Does Home Market Size Matter for the Pattern of Trade?[*]

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November 17, 2003

Abstract

Does home market size matter for the pattern of trade? Krugman started the literature, showing it does matter. Davis overturned his result, arguing that an assumption of convenience—transport costs only for the differentiated goods—conveniently obtained the result. Here we relax another persistent assumption of convenience—*two* industry types differentiated only by the degree of scale economies—and find that market size reemerges as a relevant force in determining industrial structure.

(JEL F0, 01, R1).

1 Introduction

A famous result due to Krugman (1980) is that increasing-returns industries will tend to concentrate production within large markets.[1] If a large country begins to trade with a small country, the large country will shift its industrial structure towards the production of increasing-return-type goods and export these to the small country. The small country, in turn, will shift its structure towards constant-return-type goods and export these to the large country. For example, if trade barriers are reduced between a small country such as New Zealand and a large country such as Japan, New Zealand will shift away from a scale-economy industry such as autos towards a constant-returns sector such as wool. Krugman's result is important, because the success of an economy is thought to be related, in part, to its industrial mix; New Zealand is unlikely to be rich completely specializing in wool.

In a recent paper, Davis (1998) reports a striking finding that overturns Krugman's result. In the original Krugman theory, transportation cost is assumed to be positive for the increasing-returns sector and zero for the constant-returns sector. The absence of transportation cost for the constant-returns sector was regarded as an innocuous simplifying assumption. Davis shows first that this assumption is implausible and, second, that without this assumption Krugman's result is overturned. He shows that when transportation cost is the same for the two sectors, trade has *no* effect on a country's production mix between scale-economy and constant-returns sectors. To illustrate, consider the example of New Zealand and Japan and assume that there are no differences in Ricardian comparative advantage between the two countries or differences in factor proportions. The only difference is country size. Suppose the transportation cost of shipping wool is as high as the cost of shipping automobiles and automobile parts. Then opening up trade will not lead New Zealand to shift production from autos to wool; there is no shift in the small country towards the constant-returns sector.

In this paper we revisit the issue and our results breathe new life into the idea that country size matters. We find that a seemingly innocuous simplifying aspect of the Krugman and

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[1]There is a large subsequent literature. See for example, Helpman and Krugman (1985) and Krugman and Venables (1995).

Davis models is actually crucial. These papers assume there are two types of industries: one type is pure constant returns to scale and the other type has a fixed degree of scale economies. Rather than have two industry types, our model allows for a range of industry types that vary in the degree of scale economies. We follow Davis in assuming that the transportation cost is the same across industries. We find that industries with a low degree of scale economies are never traded and thus extend the Davis result to our structure. Goods with a medium and high degree of increasing returns are traded. Our key result is that the pattern of trade *does* depend on country size. The small country exports the medium increasing-returns products, the large country exports the high increasing-returns products. Think about food processing as having moderate increasing-returns as opposed to wool (constant returns) and autos (high increasing returns). We find that opening of trade between New Zealand and Japan causes New Zealand to shift production into food processing out of autos, to export food processing and import autos, all the while leaving the wool sector alone.

To allow for variations in the degree of scale economies across industries, we are forced to step out of the standard Dixit-Stiglitz model of monopolistic competition, the workhorse model of this literature. This structure features a fixed cost and constant marginal cost. In the zero profit equilibrium, fixed costs as a share of revenues equals the markup share of price. This share depends upon the elasticity of demand but is independent of the technology parameters. Thus, increasing the fixed cost in a particular industry has no effect in increasing the equilibrium fixed cost share in that industry; the share remains constant. In fact, in the benchmark case where demand for final (composite) goods is Cobb-Douglas, a change in the fixed-cost parameter of a particular industry has zero effect in all countries on the equilibrium allocation of inputs across industries.[2] A particular concern is that as the fixed-cost parameter goes to zero, there is no sense in which the outcome gets close to the constant returns case. It is plainly the case that varying the fixed-cost parameter in the Dixit-Stiglitz world is not an interesting exercise.

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[2] If the fixed cost doubles in an industry, in the new equilibrium there are half as many firms in that industry in each country as before and each firm produces twice as much output. Thus zero profit is maintained. The Cobb-Douglas assumption leaves industry revenue fixed, so the market clearing condition continues to hold.

We develop a structure to conduct our analysis that departs in two ways from the standard Dixit-Stiglitz world. First, the set of possible differentiated products is fixed and as a result there may be multiple entry, i.e., oligopoly, for any particular product. (With Dixit-Stiglitz there is at most one producer of each product.) Entry in our model is at the *intensive* margin (more firms in the same industry) rather than at the Dixit-Stiglitz-like *extensive* margin (more single-firm industries). Second, while the technology features an initial range of increasing returns, there exists a finite *minimum efficient scale* beyond which average cost is constant. (With Dixit-Stiglitz there is a fixed cost and constant marginal cost so the minimum efficient scale is infinity.) These two departures from the standard analysis complement each other; i.e., once we have made the first departure it helps considerably to make the second departure as well. The assumption that constant returns are reached at a large enough scale implies that in large markets the perfect competition outcome can be sustained. This allows for a structure that is quite tractable but rich enough to capture the importance of scale economies in small markets, given the initial range of increasing returns to scale.

We have strong conceptual grounds for adopting our *intensive-margin* structure over the usual *extensive-margin* structure of Dixit-Stigliz.[3] When the extensive margin is the relevant margin, there is no sense that small market size is a problem for obtaining production efficiency. Production size can be set to minimum efficient scale—small country size just means there will be relatively few products. If population size decreases, we can operate along the extensive margin shutting down products, and maintain efficient scale on the remaining products. But when the intensive margin is the relevant margin, a decrease in county size tends to reduce output of a given product. To the extent output falls below minimum efficient scale, this becomes a problem for production efficiency. This is the key force in our analysis.

In our model, products vary in the minimum efficient scale of production. Our question is how the structure of production and trade varies with a product's degree of scale economies. We begin our analysis by rederiving Krugman's and Davis' results in a special case of our alternative formulation. In this special case, industries can be one of two minimum efficient

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[3]See Holmes (1999) for a further discussion of the extensive versus the intensive margins.

scale types. The first type all have minimum efficient scale equal to zero, i.e. constant returns. The second type all have a minimum efficient scale equal to an identical high level. For the Krugman case, we assume transportation cost is zero for the constant returns industry. We show that when a small country trades with a large country, the small country tends to specialize in the constant returns sector, while the large country specializes in the increasing returns goods. Thus, we rederive Krugman's home market result, though there are some subtleties as we discuss in the main text. For the Davis case, we assume transportation cost is equal for the two sectors. We rederive Davis' result that there is no trade in the constant returns sector. All trade is in the increasing returns sector. Reductions in trade barriers have no effect on the structure of production.

The main body of our analysis considers the general case where there is a continuum of industry types ranging from zero minimum efficient scale to high minimum efficient scale. We find that the equilibrium with trade is characterized by two cutoffs that divide industries into low, medium, and high returns to scale. Goods with low minimum efficient scale are not traded. Goods in the high minimum efficient scale range are not produced in the small country. The small country pays for imports of high range goods with exports of medium range goods. Thus, country size does not affect trade *between* the sector of goods with low scale economies and the sector with medium and high scale economies, analogous to Davis. But here country size does affect industrial composition *within* the latter sector.

In addition to Krugman (1980) and Davis (1998), another notable contribution in this literature is Amiti (1998). Amiti (1998) uses the standard Dixit-Stiglitz structure to determine how the pattern of trade varies with country size. One of her findings is that when sectors vary in transportation costs, smaller countries will tend to specialize in lower transportation cost goods.[4] She also considers what happens when sectors vary in the elasticity of substitution demand parameter and finds that country size matters here as well, although there is no general result about the direction of the relationship. In addition to using a different modeling structure, our paper differs from Amiti in the key comparative statics exercise that is considered. Here we are interested in how the pattern of trade varies across

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[4]In a related paper (Holmes and Stevens, 2002) we use a variant of the model in this paper to further explore the effects of differences in transportation costs alone on the structure of production.

industries that differ in technology, or more specifically, the degree of scale economies. In Amiti, the degree of scale economies is fixed across industries.

2 The Model

We begin by describing the environment and then define equilibrium.

2.1 The Environment

There is a continuum of industries indexed by i on the unit interval, $i \in [0, 1]$. Let $q(i)$ denote the consumption of good i. Consumers have Cobb-Douglas preferences,

$$U = \int_0^1 \ln(q(\tilde{i}))d\tilde{i}. \tag{1}$$

Labor is the only input to production, and it is used by firms to complete two tasks. Let ℓ_1 and ℓ_2 denote the amount of labor allocated to each task, and suppose total output is Cobb-Douglas,

$$q = \lambda \ell_1^\alpha \ell_2^{1-\alpha}$$

with λ normalized to

$$\lambda \equiv \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + \left(\frac{1-\alpha}{\alpha}\right)^\alpha$$

and $\alpha \in [\frac{1}{2}, 1)$.[5] Under this technology, the cost minimizing way to produce q units of output requires q units of labor, with a share α allocated to task 1 and a share $1 - \alpha$ allocated to task 2. However, we impose an indivisibility constraint on the quantity of labor used in task 1,

$$\ell_1 \geq \alpha\theta, \tag{2}$$

where the parameter $\theta \in \left[0, \overline{\theta}\right]$ denotes the minimum efficient scale of the industry. Therefore, given θ, the labor requirement for producing q units of output is

$$\ell = c(q, \theta) = \begin{cases} \alpha\theta + (1-\alpha)\,\theta^{\frac{-\alpha}{1-\alpha}} q^{\frac{1}{1-\alpha}}, & \text{if } q < \theta \\ q, & \text{otherwise} \end{cases} \tag{3}$$

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[5]The qualitative results hold for $\alpha \in (0, \frac{1}{2})$ as well, but restricting attention to $\alpha \geq \frac{1}{2}$ simplifies by reducing the range of cases that need to be considered.

The first term of the cost function (3) is the fixed cost of the units employed at task 1. The second term is the variable cost of the units employed at task 2. For $q > \theta$, the indivisibilty constraint is not binding and both the average cost and the marginal cost of another unit of output are constant at one labor unit. But for $q < \theta$, the constraint is binding, as the firm has to employ $\alpha\theta$ units in task 1 even though it would prefer to use less; in this case, average cost is greater than one unit and falling, while marginal cost is less than one and rising. For example, in the case where $\alpha = \frac{1}{2}$, average cost and marginal cost reduce to

$$
\begin{aligned}
AC(q,\theta) &= \frac{c(q,\theta)}{q} = \frac{1}{2}\left(\frac{q}{\theta} + \frac{\theta}{q}\right) > 1, \quad q < \theta \\
MC(q,\theta) &= \frac{\partial c(q,\theta)}{\partial q} = \frac{q}{\theta} < 1, \quad q < \theta.
\end{aligned}
\tag{4}
$$

While industries all have the same α, they vary in their minimum efficient scale, $\theta(i)$. Let $f(\theta)$ denote the density of industries of type θ in the economy. Since the total measure of industries is one,

$$
\int_0^{\bar{\theta}} f(\theta)d\theta = 1.
$$

There are two countries, one small and one large. Let L be total labor in the small country and L^* be total labor in the large country.

There is an iceberg transportation cost to ship goods from one country to the other. Assume that for industry i, $\tau(i) \geq 1$ units must be shipped for every one unit that arrives. The main focus will be the case where τ is constant across i and strictly greater than 1.

2.2 Equilibrium

Let w be the wage in the small country and let Π be the aggregate profit arising from all firms located in the small country. Then income in the small country is

$$
I = wL + \Pi.
$$

Analogously define income I^* in the large country. Let $A \equiv (w, I, w^*, I^*)$ denote the set of aggregate variables.

The structure of behavior in this model is a combination of competition and oligopoly. All individuals are small relative to the economy as a whole so they take the aggregate

variables in A as fixed. Within any given industry, prices are set by oligopolist firms who take the aggregate variables A as given. Since demand in each industry is unit elastic, prices set in other industries do not matter for the behavior of this industry, so the vector A with wages and aggregate income is all that is needed.

There is a two stage game that is played in every industry i. In stage 1, firms simultaneously make entry decisions. Let n_i and n_i^* be the number of firms in industry i that commit to locating in the small and large countries. At the point of entry, firms commit themselves to satisfying the indivisibility constraint, $\ell_1 \geq \alpha\theta(i)$. In stage 2, firms simultaneously set prices, competing in a Bertrand fashion.

Our discussion of equilibrium has three parts. First we discuss the equilibrium of stage 2, the price-setting stage. Next we discuss the equilibrium of stage 1, the entry stage. Finally, we discuss the economy-wide equilibrium.

2.2.1 The Price-Setting Stage

In stage 2, the entry decisions (n, n^*) have already been determined (To simplify notation, the subscript for industry i is implicit here). Firms observe entry as well as the aggregate variables $A \equiv (w, I, w^*, I^*)$. Firms can price discriminate and set a different price in each market. Let p^L and p^E be the local and export prices set by a firm in the small country. The *delivered* export price is τp^E. Let q^L and q^E denote a small country firm's output for the local and export markets and let $q = q^L + q^E$ be the total output. Analogously define p^{L*}, p^{E*}, q^{L*}, q^{E*}, and q^* for a firm in the large country.

We restrict attention to symmetric equilibria where firms that locate in the same industry and the same country behave symmetrically. The price set by any given firm in stage 2 must maximize the firm's profits, taking as given the prices set by the other firms and the aggregate variables. In a separate appendix available on the web we show the following about equilibrium in the price subgame.[6]

Lemma 1 *Given any A and entry vector (n, n^*), a symmetric equilibrium to the price subgame exists. If there are at least two firms in a given country, $n \geq 2$ or $n^* \geq 2$, there is a*

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[6]The appendix can be accessed at http://www.econ.umn.edu/~holmes/research.html.

unique symmetric equilibrium of the price subgame. If each location has a single firm, $n = 1$ and $n^* = 1$, and if there is an equilibrium with trade, then the price subgame has a unique equilibrium.

When $n = 1$ and $n^* = 1$, there may exist a continuum of equilibria with no trade. For this case take some arbitrary equilibrium selection. For all other cases, the symmetric equilibrium is unique. Let $\pi(n, n^*)$ and $\pi^*(n, n^*)$ denote profits for a firm in the small and large country. Note these are implicitly functions of the industry i and the aggregate vector A. It is immediate that

$$
\begin{aligned}
\pi(1, 0) &= I + I^* \\
\pi(n, n^*) &\leq 0, \, n \geq 2.
\end{aligned}
$$

To understand the first statement, observe that if a firm has a monopoly in the small country and it has no competitor in the large country, it can extract the entire industry revenue as profit by making the price arbitrarily large (recall that demand is unit elastic). To understand the second statement, observe that if there are two firms at the same location, Bertrand competition drives price equal to marginal cost, which is no greater than average cost, so profits must be non-positive.

2.2.2 The Entry Stage

Turn now to stage 1. In an equilibrium, all entrants receive nonnegative profit and there is no possibility of profit from further entry. Formally, equilibrium entry (n, n^*) must satisfy

$$
\begin{aligned}
\pi(n, n^*) &\geq 0 \\
\pi(n + 1, n^*) &\leq 0
\end{aligned}
$$

with analogous conditions on profits holding in the large country. Note these conditions do not require that there be zero profit in all industries. If an industry has positive profits as a monopoly but has negative profits as a duopoly, the industry will have positive profits in equilibrium.

In general there may be multiple equilibria in the entry game. To see this, consider a simple case where there is only a single country. If one firm enters it has a monopoly and

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it extracts the income I. If two firms enter, price is driven to marginal cost. Suppose the country is large enough so that the output of both firms exceeds the minimum efficient scale. Monopoly is an equilibrium here because if a second firm enters, it has zero profit, so there is no strict positive incentive to enter. But duopoly is also an equilibrium because both firms earn zero profit. For analogous reasons, there can be multiple equilibria to the entry game with two countries. In cases of multiple equilibria, we prefer to focus on equilibria where entry forces profits to zero. Hence we use the following elimination criterion.

Elimination Criterion 1. Suppose there is an equilibrium (n, n^*) to the entry game such that $\pi(n, n^*) > 0$. Suppose also that $\pi(n + 1, n^*) = 0$ and $\pi^*(n + 1, n^*) \geq 0$, which implies that $(n+1, n^*)$ is an equilibrium. Eliminate (n, n^*) from the set of equilibria to the subgame.

Even among the set of equilibria where all firms obtain zero profit, there may exist multiple equilibria which differ by price. In particular, if I^* is large enough, if I is small enough, and if $\tau w^* \leq w$, then we can construct an equilibrium with multiple firms in one location and none in the other, as well as a second equilibrium where this is reversed. These equilibria can be Pareto ranked. We eliminate the Pareto inferior equilibrium.

Elimination Criterion 2. Suppose there are two equilibria (n, n^*) and $(\tilde{n}, \tilde{n}^*)$ to the entry game and that all profits are zero in both. If all consumers weakly prefer $(\tilde{n}, \tilde{n}^*)$, and some consumers strictly prefer it, then eliminate (n, n^*) from the set of equilibria to the subgame.

In our definition of equilibrium to the entry game, we restrict attention to equilibria that are not eliminated by criterion 1 or 2.

2.2.3 Economy-Wide Equilibrium

We turn now to a definition of equilibrium for the economy as a whole. It consists of a list of aggregate variables $A = (w, I, w^*, I^*)$ and entry decisions (n_i, n_i^*) for each industry i, and price and quantity outcomes $(p_i^L, p_i^E, p_i^{L*}, p_i^{E*})$ and $(q_i^L, q_i^E, q_i^{L*}, q_i^{E*})$ such that:

(1) Consumer demand maximizes utility.

(2) Given A, the entry and pricing decisions are a subgame perfect equilibrium of the oligopoly game that are not eliminated by criterion 1 or 2.

(3) The income taken as given for the oligopoly games equals the total income in the economy, $I = wL + \Pi$ and $I^* = wL^* + \Pi^*$.

(4) Supply equals demand in the labor market.

3 The Krugman and Davis Results

This section rederives the results in the literature for our alternative model. Instead of having a continuum of different industries, we follow the earlier literature by having two industry types, type a (agriculture) and type m (manufacturing). Assume that $\theta_a = 0$ and that $\theta_m > 0$. Thus, agricultural industries have constant returns to scale while there is a range of increasing returns with manufacturing industries. Suppose that all industries $i \in [0, f_a]$ are type a and all industries $i \in (f_a, 1]$ are type m. Therefore, a fraction f_a of industries are type a and a fraction $f_m = 1 - f_a$ are type m. Let τ_a and τ_m denote the iceberg transportation cost parameter for each industry.

3.1 Krugman

In the Krugman case there is no transportation cost for agricultural goods so $\tau_a = 1$. Manufacturers do have a transportation cost so $\tau_m > 1$. We derive conditions under which a Krugman-like result is obtained in which the small country completely specializes in agriculture.

Proposition 1 *Suppose (i) $L < \frac{\theta_m}{2}$, (ii) $L^* > 2\theta_m$, and (iii) $f_m L < f_a L^*$. Then there exists an equilibrium such that*

- $w = w^* = 1$.

- $n^*(i) = 2$, *for all i. (There is multiple entry in the large country for all goods.)*

- $n(i) = 2$, *for $i \in [0, f_a]$, $n(i) = 0$, for $i \in (f_a, 1]$. (In the small country, there is multiple entry for each agricultural good, but zero entry for each manufactured good.)*

- $p_a^* = p_m^* = 1$.

- $p_a = 1$, $p_m = \tau_m$.

Conditions (i) and (ii) ensure that the small country is sufficiently small and that the large country is sufficiently large. Condition (iii) ensures that agriculture is a large enough share of the budget to make it possible for the small country to completely specialize in agriculture. (Note that if we were to assume $f_a \geq \frac{1}{2}$, then (i) and (ii) would imply (iii).)

We sketch the proof. The agricultural sector is constant returns. Given duopoly entry for all agricultural products in both countries, Bertrand competition ensures price equals marginal cost. Given duopoly entry for all manufacturing products in the large country, price equals marginal cost there as well. The assumption $L^* > 2\theta_m$ guarantees that the two firms will produce above minimum efficient scale, so marginal cost will equal 1 and that will equal the price. The large country is in a constant returns world and its prices are unaffected by trade with the small country. Consumers face a price of 1 for both kinds of goods. Consumers in the small country pay 1 for the agricultural goods but have to pay τ_m to import manufactured goods.

The only thing that remains to be checked is the profitability of entry by a manufacturing firm in the small country. If a firm enters, its local sales will equal

$$
\begin{aligned}
q^L &= \frac{I}{p^L} \\
&= \frac{L}{\tau_m}.
\end{aligned}
$$

The first equality follows from the Cobb-Douglas assumption; the second substitutes in $I = L$ (wage is 1 and there are no profits) and the fact that $p^L = \tau_m$ (since the firm will limit price, matching the delivered import price). In the export market, the entrant would set an export price of $p^E = \frac{1}{\tau_m}$, so that with transport costs it would match the price offered by the local firms in the large country. At this price, the entrant could choose to export any quantity q^E that did not exceed τL^*. (L^* is the local demand in the large country at the local price of one; if the small-country firm exports τL^* units then L^* units are delivered.) The problem of an entrant is

$$
\begin{aligned}
\pi &= \max_{q^E \in [0, \tau L^*]} p^L q^L + p^E q^E - c(q^L + q^E) \qquad (5) \\
&= \max_{q^E \in [0, \tau L^*]} L + \frac{1}{\tau_m} q^E - c\left(\frac{L}{\tau_m} + q^E\right).
\end{aligned}
$$

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Note that even though the export price is less than the minimum average cost of 1, the entrant still might want to export because its marginal cost will be less than 1. But condition (i), $L < \frac{\theta_m}{2}$, is a sufficient condition for the maximized profit to be strictly negative, so there is no incentive to enter. It is clear that profit will be negative if L is small. When the local market is close to zero, virtually all the entrant's revenues will come from exports. But the export price is below minimum average cost, so the entrant cannot break even with exports alone.

Proposition 1 confirms the Krugman intuition for this model when a small country is paired up with a large county. A subtlety arises when a *medium*-sized country is paired with a large country. Here a reversal is possible. We illustrate this with an example. The smaller of the two countries will be called the "medium" country because the size is intermediate between $\frac{\theta_m}{2}$ and θ_m.

Example. Suppose the parameters are such that

1. $\frac{\theta_m}{2} < L < \theta_m$,

2. $3\theta_m < L^*$,

3. $\tau_m^2 < \left[2 - \frac{\theta_m}{L}\right]^{-1}$,

4. $\left[\frac{\theta_m}{L} - 1 + \frac{1}{\tau_m^2}\right]^{-1} < f_m$.

Then there exists an equilibrium where the medium-sized country completely specializes in manufacturing, and the wage in the medium country exceeds the wage in the large country, $w > w^*$. A subset of the manufactured products are produced in the medium country by a monopolist (i.e. $n(i) = 1$ for i in some subset of $(f_a, 1]$). The remaining manufactured products are not produced in the medium country, nor are agricultural products produced, $n(i) = 0$, $i \in [0, f_a]$. The large country has duopoly producers for all goods.

In this example, there are monopoly producers of manufactured goods in the medium country, but they get zero profit (the wage goes above 1 to drive this profit to zero). The medium country exports some manufactured goods and imports the other manufactured goods, as well as all the agricultural goods. It is easy to understand why this happens. As we increase L, the size of the smaller country, eventually there will be production of

some manufactured goods in the smaller country. But for a medium-sized country, the local demand might be on the short side. By exporting, the medium-sized country can get to efficient scale. This does not adversely affect producers in the large country because they are already well above efficient scale.

3.2 Davis

Davis considers the case where agricultural and manufactured goods have the same transportation cost, $\tau_a = \tau_m = \tau > 1$. He shows in his model that there can never be equilibrium trade in agricultural goods. The intuition of his result is quite simple. Suppose the small country were to export agricultural goods to the large country. The small country must then have the lowest cost to produce the agricultural good, i.e., $\tau w \leq w^*$. Since the large country imports agricultural goods, and because there can be no two-way trade in agriculture, it must be that the large country is exporting manufactured goods. But now suppose a manufacturer from the large country shifts production to the small country. The costs of serving the consumers in the large country will not increase, as the wage advantage more than compensates for the additional transportation costs. But the costs of serving consumers in the small country go down because of the wage advantage and the fact that transport costs are now avoided. So profit strictly increases, contradicting the optimality of the manufacturer locating in the large country.

Extending his result to our model involves some complications because our technology is different and because we have an oligopoly for any particular differentiated product as opposed to monopoly. Nonetheless, the same basic argument applies here. We state our formal proposition here and relegate the proof to the separate appendix.

Proposition 2 *With equal transportation costs, $\tau_a = \tau_m = \tau > 1$, there is no equilibrium under the Pareto dominance selection criterion with trade in agricultural goods. In both countries, the labor share in manufacturing equals f_m, so the industrial structure is the same.*

4 Our Continuum Model

We now return to our general model with a continuum of industry types θ on the range $[0, \overline{\theta}]$. We assume, like Davis, that the transportation cost parameter is a constant τ for all industries.

From the discussion of the Krugman model, we know that complications arise when the two countries are relatively close in size (recall the discussion of the case where a medium-sized country trades with a large country). To sharpen the analysis we focus on the case where the smaller country is quite small, and the larger country is quite large. The upper bound on the small country size is

$$L < \int_0^{\overline{\theta}} \alpha \theta f(\theta) d\theta. \tag{6}$$

Under this bound, the total labor in the small country is insufficient to pay the fixed cost of opening positive production in all of the industries. It is worth noting that we later discuss numerical examples that do not impose (6), and we obtain the same characterization as in our formal proposition below. The lower bound on the large country size is

$$L^* > 3\overline{\theta}. \tag{7}$$

This assumption implies that the large country is big enough without trade to support three firms above minimum efficient scale in all the industries. By making the market big enough for three firms, we ensure that if there is a firm in the small country producing the good, there is still sufficient demand in the large country to support two firms (which we want to sustain the Bertrand outcome in the large country).

Define a *Large Country Perfect Competition Equilibrium* to be one where $n_i^* = 2$ for all industries i; i.e., there are two firms represented in every industry in the large country. In this section we restrict attention to equilibria in this class.

Proposition 3 *There exists a Large Country Perfect Competition Equilibrium. This equilibrium is characterized by autarchy in goods with low scale economies; small-country exports in goods with intermediate scale economies; and large-country exports in goods with high scale economies.*

The analysis underlying our proposition has two parts. First, we characterize how the market structure (number of firms) varies with θ.. Second, we show how the trade structure varies with θ. Below we sketch our argument and put the formal proof in the appendix.

4.1 Market Structure

The market structure in the small country is characterized by two cutoffs, $\hat{\theta}_1 < \hat{\theta}_0$. For industries with θ below $\hat{\theta}_1$ there are two firms setting price equal to marginal cost à la Bertrand. Industries between $\hat{\theta}_1$ and $\hat{\theta}_0$ have a single firm. Industries above $\hat{\theta}_0$ have zero firms. The cutoffs are illustrated in Figure 1.

Our formal characterization of the equilibrium market structure proceeds in four steps. These steps characterize the market structure and show the existence of an equilibrium.

Step one is the observation that, in any equilibrium, the small-country wage w satisfies the following bounds that depend upon the large-country wage and the transportation cost parameters,

$$\frac{1}{\tau}w^* < w < \tau w^*. \tag{8}$$

The intuition for these bounds is analogous to the discussion of the Davis result in Section 3.2. We know there must exist some producer in the large country that exports to the small country because condition (6) rules out the possibility of an equilibrium with complete autarky. If $w \leq \frac{1}{\tau}w^*$, then such a firm could shift production to the small country and strictly reduce costs, contradicting equilibrium behavior. The costs of shipping output back to the big country would be more than offset by the wage differential and the transportation costs of output meant for the small country would be completely eliminated. Analogously, we can show $w \geq \tau w^*$ leads to a contradiction.

Step two takes the aggregate state $A = (w, I, w^*, I^*)$ as given and determines the equilibrium to the oligopoly game for each type of industry θ. We have set $w^* = 1$ as the numeraire. In a Large Country Perfect Competition Equilibrium there are zero profits in the large country so total income there is wage income, $I^* = L^*$. Since the small country income equals $I = wL + \Pi$, we can represent the aggregate state by (w, Π), the small country wage and the small country aggregate profit.

Let $\pi(\theta, w, \Pi)$ be the profit of a monopolist in the small country given the industry scale parameter θ and the aggregate state (w, Π). Analogous to the earlier formula (5), it equals local revenues plus export revenues minus total cost at the optimum export level,

$$\pi(\theta, w, \Pi) = \max_{q^E \in [0, \tau L^*]} (wL + \Pi) + \frac{1}{\tau} q^E - wc(\frac{wL + \Pi}{\tau} + q^E, \theta). \tag{9}$$

It is immediate that π is nonincreasing in the scale parameter $\theta..$ Define $\hat{\theta}_0(w, \Pi)$ to be the maximum level of θ such that $\pi(w, \Pi) \geq 0$. Profit is negative for θ above $\hat{\theta}_0(w, \Pi)$ so there is zero entry above this threshold. Define

$$\hat{\theta}_1(w, \Pi) = \frac{1}{2} \left(\frac{wL + \Pi}{w} \right).$$

At this point, when there are two entrants, the equilibrium output of each firm with Bertrand competition exactly equals minimum efficient scale. For θ less than $\hat{\theta}_1$, two firms enter in equilibrium. For θ between $\hat{\theta}_1$ and $\hat{\theta}_0$ a single firm enters.

The third step solves for the value of aggregate profits Π as a function of the wage rate, taking into account that entry depends upon $\Pi..$ Specifically, define $\Pi(w)$ to be the *minimum* value of Π that solves $H(\Pi, w) = 0$ where

$$H(\Pi, w) \equiv \Pi - \int_{\hat{\theta}_1(w, \Pi)}^{\hat{\theta}_0(w, \Pi)} f(\theta) \pi(\theta, w, \Pi) d\theta.$$

The fourth step is to solve for the wage rate in the small country that clears the labor market. Define the excess demand for labor to be

$$E(w) = \int_0^{\hat{\theta}_0(w, \Pi(w))} f(\theta) c(q(\theta, w), \theta) d\theta - L,$$

where $q(\theta, w)$ is the total quantity of type θ good produced in the small country given wage w and profit $\Pi(w)$. In the appendix we show that $E(w)$ is strictly positive near the lower bound of $w = \frac{1}{\tau}$ and strictly negative near the upper bound of $w = \tau$; continuity ensures that an equilibrium wage exists. Finally, note that in equilibrium $\hat{\theta}_0 < \overline{\theta}$ as otherwise assumption (6) would imply the resource constraint is violated.

We do not have a general result that the equilibrium is unique. In the numerical examples we discuss below, we have verified that there is a unique pair (Π, w) satisfying the above two equilibrium conditions.

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4.2 Trade Structure

The equilibrium trade structure is characterized by two cutoffs, $\hat{\theta}_E$ and $\hat{\theta}_0$. The cutoff $\hat{\theta}_0$ is the same cutoff from above that separates the single-firm entry and zero entry cases. The cutoff $\hat{\theta}_E$ satisfies $\hat{\theta}_E \geq \hat{\theta}_1$ and $\hat{\theta}_E < \hat{\theta}_0$. For industries $\theta < \hat{\theta}_E$ there is no trade. For industries in the range $\theta \in (\hat{\theta}_E, \hat{\theta}_0)$ the small country producers all export. For industries $\theta > \hat{\theta}_0$, there is no production in the small country and all the small country demand is met by imports. These cutoffs are illustrated in Figure 1.

Recall that for $\theta < \hat{\theta}_1$, there are two producers in the small country and both firms produce in the constant-returns-to-scale region of the production function. There is no trade in these goods following the original Davis argument. The price of such goods in the small country equals w, minimum average cost. Since the equilibrium wage w lies between $\frac{w^*}{\tau}$ and τw^*, there is no scope for trade.

For θ between $\hat{\theta}_0$ and $\overline{\theta}$, there are no producers in the small country. So demand is met by imports.

Now consider the range between $\hat{\theta}_1$ and $\hat{\theta}_0$ where there is a single producer in the small country. Examining the firm's problem (9), the firm will choose to export if the export price exceeds marginal cost at zero exports; i.e., if

$$\frac{1}{\tau} > w\frac{\partial c(q_L, \theta)}{\partial q}$$

for $q_L = \frac{wL+\Pi}{\tau}$. For $\theta < q_L$, marginal cost is constant in θ, while it decreases in θ for $\theta > q_L$. (See equation (4).) Thus, if any θ° chooses to export, all $\theta > \theta^\circ$ also strictly prefer to export. The export decision will have the form of a cutoff where all θ above the cutoff choose positive exports. Because there is a positive measure of imports there must be a positive measure of exports. Therefore, there must exist a cutoff $\hat{\theta}_E < \hat{\theta}_0$ where all θ above $\hat{\theta}_E$ export.

4.3 A Numerical Example

Table 1 illustrates the equilibrium for a few numerical examples. The table reports the equilibrium for two values of the transport cost parameter τ and for a variety of values of

the small county size L. We assume the minimum efficient scale θ is uniform between zero and one and that $\alpha = \frac{1}{2}$. Applying (6), the formal characterization in Proposition 3 requires $L \leq .25$ for this case. By turning to the computer, we are able to calculate the equilibrium without imposing this constraint.

Consider first the comparative statics with the small county size L for fixed τ. The cutoffs $\hat{\theta}_1$, $\hat{\theta}_E$ and $\hat{\theta}_0$ all monotonically increase in L. Thus the range of autarky goods $[0, \hat{\theta}_E]$ strictly increases and the range of imported goods $[\hat{\theta}_0, 1]$ strictly decreases. There exists a critical size level $\hat{L}$ where $\hat{\theta}_E$ and and $\hat{\theta}_0$ both hit one at exactly the same time. These critical values are reported in the table as the highest value of L for the given τ. For example, in the case of $\tau = 1.1$, $\hat{L} = .845$. If $L > \hat{L}$, the small country remains in autarky.

Notice that the set of exported goods $[\hat{\theta}_E, \hat{\theta}_0]$ first increases then decreases as L increases. The effect is ambiguous because of two offsetting forces. On one hand, larger market size induces new firms to enter who will need to export to obtain sufficient scale. On the other hand, existing firms already in have less incentive to export because a larger local market enables them to obtain sufficient scale. The first factor is more important when L is small while the second factor is more important when L is large.

Regarding market structure, the range $[0, \hat{\theta}_1]$ of goods produced by duopoly increases while the range $[\hat{\theta}_0, 1]$ produced by zero firms decreases with L. The range produced by monopoly first increases and then decreases with L. When L is initially small, the predominate effect of an increase in L is that monopolists enter industries not previously covered. But as L gets large, the predominate effect of further increases in L is duopolies replacing monopolies.

The nominal wage w increases with L and for high enough L the wage exceeds one, the nominal wage in the large county. It is worth noting that the real small country wage (not reported) is never higher than the real wage in the large county.

Next consider comparative statics with τ. As one would expect, the range of goods produced $[0, \hat{\theta}_0]$ increases with τ and the range of imports $[\hat{\theta}_0, 1]$ decreases. Somewhat counterintuitively, in some cases an increase in τ actually *increases* the range $[\hat{\theta}_E, \hat{\theta}_0]$ of exported goods. An increase in τ enables monopolists to raise prices set to local consumers, contracting local sales. This decreases marginal labor costs for those monopolists operating

at less than minimum efficient scale. This decrease in marginal labor cost raises the incentive to export and can more than offset the negative effect of higher τ on the incentive to export.

5 Concluding Remarks

Our analysis assumes the structure of demand is the same across products and allows the degree of scale economies to vary across products. It is straightforward to see that analogous results would be obtained if the cost structure were the same across products but demand structure were to differ. In particular, suppose that consumers have relatively greater demand for some products relative to others in the sense that they would choose to buy a larger quantity at the same price. For goods with a high enough level of demand, it might be possible for firms to get to the constant returns to scale region even in the small country, so these goods would not be traded, analogous to the way there is autarchy in our original model for the goods with low minimum efficient scale. Following the same logic as our original model, the small country will tend to export goods of intermediate level of demand and import goods with low levels of demand. One can think of the products with low level of demand as "boutique" or "niche" items. Only the large market can sustain production of these "unusual" items.[7]

For the sake of tractability, we chose a production technology that features constant returns above some minimum efficient scale and we focus on the case where the large country is sufficiently large to have perfect competition throughout all the industries. Alternatively, we could have conducted an analysis with a constant marginal cost and a fixed cost that varied across industries.[8] With this alternative structure, there would be at most one producer of a given product at each location, because of the usual Bertrand logic. It is intuitive that for low fixed cost products, there would be entry at both locations, so such goods would not be traded, analogous to our results here. For higher fixed-cost goods, production would be only at one location. As is typical in oligopoly games, there are multiple equilibria in this environment regarding the country in which a given industry

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[7]See Holmes (1999) for a related analysis.

[8]If we had gone this way, we would also have assumed an elasticity of substitution between products that is greater than one, rather than equal to one, to bound the optimal monopoly price.

locates. Thus, there is no hope that a clean partition of industry and trade structure as in our proposition would be the equilibrium here. Nonetheless, we expect that a qualitative aspect of our result would continue to hold. In particular, we conjecture that if the highest fixed cost is high enough and if the large country is large enough, the highest fixed-cost goods would only be produced in the large country. Thus the structure of production and trade would depend in systematic ways upon the size of the home market.

6 References

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Figure 1: Small Country Market and Trade Structures in the Continuum Model

$$0 \qquad\qquad \hat{\theta}_1 \qquad\qquad\qquad \hat{\theta}_0 \qquad\qquad\qquad \bar{\theta}$$

Market
Structure $n = 2$ $\qquad\qquad$ $n = 1$ $\qquad\qquad$ $n = 0$

Trade
Structure   Autarchy $\qquad$ Exporter $\qquad$ Importer

$$0 \qquad\qquad\qquad \hat{\theta}_E \qquad\qquad \hat{\theta}_0 \qquad\qquad\qquad \bar{\theta}$$
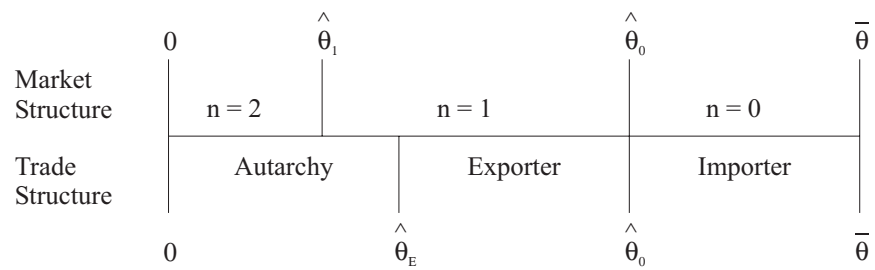
Table 1
Equilibrium in Selected Numerical Examples

Exogenous Parameters		Endogenous Variables				Market Structure			Trade Structure		
						n=2	n=1	n=0	Autarky	Exports	Imports
τ	L	w	$\hat{\theta}_1$	$\hat{\theta}_E$	$\hat{\theta}_0$	$[0,\hat{\theta}_1]$	$[\hat{\theta}_1,\hat{\theta}_0]$	$[\hat{\theta}_0,1]$	$[0,\hat{\theta}_E]$	$[\hat{\theta}_E,\hat{\theta}_0]$	$[\hat{\theta}_0,1]$
1.1	.100	.948	.052	.093	.445	.052	.394	.555	.093	.353	.555
1.1	.200	.967	.104	.194	.618	.104	.514	.382	.194	.424	.382
1.1	.500	1.016	.259	.535	.898	.259	.639	.102	.535	.363	.102
1.1	.800	1.073	.405	.933	.998	.405	.593	.002	.933	.066	.002
1.1	.845	1.083	.426	1.000	1.000	.426	.574	.000	1.000	.000	.000
1.3	.100	.856	.054	.079	.459	.054	.405	.541	.079	.380	.541
1.3	.200	.905	.110	.180	.645	.110	.535	.355	.180	.465	.355
1.3	.500	1.058	.273	.612	.946	.273	.673	.054	.612	.334	.054
1.3	.600	1.124	.322	.812	.989	.322	.667	.011	.812	.177	.011
1.3	.678	1.187	.355	1.000	1.000	.355	.645	.000	1.000	.000	.000