Equations from Nocke and Schutz

- Discussion of Arcidiacono and Miller (2011) of a way to do Rust bus problem

- Use $d_{1t} = 1$ replace at or $d_{2t} = 1$ if don't replace bus engine

- State x_t is age, s other characteristics

- Choice specific value function (net of ε_{jt})

$$
\begin{aligned}
v_j(x, s) &= \beta V(0, s), \text{ if } j = 1 \\
&= \theta_1 x + \theta_2 s + \beta V(x + 1, s), \text{ if } j = 2
\end{aligned}
$$

- Probability of replacement

$$p_1(x, s) = \frac{1}{1 + \exp\left[v_2(x, s) - v_1(x, s)\right]}$$

- Indirect utility from choice $i: y + v_i(p_i) + \varepsilon_i$

- Roy's identify purchases $-v_i(p_i)$ units of good i, the conditional demand.

$$P_i(p) = \frac{e^{v_i(p_i)}}{\sum_{j \in N} e^{v_j(p_j)}} = \frac{h_i(p_i)}{\sum_{j \in N} h_j(p_j)}$$

- Expected demand

$$\begin{aligned} & \frac{h_i(p_i)}{\sum_{j \in N} h_j(p_j)} (-v_i(p_i)) \\ = & \frac{-h_i'(p_i)}{\sum_{j \in N} h_j(p_j)} \end{aligned}$$

- Special cases:

- MNL if $h_j(p_j) = \exp\left(\frac{a_j - p_j}{\lambda}\right)$
- CES if $h_j(p_j) = a_j p_j^{1-\sigma}$, $\sigma > 1$

- IIA property. Ratio of demands for j and k depends only on p_j and p_k.

- Show Equivalent to quasilinear preferences of representative consumer.

- Partition set of products.

$$\Pi^f(p) = \sum_{\substack{k \in f \\ p_k < \infty}} (p_k - c_k) \frac{-h_k'(p_k)}{\sum_{j \in N, p_j < \infty} h_j(p_j) + \sum_{j \in N, p_j = \infty} h_j}, \text{ for all } p \in (0, \infty]^N$$

- Note funky stuff with infinity, not an issue

- Problems with standard approaches

  - Π^f not quasi concatve

  - Π^f not upper semi-continuous

  - Not supermodular

- FONC

$$
\begin{aligned}
\frac{\partial \Pi^f}{\partial p_k} &= \frac{-h_k'(p_k)}{H} + \sum_{j \in f} \left(p_j - c_j\right) \frac{\left(-h_k'\right)\left(-h_j'\right)}{H^2} - (p_k - c_k)\frac{h_k''}{H} \\
&= \frac{-h_k'}{H}\left[1 - \left(\frac{p_k - c_k}{p_k}\right) p_k \frac{-h_k''}{-h_k'} + \sum_{j \in f} \left(p_j - c_j\right) \frac{\left(-h_j'\right)}{H}\right]
\end{aligned}
$$

- Now let

$$
\iota_k(p_k) = p_k \frac{-h_k''(p_k)}{-h_k'(p_k)}
$$

which is the elasticity of $-h_k'(p_k)$.

- Claim, have a pricing equilibrium if and only if

$$
\left(\frac{p_k - c_k}{p_k}\right) \iota_k(p_k) = 1 + \sum_{j \in f} \left(p_j - c_j\right) \frac{-h_j'(p_j)}{H}
$$

- For any f, right hand side independent of the particular product. Therefore if k and l same firm

$$\left(\frac{p_k - c_k}{p_k}\right)\iota_k(p_k) = \left(\frac{p_l - c_l}{p_l}\right)\iota_l(p_l) = \mu_f \text{ (firm markup)}$$

- Rewrite above

$$\begin{aligned}
\mu^f &= 1 + \frac{1}{H}\sum_{j\in f}\frac{\left(p_j - c_j\right)}{p_j}p_j\frac{h_j''}{-h_j'}\frac{h_j'^2}{h_j''} \\
&= 1 + \frac{1}{H}\sum_{j\in f}\frac{\left(p_j - c_j\right)}{p_j}\iota_j(p_j)\gamma_j(p_j) \\
&= 1 + \frac{1}{H}\sum_{j\in f}\mu^f\gamma_j(p_j)
\end{aligned}$$

Or

$$\mu^f \left(1 - \frac{1}{H} \sum_{j \in f} \mu^f \gamma_j(p_j)\right) = 1$$

- Suppose $p_k \to \frac{p_k - c_k}{p_k} \iota_k(p_j)$ is one to one and let inverse be $r_k(.)$, the pricing function for product k

- Then

$$\mu^f \left(1 - \frac{1}{H} \sum_{j \in f} \mu^f \gamma_j(r_j(\mu_f))\right) = 1$$

- Suppose unique solution μ^f denoted $m^f(H)$, firm f's fitting in function. Then equilbrium is find H such that

$$H = \sum_f \sum_{j \in f} h_j(r_j(m^f(H))) \equiv \Gamma(H)$$

- Unique equilibrium if $\Gamma'(H) < 1$ whenever $\Gamma(H) = H$.

- Theorem

- $\bar{\mu}_k = \lim_\infty i_{k,}$ the highest markup that product k can support.

- Suppose products ranked $\bar{\mu}_1 > \bar{\mu}_2 > ... > \mu_{N^f}$.

- Suppose have CES and heterogenous σ. Then $\iota_j(p_j) = \sigma_j$.

- Suppose H small. Then sell only lowest elasticity....