$, Q total output. $P'(Q) < 0$
- Fixed cost ϕ
- Variable costs $c(q)$, $c(0) = 0$, $c'(q) \geq 0$, $c''(q) \geq 0$.
- Second stage, output per entrant is determined. Let q_N be equilibrium output per firm, given N entrants (you pick model of competition). But assume (easy to check this is satisfied with Cournot and $P''(Q) \leq 0$):
  - $Nq_N > \hat{N}\hat{q}_N$, $N > \hat{N}$ and $\lim_{N \to \infty} Nq_N = M < \infty$
  - $q_N < q_{\hat{N}}$, for $N > \hat{N}$.
  - $P(Nq_N) - c'(q_N) > 0$ for all N.
- First stage entry: N^e, then $\pi_{N^e} \geq 0$, and $\pi_{N^e+1} < 0$.

Social Planner

- Planner controls entry but not pricing given entry.
- Maximizes total surplus. So problem is

$$\max_N W(N) = \int_0^{Nq_N} P(s)ds - Nc(q_N) - N\phi$$

- Ignore integer constraint, for now. The Planner's FONC is

$$
\begin{aligned}
W'(N^*) &= P(Nq_N)\left[N\frac{\partial q_N}{\partial N} + q_N\right] - c(q_N) - Nc'(q_N)\frac{\partial q_N}{\partial N} - \phi \\
&= [Pq_N - c - \phi] + N\left[P - c'\right]\frac{\partial q_N}{\partial N} \\
&= \pi_N + N\left[P - c'\right]\frac{\partial q_N}{\partial N} \\
&= 0
\end{aligned}
$$

- Evaluate at N^e, observe that $\pi_{N^e} = 0$, so $W'(N^e) < 0$, (since $P > c'$, and $\frac{\partial q_N}{\partial N} < 0$. Excessive entry.
- Intuition
- If impose the integer constraint then $N^e \geq N^* - 1$.

Model 2: Dixit Stiglitz Model

- Unbounded set of possible goods, $x \in [0, \infty]$
- Utility function of representative consumer

$$
\begin{aligned}
U &= \left(\int_0^\infty q(x)^{\frac{1}{\mu}} \, dx \right)^\mu \\
\sigma &= \frac{\mu}{\mu - 1}
\end{aligned}
$$

for $\mu > 1$.

- L time endowment for economy
- Technology: ϕ fixed cost (labor) to setup a product. Constant marginal cost of β units of labor
- Let labor be numeraire, $w = 1$
- Let $[0, N]$ be interval of goods produced in the market. Let $p(x)$ be price of good x.

Definition of Equilibrium
$\{N, p(x), q(x), x \in [0, N]\}$ such that

1. Consumer demands $q(x)$ maximize utility given the budget constraint

2. $p(x)$ is the profit maximizing price of firm x, taking as given the prices of all other firms

3. Firms that enter make nonnegative profit

4. No incentive for further entry

(Note (3+4)$\Rightarrow$ zero profit).

- Problem of Consumer:

$$\max_{q(\cdot)} \left[\int\limits_0^N q(x)^{\frac{1}{\mu}} dx \right]^{\mu} \tag{1}$$

subject to

$$\int_0^N p(x)q(x)dx = L$$

- MRS condition: goods x_1 and x_0.

$$
\begin{aligned}
\frac{\mu \, []^{\mu-1} \left(\frac{1}{\mu}\right) q_1^{\frac{1}{\mu}-1}}{\mu \, []^{\mu-1} \left(\frac{1}{\mu}\right) q_0^{\frac{1}{\mu}-1}} &= \frac{p_1}{p_0} \\
\left(\frac{q_1}{q_0}\right)^{-\frac{1}{\sigma}} &= \frac{p_1}{p_0} \\
q_1 &= p_1^{-\sigma} \left(p_0^{\sigma} q_0\right) \\
&= p_1^{-\sigma} k
\end{aligned}
$$

Firm's problem
$$\max_{p_1} (p_1 - \beta) q_1(p_1) - \phi$$

The FONC of firm 1

$$
\begin{aligned}
p_1^{-\sigma} k - \sigma (p_1 - \beta) p_1^{-\sigma-1} k &= 0 \\
p_1 &= \sigma (p_1 - \beta) \\
\frac{p_1 - \beta}{p_1} &= \frac{1}{\sigma} \\
p_1 &= \mu \beta
\end{aligned}
$$

Constant markup over cost.

- Zero-profit condition

$$\mu \beta q - \beta q - \phi = 0, \tag{2}$$

$$\beta (\mu - 1) q = \phi$$

So

$$q^e = \frac{\phi}{\beta(\mu - 1)}$$

- Use resource constraint to determine number of products:

$$
\begin{aligned}
N\left(\beta q^e + \phi\right) &= L \\
N^e &= \frac{L}{\beta q^e + \phi} = \frac{L}{\frac{\phi}{(\mu-1)} + \phi} \\
&= L\left(\frac{\mu - 1}{\mu}\right)\frac{1}{\phi} = \frac{L}{\sigma\phi}
\end{aligned}
$$

- Consumer Welfare (per capita)

$$\text{equilibrium utility per capita} \quad = \quad \frac{\left(\int_0^\infty q(x)^{\frac{1}{\mu}} dx\right)^\mu}{L}$$

$$= \quad \frac{\left(Nq^{e^{\frac{1}{\mu}}}\right)^\mu}{L}$$

$$= \quad \frac{N^\mu q^e}{L} = \frac{\left(\frac{L}{\sigma\phi}\right)^\mu q^*}{L}$$

$$= \quad \frac{\phi^{1-\mu}\sigma^{-\mu}}{\beta(\mu-1)} L^{\mu-1}$$

Increasing in L(love of variety).

Social Planner's Problem with Dixit Stiglitz

- Given number of firms N, optimal for each to produce same output and use up labor endowment. (So total welfare same whethe $p = \mu\beta$ (with redistribution of monopoly profit) or $p = \beta$
- So given N,

$$q = \frac{L - \phi N}{N}$$

$$\text{utility per capita} = \frac{N^\mu q}{L} = \frac{N^\mu}{L}\left(\frac{L - \phi N}{N}\right)$$

$$= \frac{1}{L}N^{\mu-1}(L - \phi N)$$

- The FONC is

$$0 = \frac{1}{L}(\mu - 1)N^{\mu-2}(L - \phi N) - \phi\frac{1}{L}N^{\mu-1}$$

$$0 = (\mu - 1)(L - \phi N) - \phi N$$

$$(\mu - 1)L = \mu\phi N$$

$$N^* = \frac{(\mu - 1)}{\mu\phi}L = N^e!$$

- Discuss logic of why get first best in Dixit-Stiglitz, but not Mankiw-Whinston
- What about issue of distortions conditional on entry? Does this matter?
- Why in general there might be too little or too much entry? MW says too much, DS just enough. But in general? Think about externalities of entry, hurts other firms, helps consumers, try cooking things to helps consumers more than hurt competitors
  - What if heterogeneity of consumers makes a firm almost indifferent between a high price and low price...
  - Maybe entry trips it to lw price...

Link CES and Discrete Choice Model (from Anderson dePalma Thiss 1992 book)

- Start with discrete choice:

Type 1 extreme value has double exponential distribution

$$F(x) = \exp\left[-\exp\left(\frac{x - \eta}{\mu}\right)\right]$$

where η is location parameter and μ is scaling parameter. If choose

$$\max\{\delta_i + \varepsilon_i\}$$

Then

$$P_i = \frac{\exp(\frac{\delta_i}{\mu})}{\sum_{j=1}^{n}\exp(\frac{\delta_j}{\mu})}$$

and maximized utility is

$$V = \mu\gamma + \mu \ln \left(\sum_{i=1}^{n} \left(\frac{\delta_i}{\mu} \right) \right) - \eta$$

Now back to CES, with a discrete number of goods:

$$U = \left(\sum_{i=1}^{n} X_i^{\rho} \right)^{1/\rho} X_0^{\alpha}$$

Demand is

$$X_i = \frac{p_i^{-1/(1-\rho)}}{\sum_{j=1}^{n} p_j^{-1/(1-\rho)}} X_0$$

where

$$X_0 = \alpha \hat{Y} \equiv \alpha \frac{Y}{1+\alpha}$$

Taking monotone transformation

$$V = (1+\alpha) \ln Y + \frac{1-\rho}{\rho} \ln \left(\sum_{i=1}^{n} p_i^{-1/(1-\rho)} \right)$$

Now set up a discrete choice world where suppose consumer picks:

$$\tilde{U}_i = \ln x_i + \alpha \ln x_0 + \varepsilon_i$$

Given pick x_i, solve

$$\delta_i = \max_{(x_i, x_0)} \ln x_i + \alpha \ln x_0$$

Subject to

$$x_0 + p_i x_i = y$$

But spend a share $1/(1+\alpha)$ on inside good.

$$p_i x_i = \frac{1}{1+\alpha} y$$

$$x_i = \frac{1}{p_i} \frac{1}{1+\alpha} y$$

$$x_0 = \frac{\alpha}{1+\alpha} y$$

Then maximized utility conditional on choice i

$$
\begin{aligned}
\delta_i &= \ln\left(\frac{1}{p_i}\frac{1}{1+\alpha}y\right) + \alpha\ln\left(\frac{\alpha}{1+\alpha}y\right) \\
&= (1+\alpha)\ln y - \ln(p_i) + \alpha\ln(\alpha) - (1+\alpha)\ln(1+\alpha) \\
&= -\ln p_i + K_1
\end{aligned}
$$

For

$$
K_1 = (1+\alpha)\ln y + \alpha\ln(\alpha) - (1+\alpha)\ln(1+\alpha)
$$

Probability of choosing i is

$$
\begin{aligned}
P_i &= \frac{\exp\left((-\ln p_i + K_1)/\mu\right)}{\sum_{j=1}^{n}\exp\left((-\ln p_j + K_1)/\mu\right)} = \frac{\exp\left((-\ln p_i)/\mu\right)}{\sum_{j=1}^{n}\exp\left((-\ln p_j)/\mu\right)} \\
&= \frac{\exp\left(-\ln p_i^{\frac{1}{\mu}}\right)}{\sum_{j=1}^{n}\exp\left(-\ln p_j^{\frac{1}{\mu}}\right)} = \frac{p_i^{-\frac{1}{\mu}}}{\sum_{j=1}^{n}p_J^{-\frac{1}{\mu}}}
\end{aligned}
$$

So expected demand is

$$X_i = P_i \frac{1}{p_i} \frac{1}{1+\alpha} y$$

Set

$$\mu = \frac{1-\rho}{\rho}$$

Substitute in $\delta_i = -\ln p_i + K_1$, to get back to CES

A Different Model Eaton And Kortum (2002)

- $z_i(j)$ efficiency in producing good in country i

- c_i is labor cost in country

- Unit cost to produce j in i is $\frac{c_i}{z_i(j)}$

- iceberg cost d_{ni} cost of i to n. $d_{ii} = 1$. $d_{ni} > 1$, $n \neq i$

- Perfect competition (generalize in BEJK to oligopoly)

$$p_{ni}(j) = \left(\frac{c_i}{z_i(j)}\right) d_{ni}$$

- Price of good j in country n

$$p_n(j) = \min\{p_{n1}(j); i = 1, ..., N\}$$

- Consumers purchase individual goods in amounts $Q(j)$ to maximize

$$U = \left[\int_0^1 Q(j)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Technology

- $z_i(j)$ random variable drawn a certain way to make everying work out really easily

  - Frechet (also called Type II extreme value)

  - $F_i(z) = e^{-T_i z^{-\theta}}$

  - T_i is a country specific. Bigger T get better productivity draws

  - θ governs extent of Ricardian comparative advantage. Bigger θ less variability

  - $\log z$ has standard deviation $\dfrac{\pi}{\theta sqrt(6)}$

- Country i presents country n with a distribution of prices

$$
\begin{aligned}
G_{ni}(p) &= \Pr\left(P_{ni} \leq p\right) = 1 - F_i\left(\frac{c_i d_{ni}}{p}\right) \\
&= 1 - e^{-T_i(c_i d_{ni})^{-\theta} p^{\theta}}
\end{aligned}
$$

- Lowest price will be less than p, unless each source's price is greater than p. So $G_n(p) = \Pr\left(P_n \leq p\right)$ is

$$
\begin{aligned}
G_n(p) &= 1 - \prod_{i=1}^{N} \left(1 - G_{ni}(p)\right) \\
&= 1 - \prod_{i=1}^{N} e^{-T_i(c_i d_{ni})^{-\theta} p^{\theta}} \\
&= 1 - e^{-\Phi_n p^{\theta}}
\end{aligned}
$$

for

$$\Phi_n = \sum_{i=1}^{N} T_i \left(c_i d_{ni} \right)^{-\theta}$$

- Price parameter Φ_n.

  - If $d_{ni} = 1$, then Φ_n the same everywhere.

  - $d_{ii} = 1$, $d_{ni} = \infty$, $n \neq i$, the $\Phi_n = T_n c_n^{-\theta}$

- Probability that country i provides a good at the lowest price in country n is

$$
\begin{aligned}
\pi_{ni} &= \int_0^\infty \prod_{s \neq i} [1 - G_{ns}] \, dG_{ni}(p) \\
&= \int_0^\infty \prod_{s \neq i} e^{-T_s (c_s d_{ns})^{-\theta} p^\theta} \, dG_{ni}(p)
\end{aligned}
$$

$$= \int_0^\infty \prod_{s \neq i} e^{-T_s(c_s d_{ns})^{-\theta} p^\theta} \left[T_i \left(c_i d_{ni} \right)^{-\theta} \theta p^{\theta-1} \right] e^{-T_i(c_i d_{ni})^{-\theta} p^\theta} \, dp$$

$$= T_i \left(c_i d_{ni} \right)^{-\theta} \int_0^\infty \prod_s e^{-T_s(c_s d_{ns})^{-\theta} p^\theta} \left[\theta p^{\theta-1} \right] \, dp$$

$$= T_i \left(c_i d_{ni} \right)^{-\theta} \int_0^\infty e^{-\left(\sum_s T_s(c_s d_{ns})^{-\theta} \right) p^\theta} \left[\theta p^{\theta-1} \right] \, dp$$

$$= T_i \left(c_i d_{ni} \right)^{-\theta} \int_0^\infty e^{-\Phi_n p^\theta} \left[\theta p^{\theta-1} \right] \, dp$$

$$= T_i \left(c_i d_{ni} \right)^{-\theta} \left[-\frac{1}{\Phi_n} e^{-\Phi_n p^\theta} \right]_0^\infty$$

$$= \frac{T_i \left(c_i d_{ni} \right)^{-\theta}}{\Phi_n}$$

- Conditional distribution of price paid (condition upon country of origin) is same as unconditioned, $G_n(p)$. Given the CES preferencs, the above probability also provides the distribution of sales across locations.

- Spending share is the same.

So just like CES, only what matters is θ instead of $\sigma - 1$.

Broda and Weinstein

- Builds on Feenstra (1994) gains fro mvariety.

- Relative to that paper, work with a nested CES allowing for different elasticities at each point

  - not just counting varieties, they use expendititure shares. Can get a big increase in counts but if expenditure shares don't change much then elastiticy probably not too big.

  - in list, (page 562) it argues for point two that it is great because it allows for differences in elasticities. This seems like point one.

  - Third reason great is that if categories are merged no problem, picks things up in the shares. This is very sensible. I like very much the thinking in this paper.

- Estimation. The paper is basically regressing changes in spending shares on changes in unit prices. Or relative spending shares on relative unit prices.

Some econometric issue about how exactly this works. Assumptions about the shocks. Don't quite understand it.

Feenstra and Weinstein (forthcoming JPE)

- Note past literature has counted products to product estimates of variety gains. Estimates of gains from lower mark-ups use dif-dif because CES has constant mark-ups. This paper redoes things with translog preferences. Elasticity is inversely related to a products market share. More firms enter, firms share does down, then demand is more elastic, so mark-up is lower.

- Paper picks up a few things missed in Broda and Weinstein.

  - First, as foreign firms enter and shares of each one get smaller, you have the pro-competitie effect of lower mark-ups from imports.

- Plus domestic firms have lower mark-ups as their shares decrease. However, some domestic firms exit.

- ddress 3 potential criticisms of Broda and Weinstein:

  * CES overstates gains because assume reservation price is infinite,

  * product space crowded, so diminishing returns to variety

  * take into account exist of domestic firms.

- Bottom line: welfare gains are same as CES (where variety is only issue), and this is a 0.86 percent gains. But now half of welfare gain is due so the impact of new competitors on mark-ups.

- In discussion of data look at $H_{it}^k s_{it}^k$ where H_{it}^k is herf for country i in sector k and s_{it}^k of country i in sector k.

$$H_{it}^i = \sum_j \left(s_{jt}^{ik} \right)^2$$

  Note the Herf is the average charge conditional on country, so this becomes is share unconditioned on country. They make a big point that this share is declinining for US firms, (so mark-up should go down). Also, while for some countries things get more concentrated, the average Herf declines, so plugging into the translog, mark-ups go down.

- Note again, since US Herf rose, variety down, but when weighted times US share, US firms share down, so mark-ups down.

- Get HERF data from PIERs import data on waterbourne imports