

— Technical Appendix —

Does Home Market Size Matter for the Pattern of Trade?*

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August 15, 2003

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This appendix proves Lemma 1 and Propositions 1-3 from the paper. Also included are proofs of two claims related to subtleties raised in the text.

1 Proof of Lemma 1

This section proves the existence of an equilibrium in stage 2, the price-setting stage. It also provides the conditions under which the equilibrium is unique and offers an example of multiple equilibria when those conditions are not met. We need a few preliminary results before proving Lemma 1.

Lemma 1.1 *Any equilibrium with $n \geq 1$ and $n^* \geq 1$ has the following properties.*

1. $q^L > 0$ and $q^{L^*} > 0$.
2. $p^L \leq \tau w^* c'(q^*)$ and $p^{L^*} \leq \tau w c'(q)$
3. If $q^{E^*} > 0$, then $p^L = \tau p^{E^*} = w c'(q) = \tau w^* c'(q^*)$ with an analogous condition if $q^E > 0$.

Proof. These conditions follow immediately from standard Bertrand reasoning. ■

Lemma 1.2 *In the price subgame, given A and (n, n^*) , there is no two way trade (i.e., both $q^E > 0$ and $q^{E^*} > 0$ cannot hold simultaneously).*

Proof. This is only relevant if $n \geq 1$, $n^* \geq 1$, so assume it is true. Take A as given. Look at the symmetric equilibrium of price competition. From the previous lemma, if $q^{E^*} > 0$, then

$$p^L = \tau p^{E^*} = w c'(q) = \tau w^* c'(q^*)$$

Analogously, if $q^E > 0$, then

$$\tau w c'(q) = \tau p^E = p^{L^*} = w^* c'(q^*).$$

Since $\tau > 1$, this is a contradiction. ■

Lemma 1.3 *Suppose $n = 1$ and $n^* = 1$. Suppose we have two equilibria with quantities (q, q^*) and $(q', q^{*'})$. Then $q < q'$ and $q^* \leq q^{*'}$ cannot both be true. Analogously, we cannot have $q^* < q^{*'}$ and $q \leq q'$.*

Proof. Observe that when $n = 1$ and $n^* = 1$, it must be the case that $p^L = \tau w^* c'(q^*)$ and $p^{L^*} = \tau w c'(q)$. Suppose there is a second equilibrium where $q' > q$ and $q^{*'} \geq q^*$. This implies $p^{L'} \geq p^L$ and $p^{L^{*'}} \geq p^{L^*}$. This implies $Q' \leq Q$ and $Q^{*' } \leq Q^*$ where uppercase Q denotes demand. Thus total demand is no greater than in the original equilibrium at each location, but total supply of output is strictly greater, a contradiction. ■

We can now prove the existence of a stage 2 equilibrium.

Lemma 1 (from the text) *Given any A and entry vector (n, n^*) , a symmetric equilibrium to the price subgame exists. If there are at least two firms in a given country, $n \geq 2$ or $n^* \geq 2$, there is a unique symmetric equilibrium of the price subgame. If each location has a single firm, $n = 1$ and $n^* = 1$, and if there is an equilibrium with trade, then the price subgame has a unique equilibrium.*

Proof. Consideration of the number of firms at each location implies four possible cases:

Case 1 $n \geq 2$ and $n^* \geq 2$.

Solve for perfect competition; it is clear that a symmetric equilibrium exists and is unique.

Case 2 $n \geq 2$ and $n^* = 1$.

Bertrand equilibrium in the small country implies

$$p^L = w c'(q)$$

and

$$p^E = w c'(q).$$

Now look at the large country monopolist. It is immediate that $p^{L^*} = \tau p^E$. If $q^E > 0$, then $\tau p^{E^*} = p^L$. We can solve the equilibrium as follows. First, for a given q , calculate p^L and p^E as above. Then the monopolist solves the following problem:

$$\max_{q^{L^*}, q^{E^*}} \tau p^E q^{L^*} + \frac{p^L}{\tau} q^E - w^* c(q^{L^*} + q^{E^*})$$

subject to

$$\begin{aligned}\frac{q^{E*}}{\tau} &\leq \frac{I}{p^L} \\ q^{L*} &\leq \frac{I^*}{\tau p^E}\end{aligned}$$

Next, see if given q , the solution is feasible. For q small enough, when we solve this problem we will have excess demand. It is clear that as we raise q , there is a unique case where the solution to this problem satisfies market clearing. It is immediate that this is a Bertrand equilibrium. Furthermore, any Bertrand equilibrium must be a solution to this construction. Hence there is a unique Bertrand equilibrium.

Case 3 $n = 1$ and $n^* \geq 2$.

This case is symmetric to Case 2.

Case 4 $n = 1$ and $n^* = 1$.

This is the most difficult case. We know that both must produce, and both must sell locally, $q^L > 0$, $q^{L*} > 0$. Also, price must match the marginal cost of the firm in the other location,

$$\begin{aligned}p^L &= \tau p^{E*} = \tau w^* c'(q^*) \\ p^{L*} &= \tau p^E = \tau w c'(q)\end{aligned}$$

But then we know Q and Q^* . Summarizing the conditions, we have

$$\begin{aligned}w c'(q) &\leq \tau w^* c'(q^*) \\ w^* c'(q^*) &\leq \tau w c'(q) \\ Q &= \frac{I}{\tau w^* c'(q^*)} \\ Q^* &= \frac{I^*}{\tau w c'(q)} \\ q &\leq Q \\ q^* &\leq Q^*\end{aligned}$$

And if $q < Q$, then

$$\begin{aligned} wc'(q) &= \tau w^* c'(q^*) \\ Q &= q + \frac{Q^* - q^*}{\tau} \end{aligned}$$

with analogous conditions if $q^* < Q^*$.

Existence

We use a fixed point technique. Take $A = (q^L, q^{E*}, q^{L*}, q^E)$ as given. Then create a new vector as follows. $A' = F(A)$ where $(q^{L'}, q^{E*'})$ is an equilibrium given (q^{L*}, q^E) and $(q^{L*'}, q^{E'})$ is an equilibrium given (q^L, q^{E*}) .

Look at the solution. More generally, take a market with demand I/p . Let x and x^* be the amount delivered to the location. Given q^{L*} and q^E let

$$\begin{aligned} h(x) &= wc(x + q^E) \\ h^*(x^*) &= w^* c(q^{L*} + \tau x^*) \end{aligned}$$

be the cost functions. Suppose $h(x)$ and $h^*(x^*)$ are weakly convex. What is the Bertrand equilibrium? Suppose $x > 0$ and $x^* > 0$. Then, from Lemma 1.1, marginal costs must be equal

$$h'(x) = h^{*'}(x^*)$$

and the amount delivered must equal demand

$$x + x^* = \frac{I}{h'(x)}.$$

If $x^* = 0$, the Lemma 1.1 also implies

$$x = \frac{I}{h^{*'}(x^*)}$$

and

$$h'(x) \leq h^{*'}(x^*).$$

Consider the following algorithm: Start with $x = x^* = 0$ and check the marginal cost. Assume W.L.O.G. that $h'(0) \leq h^{*'}(0)$. Suppose $h'(x) < h^{*'}(0)$ for all x , then $p = h^{*'}(0)$. Suppose for large enough x , that $h'(x) \geq h^{*'}(0)$; let x be smallest such x for which this is

true. Suppose that $x \leq I/h^{*'}(0)$. Then the equilibrium price is $p = h^{*'}(0)$. Otherwise, find an (x, x^*) such that markets clear and marginal cost is the same.

This mapping is continuous and compact. Suppose we only had each market on its own and constrained the other to be zero. Define $(x^\circ, x^{*\circ})$ by $(x^\circ, x^{*\circ}) = F_{part}(0, 0)$. It is clear that F maps this set to within this set.

Uniqueness

Now equilibrium is not necessarily unique in the region where both (q, q^*) are on the constant marginal cost part of the curves. If $\tau w = w^*$, then we can have different q where large country firms gets zero profit. (See Claim 1 below for an example of this.) However, if there is an equilibrium with trade, then the equilibrium is unique with respect to prices.

To see this, suppose we start with an equilibrium (q, q^*) with trade. Suppose we have another equilibrium $(q', q^{*'})$ with different prices. W.L.O.G. assume that $q' < q$. From Lemma 1.3 we know that $q^{*'} > q^*$. We know that $p^{*'} \leq p^*$ and $p' \geq p$. Moreover, since the prices are different, at least one of the inequalities is strict. Therefore, at least one of the quantities, $q, q', q^*, q^{*'}$, must be strictly below θ . From the equations for Q and Q^* , we know that $Q' \leq Q$, and $Q^{*' \geq Q^*$.

We claim first that $q \geq Q$. Suppose not, then $q < Q$. It follows that $w c'(q) = \tau w^* c'(q^*)$ and since one of the quantities is less than θ ,

$$w c'(q) > \tau w^* c'(q^{*'})$$

This contradicts equilibrium, so $q \geq Q$. Analogously, $q^{*' \geq Q^{*'}$. These imply (by Lemma 1.2) that $q^* \leq Q^*$ and $q' \leq Q'$.

Since we suppose there was positive trade with the initial equilibrium, $q > Q$ and $q^* < Q^*$ must hold. Since there are strictly positive imports into the large country, price in the large country initially must equal marginal cost for the large firm,

$$\begin{aligned} w^* c'(q^*) &= p^* \\ &\geq p^{*' \\ &\geq w^* c'(q^{*'}) \end{aligned}$$

Recall that either $p^{*' < p^*$ or $p' > p$ (one is strict). Suppose $p^{*' < p^*$. Then the first inequality above is strict implying that $c'(q^*) > c'(q^{*')$ or $q^* > q^{*'}$. But this is a contradiction. Suppose

instead that $p' > p$. Now since $q^* < q^{*'}$, the only way the above inequality can hold is if $q^* \geq \theta$. This implies that

$$w^* c'(q^*) = w^* c'(q^{*'})$$

or

$$p = \tau w^* c'(q^*) = \tau w^* c'(q^{*'}) = p',$$

a contradiction to $p' > p$. ■

Claim 1 *In the $(n, n^*) = (1, 1)$ case, without trade there exist multiple equilibria.*

When there is no trade we can construct examples where there are multiple equilibria. Assume $1 = I = I^* = w = w^*$. Assume $\theta > 1$. Let $q \in (\frac{1}{\tau}, 1)$. Then we have

$$\begin{aligned} p^* &= \tau c'(q) = \frac{\tau q}{\theta} \\ p &= \tau c'(q^*) = \frac{\tau q^*}{\theta} \\ q^* &= \frac{1}{p^*} = \frac{\theta}{\tau q} \end{aligned}$$

Is this an equilibrium? The values of p^* , p , and q^* are all optimal given q . It remains to be checked that no firm wishes to export at these prices. A firm at the small location would face an export price of p^*/τ . The marginal cost of production in the small location is

$$c'(q) = \frac{q}{\theta} = \frac{p^*}{\tau}$$

and so there is no strict incentive to export. Similarly, the large location faces an export price of p/τ and a marginal cost of

$$c'(q^*) = \frac{q^*}{\theta} = \frac{p}{\tau}..$$

This applies for all $q \in (\frac{1}{\tau}, 1)$ as

$$q < \theta$$

and

$$q^* = \frac{\theta}{\tau q} < \frac{\theta}{\tau \frac{1}{\tau}} = \theta..$$

Hence we have multiple equilibria without trade.

2 Proof of Proposition 1 (Krugman Result)

Proof. Recall that the agricultural sector is constant returns to scale, therefore, given the multiple entry at each location price equals marginal cost. In the manufacturing sector, multiple entry in the large country implies price equals marginal cost there as well. Under assumption (ii) that $L^* > 2\theta_m$, two firms can get to minimum efficient scale just based on the demand of the large county, so marginal cost is 1. With the small country completely specializing in agricultural goods, its imports of manufactured goods will have value $f_m L$ which must equal the value of its agricultural exports. Assumption (iii) that $f_m L < f_a L^*$, guarantees that the large country can absorb these exports, and so supply equals to demand.

All that remains to be shown is that it is not profitable for a manufacturing firm to enter at the small location. The local demand for good m in the small country is

$$q^L = \frac{L}{\tau}.$$

Marginal cost at q^L is

$$c'(q^L) = \left(\frac{L}{\tau\theta}\right)^{\frac{\alpha}{1-\alpha}},$$

and the export price is

$$p^E = \frac{1}{\tau}.$$

Price is greater than marginal cost at q^L since

$$\begin{aligned} \left(\frac{L}{\tau\theta}\right)^{\frac{\alpha}{1-\alpha}} &< \left(\frac{1}{\tau}\right)^{\frac{\alpha}{1-\alpha}} \\ &< \frac{1}{\tau} \end{aligned}$$

where the inequalities hold since $\alpha \geq \frac{1}{2}$ (so the exponent is greater than one) and $L < \frac{\theta}{2}$. Therefore, if the firm enters it wants to export. The total quantity produced solves price equal to marginal cost,

$$q = \theta\tau^{-\frac{1-\alpha}{\alpha}}.$$

Profit at this quantity is

$$\begin{aligned} \pi_m &= p^L q^L + p^E (q - q^L) - c(q) \\ &= L + \frac{1}{\tau} \left[\theta\tau^{-\frac{1-\alpha}{\alpha}} - \frac{L}{\tau} \right] - \left[\alpha\theta + (1-\alpha)\theta^{\frac{-\alpha}{1-\alpha}} q^{\frac{1}{1-\alpha}} \right] \\ &= (1 - \tau^{-2})L - \alpha\theta \left(1 - \tau^{-\frac{1}{\alpha}}\right). \end{aligned}$$

Profit will be negative, and so it will not be profitable to enter at the small location, if

$$L < \frac{\alpha\theta \left(1 - \tau^{-\frac{1}{\alpha}}\right)}{(1 - \tau^{-2})}.$$

The inequality holds at $\alpha = \frac{1}{2}$ as $L < \frac{\theta}{2}$. For $\alpha > \frac{1}{2}$ the next lemma shows that the RHS of the inequality is increasing in α . ■

Lemma 2.1 For $\tau > 1$ and $\alpha > \frac{1}{2}$,

$$g(\alpha, \tau) \equiv \alpha \left(1 - \tau^{-\frac{1}{\alpha}}\right)$$

is an increasing function of α .

Proof. The slope of g is

$$\begin{aligned} \frac{\partial g}{\partial \alpha} &= \left(1 - \tau^{-\frac{1}{\alpha}}\right) - \frac{1}{\alpha} \ln(\tau) \tau^{-\frac{1}{\alpha}} \\ &= \tau^{-\frac{1}{\alpha}} \left[\tau^{\frac{1}{\alpha}} - 1 - \frac{\ln(\tau)}{\alpha} \right] \end{aligned}$$

Evaluating this at $\tau = 1$ we have

$$\frac{\partial g}{\partial \alpha} = 0$$

and so it is sufficient to show that the bracketed term above is increasing in τ for $\alpha > 0$.

This slope is

$$\frac{d}{d\tau} \left(\tau^{\frac{1}{\alpha}} - 1 - \frac{\ln(\tau)}{\alpha} \right) = \frac{1}{\alpha} \left(\tau^{\frac{1}{\alpha}-1} - \tau^{-1} \right)$$

which is positive for $\alpha > 0$. ■

One subtlety noted in the text is that we can construct an example where a *medium* size country specializes in manufacturing.

Claim 2 Suppose the parameters are such that

1. $\frac{\theta_m}{2} < L < \theta_m$,

2. $3\theta_m < L^*$,

3. $\tau^2 < \left[2 - \frac{\theta_m}{L}\right]^{-1}$

$$4. \left[\frac{\theta_m}{L} - 1 + \frac{1}{\tau^2} \right]^{-1} < f_m$$

Then there exists a zero profit equilibrium where the medium size country completely specializes in manufacturing.

Proof. In the equilibrium we construct, there are two producers of each product in the large country. There the price of every good equals $p^{L*} = w^* = 1$. For the case where $\alpha = \frac{1}{2}$, marginal cost is $c'(q) = q/\theta$. If a single firm enters a particular manufacturing industry in the small county, its export price is $\frac{1}{\tau}$. Setting total output so that price equals marginal cost in the export market, its total output solves

$$w \frac{q}{\theta_m} = \frac{1}{\tau}$$

so $q = \frac{\theta}{w\tau}$. The profit of such a firm is

$$\begin{aligned} \pi &= p^L q^L + p^E (q - q^L) - wc(q) \\ &= wL + \frac{1}{\tau} \left[\frac{\theta_m}{w\tau} - \frac{wL}{\tau} \right] - w \left[\frac{\left(\frac{\theta_m}{w\tau} \right)^2}{2\theta_m} + \frac{\theta_m}{2} \right] \\ &= \frac{\tau^2 - 1}{\tau^2} wL + \frac{\theta_m}{w2\tau^2} - \frac{w\theta_m}{2} \end{aligned}$$

At $w = 1$, profit is strictly positive by assumption (1) in the claim. Profits are zero at

$$w = \sqrt{\frac{\theta_m}{\tau^2 \theta_m - (\tau^2 - 1) 2L}} > 1.$$

If price exceeds marginal cost, then exports are strictly greater than zero at this wage. We verify that this is satisfied by checking the following inequality

$$\begin{aligned} \frac{1}{\tau} &> wc'(q^L) \\ &= w \frac{L}{\tau \theta_m}. \end{aligned}$$

Substituting for w and rearranging we find

$$\frac{1}{\tau^2 \frac{\theta_m}{L} - (\tau^2 - 1) 2} L < \theta_m.$$

This inequality holds since the fraction term is less than 1 and $L < \theta_m$.

The total quantity produced solves

$$\begin{aligned}\frac{1}{\tau} &= wc'(q) \\ &= w\frac{q}{\theta_m}\end{aligned}$$

and is equal to

$$q = \sqrt{\theta_m^2 - 2L\theta_m + \frac{2L\theta_m}{\tau^2}} < \theta_m.$$

This quantity costs

$$\begin{aligned}c(q) &= \frac{\theta_m}{2} + \frac{q^2}{2\theta_m} \\ &= \frac{\theta_m}{2} + \frac{\theta_m^2 - 2L\theta_m + \frac{2L\theta_m}{\tau^2}}{2\theta_m} \\ &= \theta_m - L + \frac{L}{\tau^2}.\end{aligned}$$

Recall that we have assumed zero profits. Since $w > 1$ and since $\tau_a = 1$, there is no agricultural sector. There must be enough manufacturing products to apply all the labor. That is,

$$L < f_m c(q)$$

where the RHS is the labor requirement to produce all manufacturing goods. Rearranging,

$$\frac{1}{\frac{\theta_m}{L} - 1 + \frac{1}{\tau^2}} < f_m.$$

The LHS is less than 1 by assumption (3) in the claim. By assumption (4), f_m satisfies the above inequality.

Now check the entry conditions. There is no entry in the agricultural sector since $w > w^* = 1$ and $p_a = 1$ (as $\tau_a = 1$). We have free entry into the manufacturing sector, but not all goods are produced. Since the resource constraint is satisfied, Walras law means the market clearing conditions are satisfied. Profits are zero in the large country, so the entry conditions hold there. ■

3 Proof of Proposition 2 (Davis Result)

Proof. We show first that the small country never exports agricultural goods to the large country. Suppose to the contrary that there were such exports. Then

$$\tau w \leq w^*. \quad (1)$$

Otherwise, since there is constant returns to scale, it would be cheaper to produce agricultural goods in the large country and pay w^* per unit, than import them from the small country and pay τw per unit including the transportation cost.

Since agriculture is constant returns to scale and has transportation cost, it is obvious that there is no two way trade in agriculture. Since the large country imports agricultural goods, there must be some manufacturing products produced in the large country that are exported. Let i denote one such product. Let q_i^{L*} and q_i^{E*} denote the local and export output for firms in the large country and let q_i^* be total output.¹ As already noted, exports are positive, $q_i^{E*} > 0$.

Suppose that $n_i \geq 1$, so there also exists a firm in the small country. Lemma (1.2) shows the intuitively obvious result that there can be no two-way trade in any given product i . Hence $q_i^{E*} > 0$ implies $q_i^E = 0$. Of course $q_i^E = 0$ implies $q_i^{L*} > 0$. Suppose a firm in the small country were to capture the sales q_i^{E*} and q_i^{L*} of a firm in the large country. We first claim that the increment in cost to the small-country firm of capturing these sales is strictly less than the total cost to the large-country firm, i.e.,

$$wc(q_i^L + \frac{q_i^{E*}}{\tau} + \tau q_i^{L*}) - wc(q_i^L) < w^*c(q_i^{E*} + q_i^{L*}). \quad (2)$$

Observe that if the small-country firm takes over the large-country firm's production, it avoids transportation cost on deliveries to the small country (the $\frac{1}{\tau}$ term) but incurs transportation cost on the deliveries to the large country (the τ term). We have that

$$\begin{aligned} wc(q_i^L + \frac{q_i^{E*}}{\tau} + \tau q_i^{L*}) - wc(q_i^L) &\leq wc(\frac{q_i^{E*}}{\tau} + \tau q_i^{L*}) \\ &< wc(\tau q_i^{E*} + \tau q_i^{L*}) \end{aligned}$$

¹Note that we restrict attention to symmetric equilibria where firms in the same country and the same industry set the same prices and have the same output.

$$\begin{aligned}
&\leq \tau w c(q_i^{E*} + q_i^{L*}) \\
&\leq w^* c(q_i^{E*} + q_i^{L*}).
\end{aligned}$$

The first inequality follows from subadditivity of $c(\cdot)$. The second follows from $\tau > 1$ and $c' > 0$. The third follows since average cost is non-increasing. The fourth follows from (1) $\tau w \leq w^*$. This proves inequality (2). Since the large-country firm has non-negative profit,

$$w^* c(q_i^{E*} + q_i^{L*}) \leq p_i^{L*} q_i^{L*} + p_i^{E*} q_i^{E*}. \quad (3)$$

By the standard Bertrand argument, the small-country firm can capture the large-country firm's revenues by slightly undercutting prices. Combining (2) and (3), these additional revenues to the small-country firm are strictly greater than its incremental cost. This contradicts equilibrium in the price-game stage.

Next consider the case where $n_i = 0$. We know that $n_i^* \geq 1$. If $n_i^* = 1$, the firm has a monopoly. By the same argument used to prove (2), the monopolist could lower its cost by selecting the small country instead.

Finally, if $n_i = 0$ and $n_i^* \geq 2$, it must be that $q_i^* \geq \theta$ and $p_i^* = w^*$, and $p_i = \tau w^*$. This follows since price equals marginal cost with two firms. But if $q_i^* < \theta$, marginal cost would be less than average cost so profits would be strictly negative. This contradicts equilibrium. Thus price equals average cost and firms earn zero profit.

We claim that there must exist a second equilibrium of the oligopoly game that Pareto dominates this equilibrium. Let $\tilde{n} = n_i^*$ and $\tilde{n}^* = 0$. We construct an equilibrium where $\tilde{p} = w$, and $\tilde{p}^* = \tau w$. Note that $\tilde{p}^* = \tau w \leq w^* = p^*$, so consumers in the large country are no worse off. Also $\tilde{p} = w < \tau w^* = p$, so consumers in the small country are strictly better off. Also since $\tilde{p}^* \leq w^*$, there is no incentive for a firm to enter in the large country. We will show that the firms locating in the small country obtain zero profit. This will prove the second equilibrium Pareto dominates since profits are zero in any case, and no consumer is worse off, and the small country consumers are strictly better off.

In the new allocation, setting output per firm to total output at the new prices divided by the number of firms yields,

$$\tilde{q} = \frac{1}{n_i^*} \left(\frac{I}{w} + \tau \frac{I^*}{\tau w} \right) = \frac{1}{n_i^*} \frac{1}{w} (I + I^*).$$

Since $\tau w \leq w^*$, $w < w^*$. Hence

$$\tilde{q} > \frac{1}{n_i^*} \frac{1}{w^*} (I + I^*) = q^* \geq \theta.$$

Hence each firm is above θ in the new allocation so price equals average total cost. Profit is zero. Thus this is an equilibrium to the oligopoly game that Pareto dominates the original equilibrium. This is a contradiction since we assumed from the start we were restricting attention to equilibria of the oligopoly game that are not Pareto dominated.

The contradictions derived above show that the small country never exports agricultural goods to the large country. A parallel argument shows that the large country never exports to the small country. ■

4 Proof of Proposition 3 (Continuum Model)

Step 1: Bounding the wage rate

Lemma 4.1 *In any equilibrium*

$$\frac{1}{\tau} w^* < w < \tau w^*.$$

Proof. Suppose we have a large-country perfect-competition equilibrium and $w \leq \frac{1}{\tau} w^*$. Using the same argument used in Proposition 2, if entry were $(n_i, n_i^*) = (0, 2)$ for some industry i , then this outcome could be eliminated by $(2, 0)$, contradicting the fact that we started with an equilibrium. Thus for every industry i , $n_i \geq 1$. This contradicts the resource constraint, equation (6) in the text. If $w \geq \tau w^*$, a similar line of argument shows that the small country has no exports, which implies it has no imports. But autarchy cannot be an equilibrium again because of the resource constraint. ■

Let wage in the large country be the numeraire, $w^* = 1$. It is immediate that in a large-country perfect-competition equilibrium, $p_i^* = 1$ for all i . There is also zero profit in the large country, $\Pi^* = 0$, and so $I^* = L^*$. We let (w, Π) denote the aggregate state.

Step 2: Solving the oligopoly game for each industry

All industries of the same type θ will have the same outcome, except for a set of measure zero where multiple outcomes are possible. For the rest of this section it is useful to use θ to keep track of the type of an industry rather than the index i of the industry. Let $n(\theta)$ be the entry of industries of type θ in the small country. (By definition, $n^*(\theta) = 2$). Let $\pi(n, \theta)$ be profit of entry into an industry of type θ given n entrants at the small location (the level of entry in the large country is implicit at $n^*(\theta) = 2$). Let $p(\theta)$ be the equilibrium price in the small country.

Take the aggregate state, (w, Π) , as given, where the small-country wage w satisfies the bounds in step 1. We characterize the equilibrium market structure in the small country by identifying the sets of θ for which the small country has a local duopoly or monopoly.

Identifying the sets of θ for which the small country has a local duopoly or monopoly amounts to identifying a set of cut-off values. Let $\hat{\theta}_1$ be the first cutoff,

$$\hat{\theta}_1 = \frac{1}{2} \left(\frac{wL + \Pi}{w} \right).$$

For $\theta \leq \hat{\theta}_1$, the unique equilibrium of the entry game has $n(\theta) \geq 2$. This follows from the fact that $w < \tau$ (so imports are more expensive than local production) as well as from our use of elimination criterion 1 (ruling out the monopoly equilibrium since duopoly is feasible).

For $\theta > \hat{\theta}_1$, $n(\theta) \leq 1$, a firm that enters has a monopoly. Depending upon the size of θ , the monopoly may or may not export and may or may not reach the efficient scale. Regardless, the entering firm takes as given that $p^{L*} = 1$ and $p^{E*} = \frac{1}{\tau}$ in the price subgame. It is clear that in the subgame

$$p^L = \tau > w.$$

Since price is greater than marginal cost, the firm sells to the entire local market, so there are no imports. The sales in the local market are

$$q^L = \frac{wL + \Pi}{p^L} = \frac{wL + \Pi}{\tau}.$$

We now characterize the monopoly firms corresponding to different values of θ .

First, the monopoly may reach the efficient scale without a need to export. Let

$$\hat{\theta}_2 = \max \left\{ \frac{wL + \Pi}{\tau}, \hat{\theta}_1 \right\}$$

denote the value of θ such that the firm can reach the efficient scale just selling locally.² As long as $2w > \tau$, there will be a positive measure of firms in this category, i.e., $\hat{\theta}_2 > \hat{\theta}_1$. Their profits are

$$\begin{aligned}\pi(\theta, w, \Pi) &= p^L q^L - wc(q^L, \theta) \\ &= (wL + \Pi) \left(1 - \frac{w}{\tau}\right).\end{aligned}$$

Second, even if a firm does not reach the efficient scale with local demand only, it may still opt not to export. Profits for these firms come from local demand only,

$$\begin{aligned}\pi(\theta, w, \Pi) &= p^L q^L - wc(q^L, \theta) \\ &= (wL + \Pi) - w \left[\alpha \theta + (1 - \alpha) \theta^{-\frac{\alpha}{1-\alpha}} \left(\frac{wL + \Pi}{\tau}\right)^{\frac{1}{1-\alpha}} \right]\end{aligned}$$

This region of θ begins at $\hat{\theta}_2$ and extends until it becomes more profitable to export. Define $\hat{\theta}_3$ to be the cutoff point such that $q^E = 0$ at this point but $q^E > 0$ for higher θ . To determine export behavior, recall that the firm can sell as much as it wants at the price $\frac{1}{\tau}$ up to the total quantity demanded, i.e. $q^E \leq \tau L^*$, so the monopoly profit is

$$\begin{aligned}\pi(\theta, w, \Pi) &= \max_{q^E \leq \tau L^*} p^L q^L + p^E q^E - wc(q^L + q^E, \theta) \\ &= \max_{q^E \leq \tau L^*} (wL + \Pi) + \frac{1}{\tau} q^E - wc\left(\frac{wL + \Pi}{\tau} + q^E, \theta\right).\end{aligned}$$

The FONC is

$$\frac{1}{\tau} - w \theta^{-\frac{\alpha}{1-\alpha}} \left(\frac{wL + \Pi}{\tau} + q^E\right)^{\frac{\alpha}{1-\alpha}} = 0$$

Solve out for the optimal export quantity yields

$$q^E = (w\tau)^{-\frac{1-\alpha}{\alpha}} \theta - \frac{wL + \Pi}{\tau}$$

The cutoff marking indifference to exporting is

$$\hat{\theta}_3 = w^{\frac{1-\alpha}{\alpha}} \tau^{\frac{1-2\alpha}{\alpha}} (wL + \Pi).$$

²The max operator rules out monopoly when duopoly is feasible—see Elimination Criterion 1 in the paper.

Maximized profits when exporting are

$$\pi(\theta) = (wL + \Pi) \left(1 - \frac{1}{\tau^2}\right) - \alpha\theta \left(w - w^{-\frac{1-\alpha}{\alpha}} \tau^{-\frac{1}{\alpha}}\right). \quad (4)$$

Given $w \geq \tau^{-1}$, it is immediate that $\pi(\theta)$ is non-increasing. We know that $\pi(\theta_1) > 0$. Define $\hat{\theta}_0$ to be the point where $\pi(\theta) = 0$,

$$\hat{\theta}_0 \equiv \frac{1}{\alpha} (wL + \Pi) \left(1 - \frac{1}{\tau^2}\right) \left(w - w^{-\frac{1-\alpha}{\alpha}} \tau^{-\frac{1}{\alpha}}\right)^{-1}.$$

Note that for an arbitrary w and $\Pi \geq 0$, we might have $\hat{\theta}_0(w, \Pi) > \bar{\theta}$, i.e. the cutoff might occur at a product level that is higher than any existing product. As we shall see, this will not occur in any equilibrium.

Step 3: Solving for aggregate profits

For a given level of $w \in [\frac{1}{\tau}, \tau]$, define a function $\Pi(w)$ to be the minimum value of Π that solves the following

$$0 = H(\Pi, w) \equiv \Pi - \int_{\hat{\theta}_1(w, \Pi)}^{\hat{\theta}_0(w, \Pi)} f(\theta) \pi(\theta, w, \Pi) d\theta,$$

where we define $f(\theta) = 0$ for $\theta > \bar{\theta}$.

Lemma 4.2 *The function $\Pi(w)$ defined above exists and is continuous and satisfies $\Pi(w) \geq 0$.*

Proof. It is immediate for $w < \tau$, that $H(0, w) < 0$. For sufficiently large Π the integral goes to zero and so $H(\Pi, w) > 0$. Since $H(\Pi, w)$ is continuous a solution exists. The continuity of $\Pi(w)$ follows from the theorem of the maximum. ■

Observe that at $w = \tau$, $\Pi(w) = 0$ must hold.

Step 4: Solving for the market-clearing wage

Lemma 4.3 *There exists a wage, $w \in (\frac{1}{\tau}, \tau)$, which clears the labor market.*

Proof. Define excess demand by

$$E(w) = \int_0^{\hat{\theta}_0(w, \Pi(w))} f(\theta) c(q(\theta, w), \theta) d\theta - L.$$

This is continuous. Consider $w = \tau$. At this wage $\Pi(w) = 0$. As w approaches its upper bound, τ , exports go to zero and autarchy prevails. This results in a labor surplus. To see this, we use the following limits,

$$\begin{aligned}\lim_{w \rightarrow \tau} I &= \tau L \\ \lim_{w \rightarrow \tau} \hat{\theta}_1 &= \frac{L}{2} \\ \lim_{w \rightarrow \tau} \hat{\theta}_k &= L, \quad \text{for } k = 2, 3, 0\end{aligned}$$

to show that

$$\lim_{w \rightarrow \tau} E(w) = LF(L) - L < 0.$$

The inequality uses $L < \bar{\theta}$ (from the resource constraint, equation (6) in the text) to show that $F(L) < F(\bar{\theta}) = 1$. On the other hand,

$$\lim_{w \rightarrow \frac{1}{\tau}} \hat{\theta}_0 = \bar{\theta}$$

which implies that as w approaches its lower bound all goods are produced at the small location. Therefore,

$$\lim_{w \rightarrow \frac{1}{\tau}} E(w) > \int_0^{\bar{\theta}} \alpha \theta f(\theta) d\theta - L > 0,$$

where the first inequality follows from looking only at the labor requirements for task 1, and the second inequality is from the resource constraint (6). Existence follows from continuity of $E(w)$. ■

This completes the existence of the equilibrium market structure. While a number of cut-offs for θ are used in calculations, there are really only two that characterize market structure (the rest have to do with where on the average cost curve a monopoly firm is). For θ below $\hat{\theta}_1$, there is duopoly, and above $\hat{\theta}_1$ (but below $\hat{\theta}_0$) there is monopoly. Beyond $\hat{\theta}_0$ there are no firms at the small location.