

# Bar Codes Lead to Frequent Deliveries and Superstores

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## **Abstract**

This paper explores the consequences of new information technologies, such as bar codes and computer-tracking of inventories, for the optimal organization of retail. The first result is that there is a complementarity between the new information technology and frequent deliveries. This is consistent with the recent move in the retail sector toward higher-frequency delivery schedules. The second result is that adoption of the new technology tends to increase store size. This is consistent with recent increases in store size and the success of the superstore model of retail organization.

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## 1. Introduction

The continuing revolution in computing technology and the emergence of the “new economy” is one of the big stories of our time. As economists, we should be interested in understanding the impact of this revolution on the organization of economic activity. Retailing is one sector for which the effects might be particularly important. Certainly the adoption of new information technology, such as bar codes and computer tracking of inventories, has been pervasive in this sector.

The new information technology has led to a substantial reduction in the time interval between when inventory levels are measured and when new orders are delivered. This paper explores the consequences of this innovation for the optimal organization of retail. There are two main results. First, the advent of the new technology induces stores to increase delivery frequency. Second, the optimal store size increases. Both of these implications are consistent with recent trends in the retail sector.

The second result is the easiest to understand. Optimal store size increases as a consequence of the first result that delivery frequency increases combined with the fact there are obvious scale economies in deliveries. If a store remains the same size but doubles its delivery frequency, then on average the trucks making the deliveries will be half as full. Thus there is an incentive for stores to get bigger to fill up the trucks and thereby economize on delivery costs. The resources used in transportation in the retail sector are substantial and in 1992 accounted for \$52 billion dollars or 4.7 percent of the value of output in the sector.<sup>2</sup>

Given the size of these costs, the link between changes in the size structure of retail and increases in delivery frequency may be of substantial quantitative significance.

This second result identifies a new factor contributing to the trend that superstores like Wal-Mart are replacing “Main Street.” Main Street is a collection of stores in a small town that might include a hardware store, a pharmacy, a clothing store, etc. Because of the small scale of these stores, the frequency of deliveries for such stores is often once a week or less.

The trucks that come into town to make deliveries to such stores tend to be dedicated to a

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<sup>2</sup> This figure includes the imputed cost of in-house trucking services as well as hired trucking services. The value of retail output is sales less the cost of goods sold. The reported figure here is from the U.S. Transportation Satellite Accounts for 1992 (Fang et al. 1998).

particular kind of product; i.e. one truck carries hardware, another truck carries clothing, etc. The function of a Wal-Mart in this paper is to fold all of these various businesses into one operation so that these businesses can share delivery trucks. The total number of trucks making deliveries in any given town may still be the same. But now each truck can contain hardware, clothing, etc., so each of the product lines can get daily deliveries. The reason this is happening now, and not before, is that the new technologies are putting a premium on having a high delivery frequency that did not exist before the advent of the new technologies.

I now explain why the new technologies have created this premium. This is the first result stated above, that the new information technology and high delivery frequency are complements. Consider two stores: one gets deliveries every day, the other only once a month. Before the advent of the new technology, when either store placed an order it would have had relatively imprecise information about its current inventories. Perhaps a store would know yesterday's inventories, but would not have had a chance yet to process yesterday's sales information, so the current inventory would be uncertain. Alternatively, even if there were no such lags at the store level, there might have been lags at the supplier level, so that an order for today would have to have been based on the inventory from yesterday. Now with the advent of the new technology, it is possible for today's order to be based on today's inventory. Clearly this information is valuable as either store can more finely tune inventories to demand, and thereby reduce stockouts as well as excess inventory. My result is that the information is **more** valuable for the daily delivery store than the monthly delivery state. Thus the new technology tends to favor stores with very frequent delivery.

To understand the result, consider how the value of the new technology changes when moving from the daily-delivery case to the monthly-delivery case. My result is the overall effect of this movement is negative; i.e., the value of the new technology declines as the cycle length increases. To obtain the result, I decompose this overall change into three distinct components.

The first is the **law-of-large-numbers** effect. Average sales per day over the cycle are more predictable with a monthly cycle than with the daily cycle, since the monthly case is

averaging 30 draws of demand. Since average demand is more predictable, information about its realization is of less value. This effect on the value of bar codes in the movement from the daily to the monthly case is then obviously negative.

The second is the **less-relevant-information** effect. In the daily delivery case, bar codes are providing information about one hundred percent of the sales in the previous order cycle. In the monthly delivery case, bar codes provide new information only about sales for day 30, the last day of cycle. By the time of the order, the store knows what has happened on days 1 through 29, even without bar codes. With a long order cycle, bar codes provide relatively less new information, as most of the information needed for the order is old news by the point of the order. Because of the less-relevant information effect, moving from the daily to the monthly case has a clear negative effect on the value of bar codes.

The third is the **increased-holding-cost** effect. The average amount of time a good is held in inventory before sale is substantially longer with monthly delivery than with daily delivery. So inventory holding costs are more of a consideration in the monthly case. As explained by Milgrom and Roberts (1988), inventories can serve as a substitute for information. In the daily delivery case, inventory holding costs are negligible, so it is cheap to use inventories to substitute for information. But this strategy is expensive in the monthly case. Thus any fixed amount of information provided by bar codes can be more valuable for the monthly case. Therefore, this third effect can be positive, offsetting the first two negative effects.

My key finding is that the negative second effect outweighs any positive contribution from the third effect. Adding in the negative first factor reinforces this for a total effect that is negative. Thus the value of bar codes declines when moving from the daily case to the monthly case. In some cases, the law-of-large numbers effect is the predominate factor in this decline; but in other cases the less-relevant-information effect is predominate.

The argument for why the second effect outweighs the third effect is complex. But it is possible to give a sketch of the reasoning. When a store gets bar codes it can avoid making a mistake in its order decision. Suppose the mistake is that the store orders one extra unit of inventory that sits in stock the entire order cycle and is not sold. The impact of effect three above is that such a mistake can be 30 times more costly in the monthly case than in the daily case, since the extra unit can sit in inventory 30 times as long. The impact

of effect two is that the advent of bar codes eliminates one mistake a month with monthly delivery compared to 30 mistakes a month with daily delivery. So it would seem that the two effects cancel each other out, and indeed they do in certain limiting cases. But more generally, an extra unit of inventory is less likely to remain unsold over the cycle in the monthly case, because the increase in average holding costs will make the optimal stockout probability higher. The expected time in stock for such an extra unit is less than a month and, if the good is sold, the profit on the sale offsets the holding cost. Thus a mistake in the monthly case where an extra unit of inventory is held is less than 30 times as costly a mistake as in the daily case, so the third effect is dominated by the second effect.

There is actually a fourth effect that is not incorporated into the model. The new technologies clearly have lowered the transaction cost of placing an order. This savings is worth less, the less frequently orders are placed. Thus, this negative fourth effect reinforces the net negative effect of the other three factors. While this effect is likely to be empirically important, there is no need to spend any more time here analyzing this effect, since it is obviously negative.

#### Related Theoretical Literature

The order decision model considered here is a variant of the “newsvendor” problem in the inventory management literature.<sup>3</sup> In this classic problem, a seller is faced with demand uncertainty. The benefit of holding an extra unit of inventory is that if demand is high enough the seller gets an additional sale it otherwise would not get. The cost is that the extra unit might not be sold if demand is low—this is a problem since holding inventory consumes resources. Thus the optimal inventory stock trades off this benefit and cost.

The operations research literature has not addressed my question of how greater precision of information about inventories affects optimal delivery frequency and the optimal store size. But this literature has studied the related issues of how reductions in the randomness of demand (see, for example, Gerchak and Mossman (1992)) and greater precision of inventory information (see, for example, Song (1994)) affect optimal profits and optimal inventory holdings. It is clear that both factors increase expected profit. It is intuitive that both factors should also reduce optimal inventory holdings and that is the general tendency (although

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<sup>3</sup> For a recent textbook treatment of the newsvendor problem see Anupindi et al (1999).

counter examples can be constructed).

Turning to the economics literature, this paper is closely related to Milgrom and Roberts (1990). They discuss a number of new practices that are being adopted in modern manufacturing, such as computer aided design, smaller batch sizes, shorter product-development times, quicker order processing, and speedier delivery. They show these various activities are complementary. My first result is in the same spirit.

Somewhat surprisingly, there have been few attempts in the economics literature to try to explain recent increases in store size and the emergence of superstores such as Wal-Mart. The advent of the car is the key explanation for increases in store size earlier this century, but presumably this technological force has played itself out and there are now other forces at work. One recent paper addressing this phenomenon is Bagwell, Ramey, and Spulber (1997). They emphasize the importance of scale economies in the adoption of new information technologies. Since small stores and even elementary school libraries are using bar codes now, it may be that the relevant scale economies are in complementary activities such as deliveries rather than in the adoption of the information technologies, *per se*. A related idea is that the adoption of the new information technology may require greater coordination of the various stages of distribution; such coordination may be easier within the same firm and can potentially account for Wal-Mart's vertical integration into wholesaling activities. The share of small independents may be declining since they lack this vertical coordination. Surely this factor and other factors unrelated to the mechanism that I highlight play important roles in accounting for why store sizes are increasing. The modest goal of this paper is to fully study one particular channel through which changes in technology affect the size structure of establishments. The important topic of assessing the relative contributions of various explanations is left to future work.

Economists defer to operations research specialists the job of how to tell firms to manage their inventories. But understanding the fundamental forces affecting establishment size is a job that economists have held onto themselves. There is a tradition of important work in this area including Lucas (1978) and Chandler (1990). This work has important implications for antitrust policy. In the retail sector, a major recent case was the proposed merger of Staples and Office Depot that was challenged by the FTC. Analysis of the appropriate

policy in such mergers requires an understanding of why superstores such as Staples and Office Depot have emerged. To the extent that the advent of new information technologies is playing a role, economists need to study the mechanics of this relationship, rather than leave it as a black box to be worked out by operations research specialists.

### Related Empirical Literature

The dramatic effect of the new information technologies on the ordering process is well documented.<sup>4</sup> Through the use of bar codes, stores can now keep track of inventories on a real time basis. Electronic data interchange (EDI) systems have been developed, connecting retailers' computers with their wholesalers' computers. This analysis will use the term "bar codes" to refer to the new technology but the term is meant more broadly to include all the new technologies that enable decision makers to have more information at the point they place orders.

Two recent papers, Abernathy et al. (1995) and Hwang and Weil (1998), examine evidence for the first result of this paper that short lead times and frequent delivery are complements. Both studies examine data on apparel suppliers. Suppliers are classified as either high- or low-frequency suppliers. The studies find that suppliers making frequent deliveries are substantially more likely to make use of the new information technologies in their relationships with retailers. For example, Hwang and Weil (1998) report that in 1988, 70 percent of high-frequency suppliers were putting bar codes on their products while only 38 percent of low-frequency suppliers were doing so. In addition to this cross-section evidence, there is strong evidence of this relationship in the time series. There was a substantial increase in the use of the new information technologies over the period from 1988 to 1992. In fact, Hwang and Weil (1998) report that use of Electronic Data Interchange (EDI) increased from 33 percent to 83 percent over this four-year period. Over this same period, there was a substantial increase in delivery frequency. Whereas in 1988 only 7 percent of sales were delivered on a weekly frequency, by 1992 the fraction of such sales had increased to 30 percent.

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<sup>4</sup> References on these various technologies include Kinsey et al. (1996); Brown (1997); Dunlop and Rivkin (1997); Kahn and McAlister (1997); and textbooks on logistics, including Coyle, Bardi, and Langley (1996). See also Raman et al. (2001) for a discussion of problems encountered in the implementation of these technologies.

In addition to this statistical evidence, one gets a clear sense from looking at both the trade and academic literature that delivery frequencies have increased substantially in recent years. Much is now written about the recent emergence of the “continuous replenishment system” in which there are daily deliveries to restock inventories. In cases where this system has been adopted, for example, in Procter and Gamble’s shipments to Wal-Mart, and Campbell Soup’s shipments to retailer distribution centers, it is clear that there has been substantial use of the new information technologies to link upstream with downstream.<sup>5</sup>

At this point, there is little formal evidence on result two of the paper concerning the connection between the new information technologies and store size. However, the anecdotal evidence concerning Wal-Mart and other superstores, such as Home Depot, is consistent with this link. The emergence of these large stores has increased store sizes, particularly in rural areas.<sup>6</sup> It is well known that superstore chains, such as Wal-Mart and Home Depot, have been pioneers of the new information technologies. They have also been leaders in increasing the frequency of deliveries. For example, according to Vance and Scott (1994, p. 93), “Its insistence on the frequent, often daily delivery of merchandise set Wal-Mart apart from rivals such as K-Mart, which generally delivered merchandise to its stores once every five days.”

## Organization

Section 2 develops the model of the order decision for a particular product, in which the information technology and the delivery frequency are both exogenously fixed. Section 3 does the main work of the paper. It shows that the value of bar codes decreases when deliveries become less frequent. Section 4 imbeds the order-decision model into a model of a store where order frequency and store size are both endogenous. Section 5 concludes.

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<sup>5</sup> See Cachon and Fisher (1997) and Kumar (1996).

<sup>6</sup> My analysis of County Business Patterns data (U.S. Bureau of the Census 1981, 1997) indicates that the percentage of retail employment in establishments with 20 or more employees increased from 57 percent in 1977 to 67 percent in 1995. In the building materials sector that is now dominated by Home Depot, the share increased from 35 to 57 percent over the same period. Even if we hold fixed the identity of the chain, stores are getting bigger. Wal-Mart is now constructing “supercenters” that also sell groceries and are substantially larger than their earlier generation stores (see the article “Consumer acceptance of one-stop shopping fuels supercenter growth,” in Discount Store News, July 13, 1998.)

## 2. The Order-Decision Model

This section sets up the model that will be analyzed in the next section. It is a model of a store where sales arrive every period and deliveries arrive every  $n$  periods. Realized demand is random and i.i.d. over time. Since deliveries arrive every  $n$  periods, the model belongs to the class of fixed order length models.

While fixed order length models get some attention in management textbooks, most of the emphasis is on models where orders do not occur at regular intervals. Rather, decision makers continuously review stocks and choose to have deliveries whenever stocks are low. Before describing the model, I need to motivate why I chose a fixed delivery time rather than allow it to depend upon the inventory stock level. Of course my assumption would not be appropriate in cases where a store orders only a single good from a particular supplier and gets deliveries shipped via Federal Express whenever inventories are low. But my assumption is appropriate in the common situation where a store is supplied by a wholesaler that carries thousands of different products. Such wholesalers commonly make regular deliveries. Since there are generally so many different items involved in a delivery, it is unlikely that the stock level of any one particular good, unless it is an exceptionally important good, will affect the timing of delivery. If a hardware store is supposed to get a delivery from its wholesaler next Friday, and if stock of a particular screw size is running low, it would be surprising if the timing of the delivery were moved up, just to raise the stock of the particular screw to the desired level.

I now turn to the details of the model. Demand in period  $t$  is the random variable  $z_t$  drawn i.i.d. over time from a distribution with mean  $\mu$ , a minimum of zero and a maximum of  $k$ . At some points in the analysis, it is convenient to use a discrete distribution,  $z_t \in \{0, 1, 2, \dots, k\}$ , where  $z_t$  is a nonnegative integer less than or equal to the maximum possible demand  $k$ . Let  $f_z$  be the probability of demand realization  $z$ ,  $\sum_{z=0}^k f_z = 1$ . At other points I will use a continuous distribution on the range  $[0, k]$  with density  $f(z)$  that is strictly positive on the interior of the support.

Each period  $t$  begins with an inventory level  $s_t$  that is called the initial stock. If period  $t$  is a delivery period, the store places an order  $x_t$  at time  $t$  for delivery the same period. The deliveries occur every  $n$  periods and without loss of generality I assume the delivery periods

are  $t = 1, t = n+1, t = 2n+1$ , and so on. The delivered goods become immediately available for purchase. The inventory on the shelves before consumers arrive is then  $r_t = s_t + x_t$ . The inventory level  $r_t$  is called the **available stock**. If period  $t$  is not a delivery period then the available stock equals the initial stock,  $r_t = s_t$ .

At the end of each period the realization of demand  $z_t$  occurs. Sales in the period are the minimum of demand and the available stock  $r_t$ ,

$$q_t = \min \{r_t, z_t\}.$$

The store begins the next period with  $s_{t+1} = r_t - q_t$  as the initial stock.

Each consumer that arrives is assumed to have an inelastic demand for one unit up to a particular reservation price. The firm sets the retail price equal to this reservation price to extract all surplus. The difference between the retail price and the wholesale price is the gross margin and is denoted by the parameter  $\gamma > 0$ . The cost of holding inventory in a period is  $\lambda$  per unit where  $\lambda < \gamma$ . This is assessed at the beginning of the period, so it is paid regardless of whether or not the good is sold.<sup>7</sup> Thus  $\gamma - \lambda$  is the net profit on a sale in a period. The description of the model so far implicitly assumes there is no backordering; if a buyers shows up and there is no merchandise, the net profit  $\gamma - \lambda$  is lost which the store never recovers. More generally, I can relabel things so that  $c_u = \gamma - \lambda$  is an **underage** cost, i.e. a loss per unit shortfall any time  $r_t$  is under  $z_t$ , and  $c_o = \lambda$  is an **overage** cost, a loss per unit excess any time  $r_t$  is over  $z_t$ . Backordering can be allowed by assuming that the underage cost  $c_u$  is not as high as the profit margin, so some profit is eventually made on customers who are initially not served.

There are three possibilities as to the available information when an order is placed in some period  $t$ . These cases are indexed by the parameter  $\ell$  which is called the **information lag**. Alternatively, I could call  $\ell$  the length of the review period, which is what it is called in the inventory management literature.<sup>8</sup> The case of  $\ell = 0$  is an extreme case where the store is actually able to observe what demand  $z_t$  will be at the end of the period when it places the order  $x_t$  at the beginning of the period. In this case the store can order to

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<sup>7</sup> The model is equivalent to a model where the holding cost  $\lambda$  is paid only if the good is not sold in the period and the gross margin is  $\gamma^0 = \gamma - \lambda$ .

<sup>8</sup> The review period measures the time between when inventory is measured and when the order is placed.

exactly match the end-of-period demand. In the case  $\ell = 1$ , the store can observe the initial inventory stock  $s_t$  but does not observe demand  $z_t$ . In the case of  $\ell = 2$ , the store must base its order only on knowledge of the available stock  $r_{t-1}$  from the previous period. The advent of bar codes will be modeled as a reduction in  $\ell$ . Under one scenario  $\ell$  falls from 1 to 0; under a second it falls from 2 to 1. My motivation for looking at the first case is that it is extremely tractable. My motivation for the second case is that it is more realistic. The results turn out to be similar for the two cases.

Assume the store selects its order policy to maximize the long-run average per-period profit. I have also considered the case where the store maximizes discounted expected profits with discount factor  $\beta < 1$  with similar results.

## 2.1 The Optimal Inventory Policy

This subsection analyses the inventory policy under optimal behavior. The objective is to calculate  $v^\ell(n)$ , the long-run average profit when the information lag is  $\ell$  and the cycle length is  $n$ . It is convenient for this section to assume that demand is discrete. This facilitates the analysis of the  $\ell = 2$  case since it ensures a finite state space, which is of use when solving the dynamic problem. The issue of a finite state space does not come up for the cases of  $\ell = 0$  and  $\ell = 1$  since the store's problem in these cases is a static problem.

The solution is trivial when the information lag  $\ell$  is 0 and the cycle length  $n$  is 1. Since the store observes the end-of-period demand  $z_t$ , it sets the order  $x_t$  equal to  $z_t$ . Daily average profit is

$$v^0(1) = \mu(\gamma - \lambda),$$

mean expected demand times the difference between gross profit minus the holding cost. When  $\ell = 0$  and  $n > 1$ , the store knows the sales in the first period of the order cycle when placing the order but does not observe sales in the later periods. I skip the derivation of this case since it is an immediate extension of the  $\ell = 1$  case.

To derive the value in the  $\ell = 1$  and  $\ell = 2$  cases, I need to introduce some additional notation. Let  $\pi_r$  be the expected profit in a period when the available stock in the period

is  $r$ ,

$$\pi_r = \gamma \int_{z=0}^{\infty} f_z \min\{z, r\} - \lambda r. \quad (1)$$

This is  $\gamma$  times expected sales minus the holding cost. Straightforward arguments show it is sufficient to restrict attention to the set of integer values  $r \in \{0, 1, 2, \dots, (1+n)k\}$ . Under optimal behavior for either  $\ell = 1$  or  $\ell = 2$ , all the states along the optimal path can be found in this set. Let  $\Pi = (\pi_0, \pi_1, \dots, \pi_{(1+n)k})$  be the vector of expected current profits for each of these possible states.

Suppose there is no delivery next period. Let  $\phi_{r,r'}$  be the probability the store will be at  $r'$  next period given it is at  $r$  in the current period. Let  $\Phi$  be the matrix formed with  $\phi_{r,r'}$  as its elements.

Let  $v_r$  be the sum of the expected average profit over the  $n$  periods in the order interval, given that the state is  $r$  in the first period of the order interval. Let  $V = (v_0, v_1, v_2, \dots, v_{n(k+1)})$  be the vector of expected average profits. The first term of  $v_r$  is the return  $\pi_r$  in day 1 divided by  $n$ . To obtain the expected return in the later periods 2 through  $n$ , we use the transition matrix  $\Phi$ . Thus the vector of average profits is

$$V = \frac{\sum_{t=1}^n \Phi^{t-1} \Pi}{n},$$

where  $\Phi^0$  is defined as the identity matrix, and  $\Phi^t$  is the  $t$ -step transition matrix,  $\Phi^t = \Phi^{t-1} \Phi$ .

Consider the easy case where  $\ell = 1$ . At the beginning of the order cycle, the store picks the available stock  $r$  to maximize the expected average profit over the order cycle,

$$r^* = \arg \max_r v_r.$$

The store's optimal order in period  $t$  is then  $x_t = r^* - s_t$ , the difference between the optimal available stock and the initial stock. The maximized expected average daily profit when  $\ell = 1$  is

$$v^1(n) = v_{r^*},$$

where the dependence on  $n$  is made explicit for later use.

Now consider the more complicated case where  $\ell = 2$ . Here the order at time  $t$  depends on the available stock  $r_{t-1}$  from the previous period. Let  $x_r$  be an order given that the

available stock the previous period was  $r$ . Let  $X = (x_0, x_1, \dots, x_{(1+n)k})$  be the policy rule containing these choices.

Now consider the choice of the optimal order policy  $X$ . Given an order policy  $X$ , let  $\omega_{r,r'}(X)$  be the probability of going from an available stock of  $r$  at the end of the last cycle to an available stock level  $r'$  at the beginning of the current cycle. Let  $\Omega(X)$  be the matrix with  $\omega_{r,r'}(X)$  in row  $r$  and column  $r'$ . Let  $p_r(X)$  be the long-run average probability that the available stock is  $r$  at the beginning of an order cycle, given a policy rule  $X$ . Let  $P(X) = (p_0(X), p_1(X), \dots, p_{(n+1)k}(X))$  be the vector formed by these probabilities. This is the unique solution to

$$P(X) = \Omega(X)' [\Phi']^{n-1} P(X)$$

Note that in periods 1 through  $n - 1$  the transition to the available stock in the next period is governed by  $\Phi$  which applies when deliveries are zero. The transition from period  $n$  to the first period of the new cycle is governed by  $\Omega(X)$  which includes deliveries.

The choice of  $X$  determines the fraction of time  $p_r(X)$  that the stock at the beginning of a cycle is at a particular  $r$ . The expected average profit in the cycle given  $r$  is  $v_r$  from above. Thus, the maximized average value over the cycle with  $\ell = 2$  is

$$v^2(n) = \max_X P(X)'V.$$

### 3. The Value of Bar Codes and Delivery Frequency

This section is the meat of the paper as it demonstrates that frequent deliveries and bar codes are complements. The advent of bar codes is interpreted as a reduction in the information lag  $\ell$ . Formally,  $\ell^B < \ell^N$  where  $\ell^B$  and  $\ell^N$  denote the information lag with bar codes and with no bar codes. Let  $\Delta(n)$  be defined as the difference in average profit per period between the bar-code and the no-bar-code case as a function of the length  $n$  of the order cycle; i.e.,

$$\Delta(n) \equiv v^{\ell^B}(n) - v^{\ell^N}(n). \quad (2)$$

Deliveries are more frequent the shorter the order cycle and the maximum delivery frequency is obtained when the order cycle is one period. This section presents several results that indicate that  $\Delta(n)$  is strictly less than  $\Delta(1)$  for  $n > 1$ . Thus a store with the highest delivery

frequency has the greatest value of bar codes; i.e., bar codes and high delivery frequency are complements. A stronger statement is that not only is  $\Delta(n)$  greater than  $\Delta(1)$ , but furthermore  $\Delta(n)$  is monotonically decreasing. Discussion of this stronger statement is deferred to the end of the section where I consider numerical examples.

As discussed in the introduction, there are three distinct reasons why the value of bar codes differs between the  $n$ -period and 1-period cases. The first two effects, the law-of-large-numbers and the reduced-information effects, tend to make  $\Delta(n)$  smaller than  $\Delta(1)$ ; these effects negatively influence the difference  $\Delta(n) - \Delta(1)$ . The third effect, the increased-holding cost effect, can work in the opposite direction, having a positive effect on the difference  $\Delta(n) - \Delta(1)$ . The section begins by formally decomposing the change to  $\Delta(n)$  from  $\Delta(1)$  into these three components. It then shows that the negative second factor outweighs any positive effect of the third factor, and so the net effect is negative from just looking at these two. Adding in the first factor makes the combined effect even more negative. Thus  $\Delta(n)$  is less than  $\Delta(1)$  as claimed.

Two different scenarios are considered:

Scenario 1. Bar codes reduce the information lag from  $\ell^N = 1$  to  $\ell^B = 0$ .

Scenario 2. Bar codes reduce the information lag from  $\ell^N = 2$  to  $\ell^B = 1$ .

Scenario 1 is analytically tractable and a number of formal theorems can be obtained that illustrate the factors at work. Scenario 2 is more realistic, but less tractable, and theorems are more difficult to come by. Scenario 2 is amenable to numerical analysis and I show that the insights gleaned from the formal analysis of the first case carry over to the second case.

### 3.1 A Decomposition

In order to provide a formal decomposition of the effect into the three factors discussed above, I introduce two hypothetical intermediate cases between the  $n$ -period case and the 1-period case.

The first hypothetical case is like the  $n$ -period cycle in that there are  $n$  draws of demand. But suppose that these draws occur in a single period and that the realized demand in the

period is divided by  $n$ . Thus the demand realization in the period is

$$\tilde{z} = \frac{\prod_{i=1}^n z_i}{n}, \quad (3)$$

where each  $z_i$  is drawn from the distribution  $F$  from before. I use the symbol “ $\sim$ ” to denote this case and call it the tilde case. Let  $\tilde{v}^\ell(n)$  denote the average value in the tilde case given information lag  $\ell$  and let the value of bar codes be  $\tilde{\Delta}(n) \equiv \tilde{v}^{\ell^B}(n) - \tilde{v}^{\ell^N}(n)$ .

The second hypothetical case is exactly like the  $n$ -period case except for the fact that the holding cost  $\lambda$  is incurred only for inventory held in the first period of the cycle. Inventory held between periods 2 and  $n$  in the cycle has no holding cost. I use the symbol “ $\hat{\cdot}$ ” to denote this case and call it the hat case. Let  $\hat{v}^\ell(n)$  denote the average value in the hat case given information lag  $\ell$ . Let the value of bar codes be  $\hat{\Delta}(n) \equiv \hat{v}^{\ell^B}(n) - \hat{v}^{\ell^N}(n)$ .

Define  $H(n)$  to be the difference between  $\Delta(n)$  and  $\Delta(1)$ . Decompose this difference into three parts by starting at the 1-period case and moving sequentially to the tilde, the hat, and the  $n$ -period case and calculating the change from each step,

$$\begin{aligned} H(n) &\equiv \Delta(n) - \Delta(1) \\ &= \overset{\text{h}}{\tilde{\Delta}(n) - \Delta(1)} + \overset{\text{h}}{\hat{\Delta}(n) - \tilde{\Delta}(n)} + \overset{\text{h}}{\Delta(n) - \hat{\Delta}(n)} \\ &\equiv H_1(n) + H_2(n) + H_3(n). \end{aligned}$$

The first component  $H_1(n)$ , the movement from the 1-period to the tilde case, is the law-of-large-numbers effect. With this change, the order cycle length stays fixed at 1, but the number of demand draws is increased to  $n$  with the total divided by  $n$  as in (3). This change obviously reduces the variance in demand and in the limit as  $n$  gets large the variance goes to zero. Bar codes obviously have zero value when the variance of demand is zero so  $\tilde{\Delta}(n)$  is zero in the limit and

$$\lim_{n \rightarrow \infty} H_1(n) = -\Delta(1) \leq 0.$$

It is intuitive that  $H_1(n)$  should be negative for all  $n$ . As stated in Proposition 1 below,  $H_1(n) < 0$  under Scenario 1 (when the lag falls from 1 to 0). For Scenario 2, counterexamples can be constructed, as discussed later. However, these examples are somewhat pathological. The force of the law of large numbers in making  $H_1(n)$  negative is clear and overwhelming.

The second component  $H_2(n)$ , the movement from the tilde case to the hat case, is the less-relevant-information effect. The tilde and hat cases are alike in every dimension except for information. In both cases there are  $n$  demand draws over the cycle. In both cases the holding cost is  $\lambda$  per unit of inventory at the start of the cycle. (In this tilde case this is because the cycle is only one period; in the hat case this is because only the first period of the cycle entails a holding cost.) For simplicity, assume Scenario 1 applies. Then without bar codes, the store faces exactly the same problem in both cases. The store has to pick an inventory level to start the cycle with, when it does not know any of the  $n$  upcoming demand realizations and when the holding cost is  $\lambda$ . With bar codes in the tilde case, the store gets to observe all  $n$  realizations, while in the hat case the store gets to observe only the first of the  $n$  observations. Since the tilde store could always choose not to use its additional information, it is immediate that  $H_2(n) \leq 0$  must hold.

The third component  $H_3(n)$ , the step from the hat case to the  $n$ -period case, is the increased-holding-cost effect. The hat and the  $n$ -period cases are the same except for one difference: In the hat case, the store pays the holding cost only in the first period, while in the  $n$ -period case it pays it every period. It is easy to see that  $H_3(n)$  can be positive. Consider the simple example where the demand realization is binary each period, equaling  $z = 1$  with probability  $f_1$  and  $z = 0$  with probability  $f_0 = 1 - f_1$ . Suppose that  $\gamma$  is so high that it is optimal to hold enough inventory to meet any possible sequence of demand realizations. The value of bar codes turns out to be the same in both Scenario 1 and 2. In either scenario, bar codes enable the store to observe a single demand realization that it otherwise would not see. Without bar codes, the store uses the conservative strategy of ordering as though the unobserved demand realization were positive (to guarantee there is always sufficient inventory to meet any possible demand). But in the event the demand realization is actually zero (which happens with probability  $f_0$ ), the store ends up carrying one excess unit of inventory over the entire cycle. This raises average holding over the  $n$ -period cycle by  $n\lambda$ . Dividing through by  $n$  to convert to a per period basis, the benefit of bar codes is

$$\Delta(n) = f_0\lambda. \tag{4}$$

In the hat case, the store uses the same strategy. But now the holding costs is only incurred

in the first period. Dividing through by  $n$  to convert to a per period basis, the benefit of bar codes is

$$\hat{\Delta}(n) = \frac{f_0\lambda}{n}. \quad (5)$$

Thus the benefit in the hat case is only one  $n$ -th as large as in the  $n$ -period case, so  $H_3(n)$  is strictly positive.

The sum of the second and third components captures the movement from the tilde case to the  $n$ -period case,

$$\begin{aligned} H_{2,3}(n) &= H_2(n) + H_3(n) \\ &= \Delta(n) - \tilde{\Delta}(n). \end{aligned}$$

The results below will show that the negative effect of  $H_2(n)$  more than outweighs any positive effect of  $H_3(n)$ . To gain an understanding of what will be driving these results, I continue the discussion of the binary demand example above. In the tilde case, there are  $n$  draws of demand and the total is divided by  $n$  to get  $\tilde{z}$ . But note that the maximum possible realization of  $\tilde{z}$  remains 1 and is independent of  $n$ . (This occurs if all  $n$  draws come up  $z = 1$ ; after dividing through by  $n$ ,  $\tilde{z} = 1$ ). The expected value of  $\tilde{z}$  is also independent of  $n$ , remaining constant at  $f_1$ . (Of course the variance of  $\tilde{z}$  declines in  $n$ .) With  $\gamma$  assumed to be high as stipulated above, a store without bar codes uses the conservative strategy of ordering as though the unobserved value of  $\tilde{z}$  was its maximum level of 1. Since the expectation of demand is only  $f_1$ , the absence of bar codes raises average inventories by  $1 - f_1 = f_0$ . Since the holding cost is  $\lambda$ , the benefit of bar codes is the expected value of the reduction of inventories,

$$\tilde{\Delta}(n) = f_0\lambda. \quad (6)$$

But this is the same as  $\Delta(n)$ , so we see that the benefit of bar codes in the tilde case is exactly equal to the benefit in the  $n$ -period case, so that  $H_{2,3}(n) = 0$ . Bar codes reduce expected inventory holdings by the same amount in both cases.

This example assumed that the gross margin  $\gamma$  is extremely high so that the store always has enough inventory to meet any demand. If  $\gamma$  is low enough, it will not be profitable to hold such high inventories and stockouts will occur. The key point is that as  $\gamma$  is reduced, the  $n$ -period case will be first to have stockouts. The holding cost over the cycle is  $n\lambda$  for

the  $n$ -period case but only  $\lambda$  for the tilde case, so it is cheaper in the latter case to avoid stockouts. Suppose  $\gamma$  is in the range where there are no stockouts in the tilde case but stockouts occur with positive probability in the  $n$ -period case. Then  $\tilde{\Delta}(n)$  remains  $f_0\lambda$  as above. But  $\Delta(n)$  must be less than  $f_0\lambda$ , as I now show. Suppose without bar codes the store places the order that would be optimal under bar codes if the demand realization were positive. A store using this strategy will start the order cycle with one extra unit in inventory (compared to the bar code level) on a fraction  $f_0$  of cycles. In the discussion above, this extra unit had no hope of being sold, so it drove up average inventory costs by  $\lambda$ . But now stockouts occur with some probability, so this extra unit might actually be sold before the order cycle ends. The expected average holding cost of this extra unit is therefore less than  $\lambda$ . In addition, if the unit is sold, the gross margin  $\gamma$  of the sale will contribute to profits. Thus the absence of bar codes results in a welfare loss that is strictly less than  $f_0\lambda$ , so  $\tilde{\Delta}(n) > \Delta(n)$  in this region.

### 3.2 Scenario 1

This subsection presents analytic results. It is assumed throughout this subsection that Scenario 1 applies where bar codes reduce the lag from  $\ell^N = 1$  to  $\ell^B = 0$ . Since the store's problem is static in both cases, I don't need to worry about the state space, and it is convenient to assume in this subsection (unless specified otherwise) that demand is continuous with strictly positive density  $f(z)$  for  $z$  in the interior of the range  $[0, k]$ .

For the 1-period case, it is possible to derive a simple formula for the value of bar codes  $\Delta(1)$  which greatly facilitates the analysis. With no bar codes and a one period cycle, the optimal available stock  $r^*$  solves the first-order condition.

$$[1 - F(r^*)]\gamma = \lambda. \tag{7}$$

This balances the additional expected revenue from the marginal unit (the probability of its sale is  $F(r^*)$ ) with the marginal holding cost. If the store has bar codes and  $z$  turns out to be lower than  $r^*$ , it will save  $(r^* - z)\lambda$  from lower holding cost. If  $z$  turns out to be greater than  $r^*$ , it will gain  $(z - r^*)(\gamma - \lambda)$  in additional profit. The value of bar codes is then

$$\begin{aligned}
\Delta(1) &= \int_0^{r^*} (r^* - z)\lambda f(z)dz + \int_{r^*}^k (z - r^*)(\gamma - \lambda)f(z)dz \\
&= r^*\lambda F(r^*) - r^*(\gamma - \lambda)(1 - F(r^*)) + \gamma \int_{r^*}^k z f(z)dz - \lambda \int_0^k z f(z)dz \\
&= \gamma \int_{r^*}^k z f(z)dz - \lambda \int_0^k z f(z)dz \\
&= \lambda \frac{\int_{r^*}^k z f(z)dz}{1 - F(r^*)} - \mu \\
&= \lambda E[(z - \mu) | z \geq r^*],
\end{aligned} \tag{8}$$

The third and fourth terms use the first-order condition (7). If  $\lambda = \gamma - \lambda$  and if  $f(z)$  is symmetric, then  $r^* = \mu$  and formula (8) reduces to

$$\Delta(1) = \lambda E|z - \mu|. \tag{9}$$

This is the unit cost of a mistake times the expected size of the mistake.

The first result concerns the first component.

**Proposition 1.** The first component is strictly negative,  $H_1(n) < 0$ .

**Proof.** It is immediate that  $v^0(1) = \tilde{v}^0(n) = \mu(\gamma - \lambda)$ . Thus to prove the result, I need to show  $v^1(1) < \tilde{v}^1(n)$ . To see this is true, suppose in the tilde case the store could divide itself into  $n$  substores with each substore receiving a different one of the  $n$  individual draws of demand that average out to  $\tilde{z}$  through (3). In this case profit would be the same as in the  $v^1(1)$  case. But by allowing inventory to be traded among the substores in response to individual substore demand realizations, profit would even be higher. Q.E.D.

The rest of the subsection looks at  $H_{2,3}(n) \equiv \Delta(n) - \tilde{\Delta}(n)$ . I begin with a limit result for  $n$ .

**Proposition 2.** Assume that  $f(z)$  is symmetric around  $\mu$ . For large enough  $n$ ,  $H_{2,3}(n) < 0$ . Moreover,

$$\lim_{n \rightarrow \infty} \frac{\Delta(n)}{\tilde{\Delta}(n)} = 0.$$

**Proof.** Since  $\Delta(n)$  and  $\tilde{\Delta}(n)$  both go to zero, rather than divide by  $n$ , I look at  $n\tilde{\Delta}(n)$  and  $n\Delta(n)$ . Let  $\tilde{r}(n)$  be the optimal starting inventory with no bar codes. Then

$$\begin{aligned}
n\tilde{\Delta}(n) &= n\lambda \int_0^{\tilde{r}} \tilde{f}(\tilde{z}) (\tilde{r} - \tilde{z}) d\tilde{z} + n[\gamma - \lambda] \int_{\tilde{r}}^k \tilde{f}(\tilde{z}) (\tilde{z} - \tilde{r}) d\tilde{z} \\
&\geq \int_0^{\tilde{r}} \tilde{f}(\tilde{z}) (n\tilde{r} - n\tilde{z}) d\tilde{z} + \int_{\tilde{r}}^k \tilde{f}(\tilde{z}) (n\tilde{z} - n\tilde{r}) d\tilde{z} \times \min\{\lambda, \gamma - \lambda\} \\
&\geq E \left[ \sum_{i=1}^n z_i - n\mu \right] \times \min\{\lambda, \gamma - \lambda\}.
\end{aligned} \tag{10}$$

The third inequality holds because setting  $\tilde{r} = \mu$  clearly minimizes the bracketed term on the second line, given the symmetry assumption. The third line is the expected absolute deviation from the mean, times the minimum of the overage and underage costs. This inequality (10) can be rewritten as

$$n\tilde{\Delta}(n) \geq \sqrt{n} E \left[ \frac{\sum_{i=1}^n z_i - n\mu}{\sqrt{n}} \right] \times \min\{\lambda, \gamma - \lambda\}. \tag{11}$$

Using the central limit theorem, the expectation term in (11) above converges to a positive constant as  $n$  gets large (the distribution of the variable  $(\sum_{i=1}^n z_i - n\mu)/\sqrt{n}$  converges to the normal distribution with the same variance as  $z$ ). Thus  $n\tilde{\Delta}(n)$  increases in the limit at least at the rate of the square root of  $n$ .

In contrast, we have

$$n\Delta(n) \leq \mu(\gamma - \lambda). \tag{12}$$

To prove (12), observe first that with bar codes, the optimal order with an  $n$ -period cycle given realized demand  $z_1$  has the form  $r(z_1) = z_1 + r'$ ; i.e., the store orders  $z_1$  for the current period plus an amount  $r'$  to have on the shelves for sales in periods 2 through  $n$ . Suppose that without bar codes the store were to employ the following hypothetical policy (that obviously is not optimal): The store orders  $r'$  and refuses to sell to any consumers that arrive in the first period. The expected return over the order cycle from this hypothetical policy, beginning at period 2, is the same as for the optimal policy with bar codes, because in both cases the inventory is  $r'$  at the beginning of period 2. The difference is that in the optimal policy under bar codes, the store has additional sales of  $z_1$  in period 1, which yields an additional expected profit of  $\mu(\gamma - \lambda)$ . The increase in profits over the order cycle from having bar codes must be bounded by  $\mu(\gamma - \lambda)$  since the maximized return without

bar codes must be at least as high as the return from the hypothetical policy. Dividing this bound by  $n$  to convert it to a per-period basis yields  $\Delta(n) \leq \mu(\gamma - \lambda)/n$ , which proves (12).

The fact that  $n\tilde{\Delta}(n)$  increases without bound while  $n\Delta(n)$  is bounded proves the claim. Q.E.D.

Proposition 2 treats the case of large  $n$ . The next result considers the opposite extreme where  $n$  is small. Note in the statement of the result the dependence of variables such as  $\tilde{\Delta}$  and  $\Delta$  on  $n$  is implicit since  $n$  is fixed at  $n = 2$ .

**Proposition 3.** Assume  $n = 2$ .

(i) **Binary Demand and General  $\lambda$ .** Assume demand is discrete and binary,  $z \in \{0, 1\}$ . Then  $H_{2,3} \leq 0$  and the strict inequality holds for  $\lambda$  close enough to  $\gamma$ .

(ii) **Continuous Demand and Small  $\lambda$ .** Assume demand is continuous on  $[0, k]$  and that  $f(k) > 0$ . For positive  $\lambda$  close enough to 0,  $\tilde{\Delta} \approx \sqrt{2}\Delta$  so  $H_{2,3} < 0$ . But  $H_{2,3}$  is negligible compared to  $H_1$ ,

$$\lim_{\lambda \rightarrow 0} \frac{H_{2,3}}{H_1} = 0.$$

(iii) **Continuous Demand and Large  $\lambda$ .** Assume demand is continuous. For  $\lambda < \gamma$  close enough to  $\gamma$ ,  $\tilde{\Delta} \approx 2\Delta(2)$  so  $H_{2,3} < 0$ . Furthermore,  $H_1$  is negligible compared to  $H_{2,3}$ ,

$$\lim_{\lambda \rightarrow \gamma} \frac{H_1}{H_{2,3}} = 0$$

The proof is available on request.

The binary demand case is extremely tractable making it possible to derive an analytic result for all  $\lambda$ . For this case, Part (i) of Proposition 3 says that the negative effect of the second component  $H_2$  (the reduced-information effect) outweighs any offsetting effect of the third component  $H_3$  (the increased-holding-cost effect). Thus the combined effect  $H_{2,3}$  reinforces the negative law-of-large-numbers effect  $H_1$ .

Parts (ii) and (iii) consider limiting values of  $\lambda$  for the continuous demand case. For these limiting cases it is not only possible to show that  $H_{2,3} < 0$ , but it is also possible to compare the magnitude of  $H_{2,3}$  to  $H_1$ . For the case of small  $\lambda$ ,  $H_{2,3}$  is small in absolute value compared to  $H_1$ , analogous to the case of large  $n$ . To see why the law-of-large-numbers effect is predominate for small  $\lambda$ , note that in this range without bar codes, the optimal inventory is close to the maximum possible demand. The probability of getting  $n$  draws close to the

maximum possible level is negligibly small compared to the probability of getting one such draw, so the possibility of averaging  $n$  demand draws can lead to a relatively substantial reduction of inventory at the expense of a relatively small increase in stockout probability.

In the case where  $\lambda$  is large, the  $H_1$  effect is negligible compared to  $H_{2,3}$ , reversing what happens  $\lambda$  is small. This illustrates that factor two, the reduced information effect, can play a significant role. The law of large numbers effect is not the whole story in understanding why stores with less frequent delivery value bar codes less.

### 3.3 Scenario 2

Now consider the case where bar codes reduce the lag from  $\ell^N = 2$  to  $\ell^B = 1$ . This subsection shows that the main message of my previous results for Scenario 1 continues to hold. For large  $n$ , the analysis is very similar to the earlier analysis. Clearly  $\Delta(n)$  and  $\tilde{\Delta}(n)$  go to zero as before so that for large  $n$ ,  $H(n)$  is negative and the first component  $H_1(n)$ , the law-of-large-numbers component, is the whole story in the limit. But for small  $n$ , there is a subtlety.

The subtlety involves some counter-intuitive examples that can arise. Recall that the law-of large numbers effect in Scenario 1 is obviously negative. Bar codes are clearly worth less when sales are the average of  $n$  draws rather than a single draw. But suppose in Scenario 2, the draw of demand is a coin flip, with a probability  $f_0 = .5$  of  $z = 0$  and a probability  $f_1 = .5$  of  $z = 1$ . If  $\gamma = 1$ , the expected gross revenue of having a unit in stock is  $.5$ . If  $\lambda = .6$ , the holding cost exceeds expected revenue so the store shuts down regardless of bar codes and  $\Delta(1) = 0$ . Now consider the tilde case with  $n = 2$  so that the store gets the average of two coin flips. Then with probability  $.75$  the store will have at least a half a unit of demand, so the store can make a profit if  $\lambda = .6$ . Thus  $\tilde{\Delta}(2) > 0$  in this case. The law-of-large numbers effect  $H_1(2) \equiv \tilde{\Delta}(2) - \Delta(1)$  is strictly positive in this example.

It turns out that for the binary demand case, the total effect  $H$  is always nonpositive, despite the fact that the first component  $H_1$  can be positive,

**Proposition 4.** Suppose Scenario 2 applies and demand is binary. For  $n \in \{2, 3\}$ , an analytic result shows that  $H(n) = \Delta(n) - \Delta(1) \leq 0$ . Numerical analysis shows the result holds for  $n \geq 4$ .

The proof is available on request.<sup>9</sup>

If demand is discrete but not binary, I can construct examples where the effect  $H(n)$  is positive. But these examples are rare as I now explain. Consider the case where there are three possible realizations of demand  $z_t \in \{0, 1, 2\}$ . I calculated  $\Delta(1)$  and  $\Delta(n)$  over a uniform grid of the simplex for  $f$  and the unit interval for  $\lambda$  (normalizing  $\gamma = 1$ ). In 99 percent of the cases,  $\Delta(n) \leq \Delta(1)$ . In most of the cases where this did not hold, the first component  $H_1$  was positive for reasons analogous to the example above.

The discreteness of demand plays a central role in the counter-intuitive example constructed above. If demand were continuous it would always be optimal to have a positive inventory and the value of bar codes in the 1-period case  $\Delta(1)$  would be strictly positive, rather than zero as above. Of course, a discrete distribution can be approximated by a continuous distribution. To have any hope of eliminating these counter-intuitive examples, conditions on the smoothness of the distribution have to be made.

The remainder of this subsection focuses on two distributions that are smooth, the (truncated) normal distribution and the uniform distribution. I used discrete approximations to both distributions.<sup>10</sup> I calculated  $\Delta(1)$  and  $\Delta(2)$  over a fine grid of the models parameters for both distributions and found that  $\Delta(2) \leq \Delta(1)$  in every case. I also looked at the breakdown  $H(2) = \Delta(2) - \Delta(1)$  into its components, though because of computational considerations, I looked at a selected range of parameters and in each case  $H_1 \leq 0$  and  $H_{2,3} \leq 0$ . Table 1 plots  $H_{2,3}$  as a fraction of the total effect  $H$  for various levels of  $\lambda$  for a fixed normal distribution of sales.<sup>11</sup> As in Proposition 3 in the previous subsection, the combined effect of factors 2 and 3 is small when  $\lambda$  is small. But just as in Proposition 3, it becomes substantial when  $\lambda$  is large.

Table 2 illustrates how the value of bar codes varies with both  $\lambda$  and  $n$ . In this example, the range is  $z \in [0, 2]$  and the mean is  $\mu = 1$ , for a distribution of  $z$  that is approximately

<sup>9</sup> The proof derives a sufficient condition for  $H(n) \leq 0$  to hold that depends upon  $f_1$  (the probability  $z = 1$ ) and  $n$  but not  $\lambda$ . For  $n \in \{2, 3\}$  I show analytically that the condition holds. I show numerically that the condition holds for a grid of  $f_1$  with increments of size less than .01 and for  $n \leq 100$ .

<sup>10</sup>Let  $f_{\text{normal}}(z)$  be the normal distribution with mean  $\mu > 0$  and variance  $\sigma^2$ . Suppose the normal distribution is truncated from the left at 0 and from the right at  $2\mu$  so that it has a symmetric distribution on the range  $[0, 2\mu]$ . Define  $k = 2\mu$  and define  $f_z = f_{\text{normal}}(z) / \sum_{j=0}^k f_{\text{normal}}(j)$ . As  $k$  gets large, this discrete distribution approximates the normal distribution.

<sup>11</sup>This uses the same distribution as below.

normal. In this example the variance of demand equals the mean demand, reflecting a substantial degree of uncertainty. To interpret this table, it is useful to first discuss what happens when the holding cost is zero. For this case, if the store has bar codes and deliveries every period it would set  $r = 2$  in each period. Its expected profit average would be mean demand (since  $\gamma = 1$  and  $\lambda = 0$ ). If the store did not have bar codes, since the holding cost is assumed zero, it would still keep enough inventory to meet any demand realization. If  $r_{t-1}$  was on the shelves before consumers arrived last period, it would order as though the maximum demand  $z = 2$  were realized, i.e. it would order  $x_t = 2 - (r_{t-1} - 2)$ . It will therefore have an available stock  $r_t = r_{t-1} - z_{t-1} + x_t = 4 - z_{t-1}$ . The expected available stock without bar codes is then  $Er_t = 4 - \mu = 3$ , which is fifty percent larger than the level with bar codes.

When  $\lambda$  is close to zero, the policy just described is approximately the optimal policy. With delivery every period and bar codes, the expected profit is then the expected demand of 1 minus  $\lambda$  times the expected inventory of 2 units. For example when  $\lambda = .01$ ,  $v^B(1) \approx 1 - .01 \times 2 = .98$ . Without bar codes, average inventory is 3 rather than 2, and average value is  $v^N(1) \approx 1 - .01 \times 3 = .97$ . The value of bar codes is the difference  $\Delta(1) = v^B(1) - v^N(1) \approx .01$ . This explains the first line in Table 2. Observe that with  $\lambda$  held fixed, the value of bar codes  $\Delta(n)$  monotonically decreases in  $n$ . For  $\lambda = .01$ , when  $n$  is increased to 4, the value falls to only a fourth of what it is when  $n = 1$ .

The value of bar codes  $\Delta(1)$  in the 1-period case substantially increases with  $\lambda$ , in the range of  $\lambda$  that is reported, rising to .0585 for  $\lambda = .2$ . The increase in value is much less pronounced for the longer cycle lengths. In fact, the values  $\Delta(n)$  for  $n > 1$  decrease between  $\lambda = .1$  and  $\lambda = .2$ . Thus the rate at which the value declines with  $n$  becomes more pronounced at the higher levels of  $\lambda$ . At  $\lambda = .2$ , the value of bar codes to a store with a 4-period cycle is less than one tenth of the value to a store with a one-period cycle.

#### 4. Superstores and Delivery Frequency

This section imbeds the model of an order decision for a particular product analyzed above into a model of a store where the choice of products carried and the delivery frequency are both endogenous variables. Given the existence of the complementarity detailed in the

previous section, the advent of bar codes will induce stores to increase delivery frequency. Given economies of scale in delivery, this will increase store size. This brief section presents one particular model of a store to make this simple point. In the model, store size increases because the breadth of the product line increases; i.e., the various shops on Main Street are folded into a Wal-Mart. The section also considers the possibility that stores might get bigger by increasing sales per product and finds there are subtle issues for this case.

## 4.1 The Model of a Store

There is a continuum of different products made up of the interval  $[0, \theta]$ . A store offers some subset of products. Let  $m \leq \theta$  denote the measure or number of products that a store sells.

The store chooses the delivery frequency  $n$ . All the different goods come in the same truck, so the delivery frequency is the same for all goods.

A key aspect of the model here is that there are economies of scale in delivery. For simplicity, I make the extreme assumption that there is a fixed cost of  $\phi$  for a delivery and that this cost does not vary with the size of the delivery or with the number of products  $m$  that are delivered. Therefore, a store with an  $n$ -period cycle has a delivery cost averaged over the  $n$  periods equal to  $c_D(n) = \phi/n$ .

Given the economies of scale in delivery, there will be an incentive to make stores big. To offset this, I assume that there is some other source of diseconomies of scale. Let  $c_S(m)$  be this other cost, which I will call the **setup** cost. Suppose this cost must be paid for each product. Assume  $c_S(m)$  is increasing and convex. This increasing cost captures factors that limit the size of a store, including unwieldy parking lots, increasing transportation cost from drawing from a wider market area, management diseconomies, and so forth.

As before, let  $v^\ell(n)$  be average **product-level** profit, given information lag  $\ell$  and interval length  $n$ . This equals gross margin times average sales minus average holding costs. If there is a proportionate change in the number of products, everything else fixed, there is a proportionate change in these revenues and costs.

The average profit per period of a store selling  $m$  different products with a cycle length  $n$ , given information lag  $\ell$  is

$$\pi^\ell(m, n) = mv^\ell(n) - \frac{\phi}{n} - mc_S(m). \quad (13)$$

The first term is the store's number of products  $m$  times the product-level expected profit. The second term subtracts the average delivery cost. This is not multiplied by  $m$  because of the scale economies. The third term subtracts the setup costs that must be paid for each of the  $m$  products

Two formulations of the objective function are considered. The first formulation is a social planner's problem. Suppose that the  $\gamma$  parameter represents the gross benefit to society when a consumer arrives at a store and obtains a unit of product that he or she desires. Then the  $\pi$  function as defined above represents the net social surplus created by a store of size  $m$  and cycle length  $n$ . Suppose that the task of the planner is to take the total set of products  $\theta$  and divide this set into different stores of size  $m$ . A choice of store size  $m$  implies that the number of different stores is  $y = \frac{\theta}{m}$ . (Allow the number of stores to be a continuous variable.) The objective of the planner is to maximize the total surplus. This is equivalent to maximizing total surplus per product (since the number of products is fixed at  $\theta$ ). Surplus per product is obtained by dividing  $\pi$  above by  $m$ :

$$w^\ell(m, n) = v^\ell(n) - \frac{\phi}{nm} - c_S(m). \quad (14)$$

The problem of the planner is to pick a store size  $m \leq \theta$  and a cycle length  $n$  to maximize the above, given the information lag  $\ell$ .

The second formulation of the objective function is that of a store maximizing the profit (13).

## 4.2 The Effect of Bar Codes

Motivated by the results of the previous section, for this section I simply assume that the value of bar codes  $\Delta(n) = v^{\ell_B}(n) - v^{\ell_N}(n)$  is decreasing in  $n$ . The result here is

**Proposition 5.** Under either formulation of the objective function, the advent of bar codes (weakly) decreases the order cycle length and (weakly) increases the optimal number of products  $m$ .

The formal proof, which is available upon request, uses the monotone comparative statics techniques discussed in Milgrom and Roberts (1990). The formal proof shows that both objective functions are supermodular in  $m$ ,  $-n$ , and  $-\ell$ . Here I give the intuition of

the result for the profit-maximization case. (The intuition for the social planner's case is similar.) Fixing  $m$  and  $\ell$ , the marginal cost of reducing the order cycle by one unit from  $n + 1$  to  $n$  is the increase in average delivery cost,

$$\frac{\phi}{n} - \frac{\phi}{(n + 1)},$$

and this does not vary with bar codes. The marginal benefit to the store of reducing the cycle length is

$$mv^\ell(n) - mv^\ell(n + 1).$$

Rearranging  $\Delta(n) > \Delta(n + 1)$  and multiplying by  $m$  yields

$$mv^{\ell_B}(n) - mv^{\ell_B}(n + 1) > mv^{\ell_N}(n) - mv^{\ell_N}(n + 1),$$

so the advent of bar codes increases the marginal benefit of reducing the cycle length. But now consider how the choice of  $n$  affects the choice of store size  $m$ . The first-order condition for the profit-maximizing choice of store size is

$$\frac{\partial \pi^j(m, n)}{\partial m} = v^j(n) - c_S(m) - mc'_S(m) = 0.$$

The choice of  $m$  balances the gain of the variable profit per merchandise line (the first term) against the decreasing returns from expanding the product line breadth (the second and third terms). A decrease in the cycle length  $n$  increases the first term, and the optimal store size  $m$  increases.

### 4.3 An Alternative Interpretation of Store Size

In the discussion so far,  $m$  has been defined as the number of products that a store carries, keeping the number of customers per product fixed. The variable  $m$  can also be interpreted as a scale parameter in a model with a single product; that is,  $m$  is simply the number of customers that a store serves. But interpreting it in this way requires some additional discussion.

Recall that demand in any period is a random draw  $z$ . If an increase in scale  $m$  is a multiplicative parameter times the original demand  $z$ , then the argument goes through. That is, it works when demand is  $mz$  where  $z$  is drawn as before. Here, increases in scale

do not allow for any law of large numbers effect. The uncertainty is at the aggregate level, so increasing scale does not get rid of it. This is appropriate if the product in question is a snow shovel and the uncertainty is about whether it is going to snow.

Suppose instead that when the store scales up it pools the random demand from  $m$  individual consumers; that is, demand in period  $t$  is  $\sum_{i=1}^m z_i$ , where for each customer  $i$  the demand  $z_i$  is drawn i.i.d. In this case, when the store is large, the idiosyncratic demands of individual customers will tend to average out. Hence, there will be two offsetting effects. On one hand, the advent of bar codes attenuates the demand smoothing incentive to get big. On the other hand, the complementarity between bar codes and delivery frequency remains and there is still the incentive to get bigger from that effect.

## 5. Conclusion

This paper shows that frequent deliveries complement the adoption of new information technologies. Given economies of scale in delivery, increasing store size is one way to accommodate increases in deliveries. Of course, there are other ways to achieve more frequent deliveries besides making stores bigger. For example, increased coordination of deliveries to establishments that are in the same geographic area may facilitate increases in delivery frequency. Chains of convenience stores that blanket an area may serve the function of increasing the coordination of deliveries. As another example, expansion of package delivery services like Federal Express may permit even small stores to work on a just-in-time delivery system. Finally, third party logistic firms have emerged that allow smaller firms to realize economies of scale in delivery (see Lee, Padmanabhan, and Whang (1997)).

Pressures to increase store size may differ between urban and rural areas. In urban areas, a high concentration of establishments in the same area may make it possible to obtain economies of scale in frequent deliveries without making establishments bigger. In rural areas, these economies of scale may only be available through expansion of store size. In this context, it is interesting to note that Wal-Mart's emergence in rural areas preceded its emergence in urban areas.

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Table 1  
Second and Third Component as Share of Total Effect  
(Demand approximately truncated normal,  $\mu = \sigma^2 = 1$ )

$\lambda$	$H_{23}/H$
.01	.133
.05	.196
.10	.246
.40	.620
.60	.911

Table 2  
Value of Bar Codes  
Various Levels of n and  $\lambda$   
(Demand approximately truncated normal,  $\mu = \sigma^2 = 1$ )

$\lambda$	n	$v^B$	$v^N$	$v^B - v^N$
.01	1	0.9800	0.9700	0.0100
.01	2	0.9650	0.9596	0.0054
.01	3	0.9562	0.9521	0.0042
.01	4	0.9478	0.9450	0.0027
.02	1	0.9600	0.9400	0.0200
.02	2	0.9342	0.9247	0.0095
.02	3	0.9164	0.9108	0.0056
.02	4	0.9028	0.8980	0.0048
.05	1	0.9000	0.8644	0.0356
.05	2	0.8465	0.8314	0.0151
.05	3	0.8137	0.8026	0.0110
.05	4	0.7851	0.7765	0.0086
.10	1	0.8000	0.7524	0.0477
.10	2	0.7215	0.7016	0.0199
.10	3	0.6702	0.6569	0.0134
.10	4	0.6236	0.6146	0.0090
.20	1	0.6238	0.5653	0.0585
.20	2	0.5183	0.5004	0.0179
.20	3	0.4477	0.4375	0.0101
.20	4	0.3871	0.3827	0.0044