Theory of the Firm

- Where draw boundaries of the firm?
  - Large literature: firms as a system of incentives.
  - IO literature on vertical integration and antitrust
  - Literature in inequality and how what a worker is paid depends on where they work (increase in outsourcing)
- Edmans and Gabaix: Literature on managerial pay
  - Incentives for a particular type of employee
  - Issue of CEO pay connected of course to this inequality issue.

What do they do?

- Start out with management as standard factor of production T in Cobb-Douglas

$$
\begin{aligned}
V &= f(K, L, T) \\
&= K^{\alpha_K} L^{\alpha_L} T^{\alpha_T}
\end{aligned}
$$

  Suppose managers vary in endowment of T (T_i is management ability of individual i). Standard results about how in competitive equilibrium equalize ratio of K and L to T. So larger firms pay more (CEO pay linear in size).

- Assignment analysis.

  – firm $n \in [0, N]$ has size $S(n)$

– CEO $m \in [0, M]$ has talent $T(m)$

$$V = S(n) + CT(m)S(n)^{\gamma}$$

– Given market wage $w(m)$, firm of type n picks CEO m to maximize

$$\max_{m} CS(n)^{\gamma}T(m) - w(m)$$

– Make Pareto assumption on distribution on firm quality and worker quality. Figure out market clearing. Get expressions for elasticity of wage with respect to firm size.

– Can go through and talk about how primitives affect pay. But doesn't have anything about incentive problems in it. Motivates section on incentives.

Static Incentives

- Production function V(a,S,ε) for action a and size S.

$$V = S + b(S)a + \varepsilon$$

- If $b(S) = b$, then no size action.

- Pay wage $c(V)$ contingent on value.
  - Limited liability on principle $c(V) \leq V$.
  - Sometimes assume limited liability on agent $c(V) \geq 0$
  - w expected wage and $w \geq 0$.

– Objective function of agent

$$E[U] = E[u\left(v(c) - g(a)\right)]$$

allow curvature

- Program (assume risk neutral)

$$\begin{aligned} \max_{c(\cdot),a} E[V(a) - c(V(a)) &\quad : \\ E[u\left(v(c(V(a)) - g(a)\right)] &\geq w \\ a &\in \arg\max_{\hat{a}} E[u\left(v(c(V(\hat{a})) - g(\hat{a})\right)\right) \end{aligned}$$

- First best, a is observable and can contract on that. Set constant wage for efficient risk sharing. Solves

$$g'(a^*_{FB}) = b(S)$$

Holmstrom and Milgrom, "Firm as an Incentive System, AER (1994)

- Build on classic moral hazard model of Holmstrom, so actions are multi-dimensional.

- Think of an insurance agent, does he work for State Farm as an employee or is he an independent contractor agent. (see NASFA web site. http://www.nasfa.com/)

- Principal utility given actions (a_1, a_2, a_3). (a_1 sales, a_2 investment in future sales, a_3 help others around the office), ε random outcome beyond the control of the agent.

$$y = f_1 a_1 + f_2 a_2 + f_3 a_3 + \varepsilon$$

- Performance measures

$$\begin{aligned} p_1 &= a_1 + \phi_1 \\ p_2 &= a_2 + \phi_2 \end{aligned}$$

where ϕ_1 and ϕ_2 random and not directly observable (so can't directly contract on a_1 and a_2)

- Risk neutral. Agent has cost cost function c, assume quadratic

$$c(a_1, a_2, a_3) = \frac{a_1^2}{2} + \frac{a_2^2}{2} + \frac{a_3^2}{2} + \frac{1}{4}(a_1 a_2 + a_1 a_3 + a_2 a_3)$$

- Consider a contract space where the wage satisfies

$$w = s + b_1 p_1 + b_2 p_2$$

and $b_1 \in [0, f_1]$, $b_2 \in [0, f_2]$.

- Risk neutral agent picks (a_1, a_2, a_3) to maximize:

$$\max_{a_1, a_2, a_3} b_1 a_1 + b_2 a_2 - c(a_1, a_2, a_3)$$

- Can verify that this is a concave problem with solution:

$$
\begin{aligned}
a_1^*(b_1, b_2) &= \frac{5}{4.5} b_1 - \frac{1}{4.5} b_2 \\
a_2^*(b_1, b_2) &= \frac{5}{4.5} b_2 - \frac{1}{4.5} b_1 \\
a_3^*(b_1, b_2) &= -\frac{1}{4.5}(b_1 + b_2)
\end{aligned}
$$

- Note:

$$
\begin{aligned}
\frac{\partial a_i}{\partial b_i} &> 0,\ i \in \{1, 2\} \\
\frac{\partial a_i}{\partial b_j} &< 0, i \neq j
\end{aligned}
$$

- Topkis's Theorem: Let $g(x, \theta)$ be continuous supermodular function of x, θ. Let

$$X(\theta) = \operatorname{arg\,max}_x g(x, \theta)$$

with least upper bound

$$x^*(\theta) = \sup X(\theta)$$

and greatest lower bound

$$x_*(\theta) = \inf X(\theta)$$

Then

$$\begin{aligned} x^*(\theta) &\in X(\theta) \\ x_*(\theta) &\in X(\theta) \end{aligned}$$

and both $x^*(\theta)$ and $x_*(\theta)$ are nondecreasing functions of θ.

- The objective function of the principal is

$$\max_{(b_1,b_2)} v = f_1 a_1 + f_2 a_2 + f_3 a_3 - c(a_1, a_2, a_3)$$

- *Lemma*: The objective function is supermodular in $(a_1, a_2, (-f_3))$.

$$\frac{\partial v}{\partial b_1} = \sum_{i=1}^{3} (f_i - c_i) \frac{\partial a_i}{\partial b_1}$$

But $b_i = c_i$, so can substitute in

$$\begin{aligned}
\frac{\partial v}{\partial b_1} &= (f_1 - b_1) \frac{\partial a_1}{\partial b_1} + (f_2 - b_2) \frac{\partial a_2}{\partial b_1} + f_3 \frac{\partial a_3}{\partial b_1} \\
\frac{\partial^2 v}{\partial b_1 \partial b_2} &= -\frac{\partial a_2}{\partial b_1} > 0 \\
\frac{\partial v}{\partial b_1 \partial(-f_3)} &= -\frac{\partial a_3}{\partial b_1} > 0
\end{aligned}$$

- Proposition: increase $(-f_3)$ (reduce importance of helping others) then optimal b_1 and b_2 increase.

- Extend the model to put in risk aversion of agent. Then optimal b_1 and b_2 depend upon variance of the noise ϕ_1 and ϕ_2. Suppose we reduce the variance of ϕ_1 then both b_1 and b_2 increase.

- So *system* of incentives. If $(-f_2)$ is big, b_1 and b_2 are big, so a separate firm, sell emloyed, not integrated. If $(-f_2)$ small, have an integrated firm.

- Only problem. Suppose increase f_1. Then increase b_1 but decrease b_2. Ooops....

The Holdup Problem

Property Rights and the Nature of the Firm

- Williamson, Transactions Costs Economics
  - Electric Power Plant and Mine (Joskow tables below)
    - Case 1: complete contracting (or existence of competition to prevent holdup). Suppose get high-powered incentives from Nonintegration. Then get nonintegration.
    - Case 2: imcomplete contracting plus not alternatives
      - relationship-specific investment (examples....)
      - ex post costs from haggling (Big enough, solve problem with VI)

- Grossman and Hart (1986), "Costs and Benefits of Ownership: A Theory of Vertical and Lateral integration," Keep incomplete contracting and relationship-specific investment. To this add:

  – math

  – focus on ex ante investments (Nash Bargaining so no ex post problems, no haggling)

  – Claim that when VI, don't solve the problem. Still have inventive issues within the firm. So VI just assigns residual rights of control.

    (Let me make my comment and then move on.)

(This is Grossman and Hart (1986), use notation from Acemoglu, Aghion, Griffight, Zilibotti (2010))

- Two agents, supplier and producer and one asset each. Example: coal mine (supplier) and electric power (producer). Coal mine has a device and electric power company has a device.

- Assign residual rights of control: who gets to walk away with an asset.

- Supplier and Buyer both make investments that are not contractable but can choose and organziational form and make transfers.

- Organization form $z \in \{VIB, NI, VIF\}$, where VIB is backward vertical integration where supplier is employed by the producer, VIF forward VI, ...(If VIB means than the producer can walk away with both assets.)

- Let $T_i(z)$ be the transfer to party i given organizational form z.

Timing

- Producer offers an organizational form $z \in \{VIB, NI, VIF\}$ and transfers $Z_P(z)$, $Z_S(z)$ (could be negative) such that

$$T_P(z) + T_S(z) = 0.$$

- Supplier decides whether to accept or reject offer. If not accepted, get payoffs O_P^{NI}, O_S^{NI} defined below. Otherwise, P and S simultaneously pick investments e_P and e_S.

- S and P bargain over the division of the revenue, Nash bargaining (will focus on case where surplus share is $\frac{1}{2}$).

- Technology

$$F(x_S, e_P, e_S) = \phi x_S(pe_P + se_S + 1) + (1 - \phi)(pe_P + 1)$$

  - x_S means the supplier provides the customized (relationship-specific) input.
  - parameters p and s govern relative importance of investments by the two parties
  - ϕ is the share of inputs accounted for by the supplier
  - Simplify by having no complementarities

- Costs of investment

$$\begin{aligned} \Gamma_P(e_P) &= \frac{1}{2}e_P^2 \\ \Gamma_S(e_S) &= \frac{1}{2}\phi e_S^2 \end{aligned}$$

With disagreement, parties get outside options

- Backward Vertical Integration (VIB) Producer owns the assets

  $O_S^{VIB}(e_P, e_S) = 0$ and $O_P^{VIP}(e_P, e_S) = F(x_S = 1, e_P, (1-\lambda)e_S)$
  where the lack of cooperation of the supplier results in a decay of $\lambda \in [0, 1)$.

- *NonIntegration*

$$\begin{aligned} O_S^{NI}(e_P, e_S) &= \theta\phi(se_S + 1) \\ O_P^{NI} &= F(x_S = 0, e_P, e_S) = (1 - \phi)(pe_P + 1) \end{aligned}$$

where $\theta \in [0, 1)$ is an inverse measure of how much the supplier loses if she sells input outside of the specific relationship.

- Forward VI (VIF)

$$O_S^{VIF}(e_P, e_S) = F(S_S = 1, (1-\lambda')e_P, e_S)$$
$$O_P^{VIF} = 0$$

where $\lambda' \in [0, 1)$ is fraction of producer's investment lost if disagreement

Nash Bargaining

$$y_i^z(e_P, e_S) = O_i^z(e_P, e_S) + \frac{1}{2}\left[F(x_S = 1, e_P, e_S) - O_P^z(e_P, e_S) - O_S^z(e_P, e_S)\right]$$

- Key point: party's share is increasing in own outside option.

- Utility of party i

$$U_i^z(y_i(e_P, e_S), e_i) = y_i^z(e_P, e_S) - \Gamma_i(e_i) + T_i(z)$$

- Now determine on the equilibrium path actions and revenues $(z^*, T_P^*, T_S^*, e_P^*, e_s^*, y_P^*, y_S^*)$

- Total surplus

$$S^z = U_S^z(y_S^z(e_P^*(z), e_S^*(z)), e_S^*(z)) + U_P^z(y_P^z(e_P^*(z), e_S^*(z)), e_P^*(z))$$

$$= \quad F(x_S = 1, e_P^*(z), e_S^*(z)) - \Gamma_P(e_P^*(z)) - \Gamma_S(e_S^*(z))$$

- Subgame perfect equilibrium picks the organization form that maximizes the surplus

$$S^{z^*} \geq S^z, \; z \in \{VIB, NI, VIF\}$$

Now Determine Equilibrium Investments Given Organization Choice

- Nash equilibrium of investment game

$$
\begin{aligned}
e_P^*(z) &= \max_{e_P} \{y_P^z(e_P, e_S^*(z)) - \Gamma_P(e_p)\} \\
e_S^*(z) &= \max_{eS} \{y_S^z(e_S^*(z), e_S) - \Gamma_S(e_S)\}
\end{aligned}
$$

- Solution (put on board)

$$
\begin{aligned}
e_P^*(VIB) &= p \text{ and } e_S^*(VIB) = \frac{\lambda}{2}s \\
e_P^*(NI) &= \left(1 - \frac{\phi}{2}\right)p \text{ and } e_S^*(NI) = \frac{1+\theta}{2}s \\
e_P^*(VIF) &= \frac{\lambda'}{2}p \text{ and } e_S^*(VIF) = s
\end{aligned}
$$

- Now

$$
\begin{aligned}
e_P^*(VIP) &> e_P^*(NI) > e_P^*(VIF) \\
e_S^*(VIP) &< e_S^*(NI) < e_S^*(VIF)
\end{aligned}
$$

- Look at Social Planner

$$
\begin{aligned}
& \max F(x_S, e_P, e_S) - \Gamma_P(e_p) - \Gamma_S(e_S) \\
= & \max \phi(pe_P + se_S + 1) + (1 - \phi)(pe_P + 1) - \frac{1}{2}e_P^2 - \frac{1}{2}\phi e_S^2
\end{aligned}
$$

- – The FONC

$$
\begin{aligned}
p - e_P &= 0 \\
s - e_S &= 0
\end{aligned}
$$

So $e_P^{**} = p$ and $e_S^{**} = s$.

- Now look at NI.
  - Problem of Producer. Given e_S, solve: (note at this point transfer is irrelevant)

$$\begin{aligned}
& \max_{e_P} O_P^z(e_P, e_S) + \frac{1}{2}\left[F(x_S = 1, e_P, e_S) - O_P^z(e_P, e_S) - O_S^z(e_P, e_S)\right] \\
= & \max_{e_P} (1-\phi)(pe_P + 1) + \frac{1}{2}\left[\begin{array}{c} \phi(pe_P + se_S + 1) + (1-\phi)\,(pe_P + 1) \\ -(1-\phi)(pe_P + 1) - \theta\phi(se_S + 1) \end{array}\right] \\
= & \max_{e_P} (1-\phi)(pe_P + 1) + \frac{1}{2}\left[\phi(pe_P + se_S + 1) - \theta\phi(se_S + 1)\right] - \frac{1}{2}e_P^2
\end{aligned}$$

The FONC is

$$(1-\phi)\,p + \frac{1}{2}\phi p - e_P$$

So

$$e_P = \left(1 - \frac{1}{2}\right)\phi p$$

(note if change the bargaining share above $1/2$ to producer, then of course it will produce more.

– Problem of Supplier

$$
\begin{aligned}
& \max_{e_P} O_P^z(e_P, e_S) + \frac{1}{2}[F(x_S = 1, e_P, e_S) - O_P^z(e_P, e_S) - O_S^z(e_P, e_S) \\
= & \max_{e_P} \theta\phi(se_S + 1) + \frac{1}{2}\left[\begin{array}{c} \phi(pe_P + se_S + 1) + (1-\phi)(pe_P + 1) \\ -(1-\phi)(pe_P + 1) - \theta\phi(se_S + 1) \end{array}\right] - \frac{1}{2} \\
= & \max_{e_P} \theta\phi(se_S + 1) + \frac{1}{2}[\phi(pe_P + se_S + 1) - \theta\phi(se_S + 1)] - \frac{1}{2}\phi e_S^2
\end{aligned}
$$

$$
\begin{aligned}
& \theta\phi s + \frac{1}{2}(1-\theta)\phi s - \phi e_S \\
e_S \;=\; & \frac{1+\theta}{2} s
\end{aligned}
$$

- But now look at how problem changes when go with VIB (so producer has all residual rights of control). Problem of producer:

  $$= \max_{e_P} \phi(pe_P + s(1-\lambda)e_S + 1) + (1-\phi)(pe_P + 1) + \frac{1}{2}[\phi(pe_P + se_S + 1) - \phi x$$

  The FONC is

  $$e_P = p$$

  But what about supplier? Outside option is zero, gets half of the surplus of gains from trade.

  $$\max_{e_S} = \frac{1}{2}[\phi(pe_P + se_S + 1) - \phi x_S(pe_P + s(1-\lambda)e_S + 1)] - \frac{1}{2}\phi e_S^2$$

  the FONC is

  $$\frac{1}{2}s\lambda = e_S$$

- etc...

Main Result

- Proposition 1: There exists $\underline{r}$, $\bar{r}$, and $\hat{r}$ such that

(1) If $\underline{r} < \bar{r}$, then see picture on the board regarding how what happens depends upon the ratio $\frac{p}{s}$

And

$$\frac{\partial \bar{r}}{\partial \phi} < 0, \frac{\partial \underline{r}}{\partial \phi} > 0 \text{ (robust?)}$$
$$\frac{\partial \bar{r}}{\partial \theta} > 0 \text{ and } \frac{\partial \underline{r}}{\partial \theta} < 0.$$