

# Step-by-Step Migrations

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## ABSTRACT

This paper considers a dynamic model of industry location in which there is a tension between two forces. First, there is the agglomerating force of preference of intermediate input variety that tends to keep an industry at its original location. Second, other location factors matter and these tend to pull the industry in the direction of a new location. A crucial aspect of the model is that the location space is continuous. When the equilibrium is unique, the migration rate is socially efficient.

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# 1 Introduction

Industries move and the process often occurs in a gradual, step-by-step process. The auto industry in the United States is a case in point. While the industry was centered in Michigan over most of the twentieth century, in the early 1980s Japanese automobile manufacturers began building “transplant” factories in states south of Michigan. Plants were first built in southern Ohio, then further south in Kentucky and Tennessee, and then finally all the way south to Alabama. Table 1 shows counts of large automobile plants by year for the corridor of states that starts with Alabama on the bottom and goes north to Michigan.<sup>1</sup> (In 2000, these six states accounted for just under half of the 77 large automobile plants in the U.S.) Michigan lost eight plants over the period while things were flat in Northern Ohio/Indiana with a net loss of one. Southern Ohio/Indiana added five plants, all but occurring on or *before* 1982. Together Kentucky, Tennessee, and Alabama added a total of six plants, all but one occurring *after* 1982. The new plant in Alabama (a Mercedes plant) was not added until the late 1990s. And in 2000 a second plant has added to Alabama by Honda that was too early in its building stages to show up in the table.

Numerous parts are used to assemble an automobile and close proximity to suppliers is considered to be a critical factor, particularly since the adoption of just-in-time production techniques. When automobile manufacturers in the 1980s first located in states like Kentucky and Tennessee, the supplier base in these states was relatively small. Since that time, a network of suppliers has emerged.<sup>2</sup> Now, when locating in Alabama, a manufacturer has access to a supplier belt in Kentucky and Tennessee, making an Alabama location relatively more attractive than it was 20 years ago.

This paper presents a theory of step-by-step migrations. Transportation costs and the desirability for access to a wide variety of differentiated input suppliers provide a force

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<sup>1</sup>Large automobile plants are defined as establishments with 500 employees or more in SIC 3711 (for years 1974-1992) and NAICS 3361 (for 2000). The source of the data is County Business Patterns and this is described in Holmes and Stevens (2003). I use the the 40.5 latitude to divide the North and South in Ohio and Indiana.

<sup>2</sup>The number of auto part establishments (SIC 3714) in these two states with at least 250 employees increased from 19 in 1974 to 56 in 1997. The source of this data is the same as for Table 1.

of agglomeration in the model. In the initial period, the industry is concentrated in a particular region of the economy. Production costs unrelated to agglomeration (e.g. wages, degree of unionization, and so forth) fall in a certain direction away from the initial region of concentration. This presents a trade-off to new entrants in the economy: By moving further away one can obtain lower wages—but only at the cost of lower agglomeration benefits.

For my results, I find that along the equilibrium path the industry never settles down or gets stuck in one place. Instead it moves in the direction of cost reductions. The step size is larger the higher the cost reduction gradient and the lower the agglomeration benefits. The step size also increases in the discount factor, as the anticipation of future industry movements feed back and leads to more movement today. My final result concerns efficiency. If the equilibrium is unique (which occurs if the cost gradient is small), the equilibrium step size is efficient.

The fact that the industry is a continuum along with the fact that the industry occupies a positive mass on the continuum (an interval) is crucial for the result that the industry does not get stuck. To explain why, consider the events that happen within a period of the model. The period begins with the location of some suppliers locked into points on an interval  $[a, b]$  because of prior decisions. Costs tend to fall in the direction towards  $b$ , everything else the same. Within the period, new suppliers enter the industry and have a one-shot opportunity to make a long-lasting location decision. New slots open up as possible location points throughout the interval  $[a, b]$  and this makes it feasible for the new generation of suppliers to operate side-by-side with the previous generation. Locations near the endpoints of the interval have the disadvantage of being further from the center of the industry and therefore have the the disadvantage of lower agglomeration benefits. But the endpoint at  $b$  has an advantage in other factors. As a result, new suppliers strictly prefer  $b$  over  $a$ , and the new generation of entering supplies occupy an interval of locations that is shifted over from that of previous generations.

The force of agglomeration highlighted here—intermediate input variety—is a force that has received extensive attention in recent years. (See Fujita, Krugman, and Venables (1999) and Fujita and Thisse (2002) for surveys.) Most of the models in this literature are static, while my model is dynamic. There are papers that do incorporate dynamics such as

Holmes (1999) and Adserà and Ray (1998) (and the related papers of Krugman (1991) and Matsuyama (1991)). But these models feature two discrete locations, while there is a continuum of locations in this model. Allowing for a continuum is crucial for the argument I just explained.

A second force of agglomeration, knowledge spillovers, has also received extensive attention in a distinct literature; see in particular Lucas (2001) and Lucas and Rossi-Hansberg (2002). I examine a variant of my model that has knowledge spillovers instead of intermediate inputs and obtain a very different welfare result. In particular, the migration rate is too fast compared to the socially efficient level. Agents do not internalize the knowledge spillover externality and move too far away. This is analogous to Rossi-Hansberg's result in a knowledge spillover model that equilibrium density is too low (Rossi-Hansberg (2002)). In my main model, where agglomeration benefits are driven by intermediate input variety, efficiency is obtained because suppliers *do* internalize the effects of their location decisions on buyers. The impacts on buyers directly affect a location's profitability. The highest profit location for a supplier is also the one that is best for buyers.

Understanding the efficiency properties of market location decisions is important because numerous government policies affect location decisions (see, for example, Holmes (1998)). In particular, there has recently there has been much discussion of so-called "smart growth" policies for cities that attempt to slow the migration of individuals away from urban cores and limit "sprawl." While my model is an industry model and not an urban model like that of Lucas and Rossi-Hansberg, my finding that the welfare analysis is very different when the agglomeration force is product variety rather than knowledge spillovers makes it likely that there are similar differences in urban models. These different welfare effects make it important to analyze the sources of agglomeration benefits as in Holmes (2002).

The trade-off in costs and benefits in this paper from increasing the step size is analogous to the trade-off found in vintage capital models (Chari and Hopenhayn (1991), Parente (1994), and Jovanovic and Nyarko (1996)). In these models, the benefits of faster adoption of new technologies must be weighed against the cost of increasing the rate of obsolescence of past investments. Jovanovic and Nyarko (1996) ask the question of whether or not it would ever be optimal to stop adopting new technologies in light of this trade-off. They find that

it may be optimal to stop at a technology level below that maximum level. In my related but different structure, it is never optimal, nor is it ever an equilibrium for the economy to get stuck in an allocation where productivity is less than its maximum possible level.

## 2 The Model

There are a continuum of locations in the economy on the positive real line. A particular location is denoted  $x \in [0, \infty)$ . Time in the model is discrete,  $t \in \{0, 1, 2, \dots, \infty\}$  and the discount factor is  $\delta < 1$ . The economy is populated by two kinds of agents, *assemblers* and *suppliers*. I first describe these two kinds of agents and then explain the assumptions on geography.

### 2.1 Assemblers

Assemblers use differentiated intermediate inputs to construct a composite intermediate good. There are a continuum of differentiated inputs and the measure of different varieties is two. Let  $y \in [0, 2]$  index a particular variety and let  $h(y)$  be the quantity of this variety. The production function for  $m$  units of the composite intermediate is

$$m = \left[ \int_0^2 h(y)^{\frac{\sigma-1}{\sigma}} dy \right]^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

where  $\sigma > 1$  is the constant elasticity of substitution. This production function is standard in the literature. Define the markup parameter  $\mu \equiv \frac{\sigma}{\sigma-1}$ . The bigger is  $\mu$ , the stronger is the preference for variety.

The assembler uses the composite intermediate to produce a final manufactured good. The production function for the final good is

$$q = m^{\frac{\phi-1}{\phi}}. \quad (2)$$

The parameter  $\phi$  is the elasticity of the derived demand for  $m$  with respect to changes in the price of  $m$ . Assume  $\phi > 1$ , so that the derived demand is elastic. Suppose that the final good is the numeraire good and refer to this numeraire good as *dollars*. The objective of the assembler here is to maximize the dollar value of output minus the dollar value of the differentiated inputs consumed.

## 2.2 Suppliers

Suppliers live for two periods in an overlapping-generations structure. In any period, a unit measure of suppliers are young and a unit measure are old, so the combined total measure of two equals the measure of the variety of goods. Each supplier has a monopoly over a particular differentiated input  $y$  for both periods of life.

There is no fixed cost for entry by suppliers. With this structure, the total product variety is fixed at two. Note this structure differs from most of the economic geography literature where there are fixed costs and the level of variety is endogenous. The focus of this paper is on *where* suppliers locate rather than on *how many* there are. The model was constructed to get at this question as directly as possible.

Each young supplier entering the economy chooses a location  $x$ . This location then becomes fixed for both periods of the suppliers life. Marginal cost to produce inputs varies *across* locations but is *constant* within a location. Let  $c(x)$  be the marginal cost (in dollars) to produce one unit of input at location  $x$ .

## 2.3 Geography

I turn now to the elements of the model that have to do with geography. These include how cost varies across locations and the nature of transportation costs. These elements also include the possibilities for location choices.

Locations with higher  $x$  are lower cost. Specifically

$$c(x) = e^{-\theta x},$$

for  $\theta > 0$ . Thus marginal cost at  $x = 0$  is one dollar and cost declines with  $x$  at a constant rate  $\theta$ .

There is a transportation cost to ship inputs from suppliers to assemblers. Suppose an assembler is located at  $x^a$  and a supplier is at  $x^s$  so the distance is  $z = |x^a - x^s|$ . In shipment, a fraction  $1 - e^{-\tau z}$  of the input is dissipated and a fraction  $e^{-\tau z}$  survives. The larger the transportation cost parameter  $\tau$  and the larger the distance  $z$ , the greater the fraction lost in transit. This *iceberg* cost formulation is standard in the literature. It does not matter

for the equilibrium whether the supplier or the assembler pays the transportation cost, so without loss of generality, I assume it is the assembler. Assume that

$$\tau > \theta, \tag{3}$$

so that the transportation cost factor is important than the exogenous differences across locations.

Land is scarce in the model in the following sense. The land requirement for production of the input is fixed at a rate of one half unit of land per unit of supplier. Thus, at any point in time  $t$ , a particular location will have either 0, 1, or 2 suppliers.

There is an adjustment process for adding a supplier to a new location. In each period, only *one* additional supplier can be added to a particular location  $x$ . I make this assumption not to attempt any degree of realism, but rather to create a model that is tractable yet captures the key tensions of interest for this paper. With this assumption, the new and old suppliers will overlap and the main issue for analysis will be the degree to which they overlap.

Define a location  $x$  as *brown* if it is currently the location of a supplier or has been in any time in the past. A location is *green* if it has never been the location of a supplier. I assume there is no possibility for *skipping* locations. Formally, a supplier cannot locate at a point  $x'$  if there is a positive measure subset of  $[0, x')$  that remains green in the period. The no-skipping assumption assures that any migration process will be a gradual one. This constraint is plausible and it makes the analysis more tractable.

In the initial condition at period 0, the old suppliers are located on the interval  $[0, 1]$ . All locations above one are green.

It remains to describe the location possibilities for assemblers. There are no restrictions. In particular, it is feasible to place the entire mass of all assemblers at a single point. Moreover, the location decisions are not fixed but are made anew each period. Like the assumption on the adjustment process, these assumptions keep the analysis simple but still maintain the tensions of interest for the paper.

Lastly, I make a comment about land markets. To keep the exposition simple, I ignore them. Land sites for suppliers are free, up to capacity. But the analysis would be no different

if I included land markets. The rents lucky suppliers get who occupy good locations would instead be shifted to land owners.

### 3 Equilibrium

Define a stationary *step-by-step* migration path with step-size  $z$  as a sequence of location decisions where each entering supplier cohort locates on an interval and this interval shifts to the right by an amount  $z$  in each period. This section characterizes equilibria and derives comparative statics for step-by-step migration paths.

Within a period, there are two stages. In stage 1, assemblers and new suppliers simultaneously make location decisions. Each agent maximizes, taking the other agents' location choices as given. In stage 2, the market in intermediate inputs takes place. This is modeled in the usual way. The assemblers are price takers and the suppliers are differentiated product monopolists. Notice that the analysis of what happens in stage 2 is a static problem. In contrast, stage 1 requires a dynamic analysis since the location decisions of new suppliers are fixed for two periods, so these decisions depend upon expectations of next-period outcomes.

Given the constant elasticity structure of the production function and the exponential function for the cost  $c(x)$ , it is sufficient to analyze the behavior at time  $t = 0$ . Period 0 begins with the locations of old suppliers fixed on the interval  $[0, 1]$ . If it is optimal for new suppliers at period 0 to locate on the interval  $[\hat{z}, 1 + \hat{z}]$ , taking as given that the supplier interval shifts by  $\hat{z}$  in all future periods, then shifting in this way is optimal for all future generations as well, so this is an equilibrium. Given the structure of the model, the payoff functions of future agents differ only a multiplicative constant and the decisions are the same.

I therefore focus in this section on analyzing what happens in period 0, treating it as a representative period. As usual, I work backwards within the period. Thus I begin in stage 2 when the market in intermediate inputs takes place, and location decisions have already been determined. Then I go back to stage 1 where the location decisions are made. In the analysis of stage 1, I first determine the location behavior of new suppliers and then determine the location behavior of assemblers. I conclude this section by putting all of this together.

### 3.1 Stage 2: The Intermediate Input Market

Consider the market for intermediate inputs in stage 2. Take as given that assemblers are located at the point  $x^a$  and calculate the profit of a supplier located at a point  $x^s$ . The marginal cost of this supplier is  $c(x^s) = e^{-\theta x^s}$ . Suppose the supplier were set a price of  $p$  before transportation costs (i.e. a F.O.B. price). The delivered price, including transportation cost, would be  $\tilde{p} = pe^{\tau|x^a-x^s|}$ . Recall that there are a continuum of firms and that the composite production function is CES. Thus, by the usual arguments (e.g. Fujita, Krugman and Venables (1999, p. 47)), the demand function for *delivered units* of a differentiated input has constant elasticity  $\sigma$  in the delivered price and takes the following form:

$$\tilde{D}_0(\tilde{p}) = \tilde{p}^{-\sigma} \frac{m_0}{v_0^{-\sigma}} = \tilde{k}_0 \tilde{p}^{-\sigma}. \quad (4)$$

Here  $v_0$  is the price index in period 0 (the minimum cost of a unit of composite),

$$v_0 = \left[ \int_0^2 p_0(y)^{1-\sigma} dy \right]^{\frac{1}{1-\sigma}}, \quad (5)$$

$m_0$  is the quantity of the composite input demanded by assemblers in period 0, and  $\tilde{k}_0$  is defined by

$$\tilde{k}_0 \equiv \frac{m_0}{v_0^{-\sigma}}.$$

The demand function relevant for the firm's problem is the *pre-shipment* quantity demand at the F.O.B. price. If  $\tilde{D}_0(\tilde{p})$  is the quantity delivered, then  $e^{\tau|x^a-x^s|}\tilde{D}_0(\tilde{p})$  is the quantity shipped. Substituting in  $pe^{\tau|x^a-x^s|}$  for  $\tilde{p}$  and simplifying, the relevant demand function is

$$D_0(p, x^s) = e^{-(\sigma-1)\tau|x^a-x^s|} \tilde{k}_0 p^{-\sigma}.$$

The firm's demand is constant elasticity with respect to the F.O.B. price  $p$ , since  $\tilde{k}_0$  is a constant from the perspective of the firm. Thus by the usual arguments, the profit maximizing price is a markup  $\mu \equiv \frac{\sigma}{\sigma-1}$  over the supplier's cost of  $e^{-\theta x^s}$  dollars. Hence, the F.O.B. price of a good produced at location  $x^s$  is

$$p(x^s) = \mu e^{-\theta x^s}, \quad (6)$$

and the delivered price of one unit of good, produced at  $x$ , delivered to  $x^a$ , is

$$p^d(x^s) = \mu e^{-\theta x^s + \tau |x^a - x^s|}. \quad (7)$$

The profit at time 0 of a firm locating at  $x^s$  on these sales is

$$\begin{aligned} \pi_0(x^s) &= [p(x^s) - c(x^s)] D_0(p, x^s) \\ &= k_0 e^{(\sigma-1)(\theta x^s - \tau |x^a - x^s|)} \end{aligned} \quad (8)$$

for

$$k_0 \equiv (\mu - 1) \mu^{-\sigma} \tilde{k}_0. \quad (9)$$

Observe that profit is increasing in  $x$  through the term with the cost coefficient  $\theta$ . This reflects the lower cost with  $x^s$ . Profit decreases in the distance  $|x^a - x^s|$  through the transportation cost term.

So far, only period 0 has been considered. Now consider period 1. Since the supplier network shifts over by an amount  $z$  and since prices (6) are proportional to cost, it is clear that the price index must fall by a factor  $e^{-\theta z}$ , i.e.,  $v_1 = e^{-\theta z} v_0$ . Furthermore, the aggregate demand for the composite in period 1 must be  $m_1 = m_0 e^{\phi \theta z}$  (recall that  $\phi$  is the elasticity of derived demand). The analog of  $\tilde{D}_0(\tilde{p})$  from (4) is then

$$\tilde{D}_1(\tilde{p}) = \tilde{k}_1 \tilde{p}^{-\sigma},$$

for

$$\tilde{k}_1 \equiv \frac{m_1}{v_1^{-\sigma}} = \frac{m_0 e^{\phi \theta z}}{e^{\sigma \theta z} v_0^{-\sigma}} = e^{(\phi - \sigma) \theta z} \tilde{k}_0. \quad (10)$$

Calculating the firm's demand for pre-shipment quantities as before and then calculating the firm's profit yields

$$\pi_1(x^s) = k_1 e^{(\sigma-1)(\theta x^s - \tau |x^a + z - x^s|)}, \quad (11)$$

where  $k_1$  is defined analogous to  $k_0$  in (9). This expression takes into account that assemblers shift location to  $x^a + z$  in the next period, so the distance between a supplier at  $x^s$  and assemblers next period is  $|x^a + z - x^s|$ .

### 3.2 Stage 1: The Location of New Suppliers

I now turn to the choice of location by new suppliers in stage 1. For the analysis of this section, I take as given that assemblers locate at  $x^a \geq \frac{1}{2}$  in period 0 which is an immediate property of any equilibrium. I solve for a stepsize  $z^*(x^a)$  that is an equilibrium “reaction” to assembler location  $x^a$ ; i.e., this stepsize is consistent with optimal location behavior by new suppliers, when each supplier takes as given that the industry evolves according to this stepsize.

To define this reaction function formally, write the single-period profit functions  $\pi_0(x^s, x^a, z)$  and  $\pi_1(x^s, x^a, z)$  as explicit functions of  $x^a$  and  $z$  as well as  $x^s$  and write the discounted sum of profit over two periods as

$$\begin{aligned}\Pi(x^s, x^a, z) &= \pi_1(x^s, x^a, z) + \delta\pi_2(x^s, x^a, z) \\ &= k_0 e^{(\sigma-1)(\theta x^s - \tau|x^a - x^s|)} \\ &\quad + \delta k_0 e^{(\phi-\sigma)\theta z} e^{(\sigma-1)(\theta x^s - \tau|x^a + z - x^s|)},\end{aligned}$$

where we substitute in formulas (8), (11), (10), (11) into the second equality. Given the pair  $(z, x^a)$ , agents take as given that assemblers will locate at  $x^a + tz$  and that new suppliers will locate on the interval  $[(t+1)z, (t+1)z + 1]$  in period  $t$ . To be consistent with equilibrium choice behavior, it must be the case that

$$\Pi(x, x^a, z) > \Pi(x', x^a, z), \quad x \in [z, z + 1], \quad x' \notin [z, z + 1]; \quad (12)$$

i.e., that the profit in each chosen location exceeds the profit of the locations not chosen. Continuity of the profit function implies that profit at the end points must be the same,

$$\Pi(z) = \Pi(z + 1),$$

unless the no-skipping condition is binding, in which case  $z = 1$  and  $\Pi(z) \leq \Pi(z + 1)$ . I define the new supplier reaction function  $z^*(x^a)$  to be a solution to (12).

It is useful to begin the analysis by discussing the easy case where firms are myopic,  $\delta = 0$ . With myopic firms, only profit in period 0 matters. Using formula (8) for period 0 profit, equality of profit at the endpoints holds if and only if

$$\theta z - \tau(x^a - z) = \theta(z + 1) - \tau(z + 1 - x^a). \quad (13)$$

Solving this condition, the new supplier reaction function  $z^*(x^a)$  equals

$$z^*(x^a) = \frac{\theta}{2\tau} + x^a - \frac{1}{2}, \quad (14)$$

unless the above exceeds one, in which case the no-skipping constraint is binding and  $z^*(x^a) = 1$ . Formula (14) has some intuitive properties. Note first that if  $\theta = 0$  so there are no cost differences across locations, then the new supplier network centers itself around the assembler location  $x^a$ . If  $\theta > 0$ , the supplier network shifts over to the right compared to the assembler location. The supplier at the left endpoint  $z$  is closer to assemblers than suppliers at the right-endpoint  $z + 1$ . This happens because the supplier at the left endpoint has higher costs. In order for indifference to hold, the production cost disadvantage must be offset by a transportation advantage.

Figures 1 and 2 plot the supplier reaction function for a case where  $\theta$  is moderate and a case where  $\theta$  is high.<sup>3</sup> Notice the kink in each figure where each reaction function hits the constraint that  $z \leq 1$ . The figures illustrate how an increase in  $\theta$  shifts up the supplier reaction

I turn now to the more complicated case with forward-looking behavior. Define the function  $S(z, x^a)$  by

$$\begin{aligned} S(z, x^a) &\equiv \frac{\Pi(z, x^a, z) - \Pi(z + 1, x^a, z)}{k_0 e^{(\sigma-1)\theta z}} \\ &= e^{-(\sigma-1)\tau|x^a-z|} + \delta e^{(\phi-\sigma)\theta z} e^{-(\sigma-1)\tau x^a} \\ &\quad - e^{(\sigma-1)(\theta-\tau|x^a-z-1|)} - \delta e^{(\phi-\sigma)\theta z} e^{(\sigma-1)(\theta-\tau|x^a-1|)}. \end{aligned}$$

The function  $S(z, x^a)$  is proportionate to the difference in payoffs between the endpoints of the interval  $[z, z + 1]$  when agents take as given that locations evolve according to  $(z, x^a)$ . In order for  $z$  to be a consistent reaction to an assembler location  $x^a$ , assuming it is interior,  $S(z, x^a) = 0$  must hold so that the return is equalized at the endpoints. Let  $z^*(x^a)$  denote a solution to  $S(z, x^a) = 0$ .

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<sup>3</sup>The values are  $\theta = .2$  in Figure 1 and  $\theta = .5$  in Figure 2. For both examples  $\sigma = 2$  and  $\tau = 1$ .

Suppose that  $\theta = 0$  and that  $x^a = \frac{1}{2}$ . It is straightforward to calculate that the unique solution to  $S(0, \frac{1}{2}) = 0$  is  $z^* = 0$ . Thus with no cost differences across locations and with assemblers locating at the center of the old supplier network, the stepsize of the new supplier network is zero.

### 3.3 Stage 1: The Location of Assemblers

The assembler location problem is simpler than the supplier problem because the assemblers readjust location in each period and therefore solve a static problem. In period 0, assemblers take as given that the old suppliers are located on the interval  $[0, 1]$  and that new suppliers are on the interval  $[z, 1 + z]$ . The center of the combined network is  $\frac{1}{2} + \frac{z}{2}$  and the endpoints are 0 and  $1 + z$ .

Assemblers will choose the location that minimizes the cost of constructing the composite intermediate. Rewriting formula (5), the price of a unit of composite at location  $x^a$  is

$$v(z, x^a) = \left[ \int_0^1 p^d(x, x^a)^{1-\sigma} dx + \int_z^{z+1} p^d(x, x^a)^{1-\sigma} dx \right]^{\frac{1}{1-\sigma}} \quad (15)$$

where again

$$p^d(x, x^a) = \mu e^{-\theta x + \tau |x^a - x|}.$$

The composite price  $v(z, x^a)$  is continuously differentiable in  $x^a$ . An interior optimal location must satisfy the first-order necessary condition,

$$A(z, x^a) \equiv \frac{\partial v(z, x^a)}{\partial x^a} = 0. \quad (16)$$

In the case where  $\theta = 0$ , things are quite simple as there are no cost differences across suppliers and therefore no price differences across suppliers in the F.O.B. price. Here the optimal location strategy is immediate. The assembler should locate in the exact center of the combined network of old and new suppliers; i.e. at  $x^{a*}(z) = \frac{1}{2} + \frac{z}{2}$  as this minimizes transportation costs.

Now consider the case where  $\theta > 0$ . It is straightforward to calculate the slope (16) and show that it is strictly positive for  $x^a \leq \frac{1}{2} + \frac{z}{2}$  so that the optimal assembler location is strictly to the right of center. To see why, suppose the assembler were to continue to locate

at the exact center of the supplier network. Observe that prices to the right of the supplier center are lower than prices to the left. Since demand is elastic, total spending on any good to the right of center is greater than on any good to the left of center. Since transportation costs are a constant fraction of spending for a fixed distance, it is straightforward to see that total spending on transportation costs is strictly greater for goods to the right than for goods to the left. It is immediate that if the assembler were to keep the delivered quantities the same but shift its location to the right a small amount, a first-order reduction in the transportation cost bill would result. Hence the optimal location solving the first-order necessary condition (16) is to the right of center; i.e.,

$$x^{a*}(z) > \frac{1}{2} + \frac{z}{2}. \quad (17)$$

In the case where  $\theta = 0$ , the function  $v(z, x^a)$  is obviously a u-shaped relationship with a single minimum. For general  $\theta$ , the expression (15) is complex and I don't have a general proof that the function is u-shaped. For numerical examples that I have looked at (such as the examples in Figures 1 and 2) the function is u-shaped implying a continuous reaction function. Assume  $x^*(z)$  is continuous in what follows. Figures 1 and 2 plot the assembler reaction functions for the two examples. Observe that an increase in  $\theta$  shifts the assembler reaction function over to the right.

### 3.4 Putting it Together

I now put it all together. The reaction function  $z^*(x^a)$  of the new suppliers and the reaction function of assemblers  $x^{a*}(z)$  can be combined to define an equilibrium. An equilibrium is a pair  $(\hat{z}, \hat{x}^a)$  such that  $\hat{x}^a = x^{a*}(\hat{z})$  and  $\hat{z} = z^*(\hat{x}^a)$ .

I start with the easy case of  $\delta = 0$ .

*Proposition 1.* Assume  $\delta = 0$ . There exists an equilibrium. If  $\theta = 0$ , the equilibrium step size is  $z = 0$  and  $x^a = \frac{1}{2}$ . If  $\theta > 0$ , the equilibrium stepsize satisfies  $\hat{z} \geq \min\left\{\frac{\theta}{2\tau}, 1\right\}$ .

Existence of equilibrium is immediate. The strict lower bound on  $\hat{z}$  obtained for  $\theta > 0$  follows from (14) and (17). Figures 1 and 2 illustrate equilibria for the two examples. For the case of moderate  $\theta$ , there is a unique equilibrium with an interior level of  $z$ .<sup>4</sup> For the

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<sup>4</sup>Note that equilibrium is unique up to rearrangements across locations of suppliers in a given generation.

case of a high  $\theta$  there are three equilibria, including one at the corner  $z = 1$  and two interior equilibria.

I turn to the more complicated case of  $\delta > 0$ . My result is

*Proposition 2.* (i) Suppose  $\theta = 0$ . There is a unique equilibrium where  $\hat{z} = 0$  and  $x^a = \frac{1}{2}$ .

(ii). There is a  $\theta' > 0$  such that if  $\theta \in (0, \theta')$ , there is a unique equilibrium where  $\hat{z} > 0$  and  $\hat{x}^a > \frac{1}{2}$

*Proof.* See appendix.

The final result of this section concerns comparative statics properties. Differentiation of the equilibrium conditions can be used to show

*Proposition 3.* Let  $(\theta, \sigma, \delta, \phi, \tau)$  be the vector of model parameters and suppose that  $\theta < \theta'$  for  $\theta'$  defined above. The equilibrium stepsize  $\hat{z}(\theta, \sigma, \delta, \phi, \tau)$  increases in  $\theta, \sigma, \delta, \phi$ , and decreases with  $\tau$ .

The model has intuitive comparative statics properties. The strength of the agglomeration economies is measured by the parameters  $\sigma$  and  $\tau$ . When  $\sigma$  is high, agglomeration economies are weak, because the elasticity of substitution is high and preference for variety is small. When  $\tau$  is high, agglomeration economies are strong because transportation costs are high. The above result says that the speed of migration increases in  $\sigma$  and decreases in  $\tau$ . Thus the stronger the agglomeration economies, the slower the migration.

The result also says that the migration is faster the greater the discount rate. New suppliers anticipate that the industry will be shifting tomorrow and this leads to bigger adjustments today. Finally, the result says that the migration rate increases with the elasticity of final demand. With a higher elasticity, the lower future prices from shifting have a larger impact on future demand. This tends to increase the weight placed on future sales and the effect is analogous to an increase in the discount factor.

## 4 Welfare

This section discusses the social efficiency of equilibrium migration paths. To keep things simple, I consider the special case where  $\delta = 0$  so I need look only at one period. The results also extend to the case of  $\delta > 0$ .

Suppose that a social planner determines the stepsize  $z$  as well as the assembler location  $x^a$ . But after these location decisions are selected, the agents go into stage 2 where there is an unregulated market in intermediate goods. Hence the prices follow equation (7). Let  $v(z, x^a)$  be the resulting price of a unit of composite given in equation (15). Suppose the planner picks  $(z, x^a)$  to maximize the total surplus of assemblers and suppliers.

Given an composite price of  $v$ , it is straightforward to use the final goods production function (2) to determine the demand for intermediate goods

$$m(v) = \left[ \frac{\phi}{\phi - 1} \right]^{-\phi} v^{-\phi},$$

and the maximized profit of assemblers

$$\pi^a(v) = \phi^{-\phi} (\phi - 1)^{(\phi-1)} v^{-(\phi-1)}.$$

Following the standard inverse elasticity rule, the profit of each supplier is a fraction  $\frac{1}{\sigma}$  of a assembler total expenditure. Aggregating, the profit of the total supplier sector is then

$$\pi^s(v) = \frac{1}{\sigma} m(v)v.$$

It follows that total surplus as a function of the composite price  $v$  has the form

$$TS(v) = \pi^a(v) + \pi^s(v) = kv^{-(\phi-1)},$$

for some constant  $k$ . Hence the planner's objective to maximize total surplus can be reformulated to minimize the composite cost  $v$ ,

$$\min_{z, x^a} v(z, x^a).$$

Let  $(z_e, x_e^a)$  be the solution to the social planner's problem, where the subscript "e" signifies the *efficient allocation*. My interest here is in comparing the efficient allocation  $(z_e, x_e^a)$  with an equilibrium allocation  $(\hat{z}, \hat{x}^a)$ . First consider the location of assemblers. By definition of equilibrium, the equilibrium assembler location  $\hat{x}^a$  minimizes  $v(\hat{z}, x^a)$  over locations  $x^a$ , taking the step size fixed at  $\hat{z}$ . Therefore, taking supplier locations as fixed, the market and the social planner would select the same assembler location. Next consider the location of suppliers. Differentiating (15) with respect to  $z$ , it is straightforward to

calculate that the social planner's first-order necessary condition for an interior choice of  $z$  is equivalent to

$$p^d(z, x^a) = p^d(z + 1, x^a);$$

i.e., that the delivered prices at the endpoints be equalized. But this condition is the same as the equilibrium condition (13) that delivered cost (and therefore profit) be identical at the endpoints. I conclude that the equilibrium supplier location  $\hat{z}$  solves the social planner problem of minimizing  $v(z, \hat{x}^a)$  over  $z$  holding the assembler location fixed at  $\hat{x}^a$ . The immediate consequence of all of this is

*Proposition 4.* Any solution to the social planner's problem  $(z_e, x_e^a)$  can be decentralized as an equilibrium allocation. Furthermore, if the equilibrium is unique, then it is a solution to the social planner's problem.

Note the qualifier that an equilibrium is necessarily efficient only if there is a unique equilibrium. In the example illustrated in Figure 2, there are three equilibria. One stable interior equilibrium, an unstable interior equilibrium, and a stable equilibrium at the corner where  $z = 1$ . In this example, the social optimum is the stable interior equilibrium.

I conclude by comparing this efficiency result to results in an analogous alternative model where the agglomeration force is knowledge spillovers like in Lucas and Rossi-Hansberg (2002) instead of intermediate inputs. Consider a model that no longer has assemblers but still has overlapping generations of suppliers with the initial generation located at  $[0, 1]$  as before. If new suppliers locate at on the interval  $[z, z + 1]$  and a particular supplier is at  $x$ , the total distance of the particular supplier from each of the rest of the suppliers is

$$h(x, z) = \int_0^1 |x - \tilde{x}| d\tilde{x} + \int_z^{z+1} |x - \tilde{x}| d\tilde{x}.$$

Suppose a supplier located at  $x$  receives a payoff of

$$\pi(x) = \theta x - \tau h(x, z).$$

The first term captures the force in the original model that when  $\theta > 0$ , locations with higher  $x$  are inherently better. The second term captures the force of agglomeration. When  $\tau > 0$ , suppliers get a higher payoff when they are closer to other suppliers.

In an interior equilibrium allocation, new suppliers must be indifferent at both endpoints  $z$  and  $z + 1$ , i.e.

$$\theta z - \tau h(z, z) = \theta(z + 1) - \tau h(z + 1, z).$$

or

$$\theta - \tau [h(z + 1, z) - h(z, z)] = 0. \quad (18)$$

This implies the step size is strictly positive as before, so the industry never gets stuck even if  $\theta$  is small. Nevertheless, the welfare properties of this alternative model are quite different.

Consider the social planner's problem of picking  $z$  to maximize total surplus

$$W = \int_0^1 (\theta x - \tau h(x, z)) dx + \int_z^{z+1} (\theta x - \tau h(x, z)) dx.$$

The slope of the welfare function is

$$\begin{aligned} \frac{dW}{dz} &= -\tau \int_0^1 \frac{\partial h(x, z)}{\partial z} dx + [\theta(z + 1) - \tau h(z + 1, z)] \\ &\quad - [\theta z - \tau h(z, z)] - \int_z^{z+1} \frac{\partial h(x, z)}{\partial z} dx \\ &= \theta - \tau [h(z + 1, z) - h(z, z)] - \tau \int_0^1 \frac{\partial h(x, z)}{\partial z} dx \\ &< \theta - \tau [h(z + 1, z) - h(z, z)]. \end{aligned}$$

The inequality holds because an increase in  $z$  has an adverse effect upon old suppliers located on the original interval  $[0, 1]$ . Notice that at the equilibrium step size solving (18), the slope of the welfare function is strictly negative. There is a negative externality associated with an increase in the step size and the market results in a stepsize that is too large.

The welfare analysis of the original intermediate input model is fundamentally different from a spillover model. In the spillover model, an increase in  $z$  has a direct negative effect on the welfare of the old suppliers. In the original intermediate goods market, new suppliers take as given the location of assemblers. When a new supplier considers moving further out to enjoy the higher productivity advantages of higher  $x$ , it internalizes the impact of this move on increasing the distance to the assembler because distance affects the profit the supplier can make on the assembler.

## 5 Concluding Remarks

This paper presents a model where agglomeration economies never prevent an industry from moving in the direction of lower costs. If the cost gradient  $\theta$  is small, the speed of migration is socially efficient. To keep things tractable, the model is highly stylized. While more work needs to be done, I believe that the results of this paper would continue to hold in a richer model with more realistic assumptions.

One highly stylized aspect of the model is that each location can only accommodate two suppliers and at most one can be added within a single period. The consequence of this structure is that suppliers spread out and that the area selected by new suppliers overlaps with that of previous suppliers. It is clear that there exist less stylized, more realistic models that would have this same feature. For example, if individual suppliers drew idiosyncratic location specific random shocks, the “spreading out” and overlapping properties would be maintained. The same forces at work in my model would be in play here.

The paper assumes assemblers can all agglomerate at a single point. This simplifies the analysis considerably but does not seem to be very important for the logic of the results. In a more general model assemblers would use land and they would spread out just like suppliers. With individual idiosyncratic draws, assemblers would be smoothed across space, mixing with suppliers. In such a more general model, new suppliers would continue to have the tendency to locate to the right and industries would migrate in each period.

The paper assumes costs fall monotonically indefinitely. By this logic, the American automobile industry will be in the Gulf of Mexico in a 100 years. Of course in the real world, underlying productivity does not always monotonically increase going in one direction. I abstracted away from this aspect of reality in order to create a model with a tractable, stationary structure. In the real world there will be multiple peaks. It is clear that the forces in my model may take the industry to a peak that may not be the highest over all the peaks. I did not go into this issue because it is already well understood from the literature on discrete locations. My interest here is in behavior within the vicinity of a peak rather than movements from peak to peak. Put in another way, my interest is in how locations change on the margin. This interest in a local analysis is similar in spirit to Lucas and

Rossi-Hansberg's (2002) analysis of the internal structure of cities.

## Appendix: Proof of Proposition 2

*Proposition 2.* (i) Suppose  $\theta = 0$ . There is a unique equilibrium where  $\hat{z} = 0$  and  $x^a = \frac{1}{2}$ .

(ii). There is a  $\theta' > 0$  such that if  $\theta \in (0, \theta')$ , there is a unique equilibrium where  $\hat{z} > 0$  and  $\hat{x}^a > \frac{1}{2}$

*Proof of (i)*

It is obvious that  $\hat{z} = 0$  and  $x^a = \frac{1}{2}$  is an equilibrium. Suppose there exists another step-by-step equilibrium where  $z > 0$ . I will prove a contradiction. From optimal assembler behavior we know that  $x^a = \frac{1}{2} + \frac{z}{2}$  or

$$z = 2(x^a - \frac{1}{2}).$$

Let  $y_t^A$  be the distance in period  $t$  to the assembler from location  $z$  (the left endpoint of the interval). Let  $y_t^B$  be the distance to the assembler from location  $z + 1$  (the right endpoint). Now

$$\begin{aligned} y_0^A &= x^a - z \\ &= 1 - x^a \end{aligned}$$

Also

$$y_1^A = x^a.$$

Analogously,

$$\begin{aligned} y_0^B &= 1 + z - x^a \\ &= x^a \\ y_1^B &= 1 - x^a. \end{aligned}$$

Thus  $y_0^A = y_1^B$  and  $y_1^A = y_0^B$ . Also,  $z > 0$  and  $x^a > \frac{1}{2}$  imply  $y_0^A < y_1^A$ . Since  $\theta = 0$ , the composite price must be constant over time. Thus the profit in a period as a function of distance from the assembler is constant over time. Thus

$$\begin{aligned} \Pi^A &= \pi_0^A + \delta\pi_1^A \\ &> \pi_0^B + \delta\pi_1^B \\ &= \Pi^B, \end{aligned}$$

where the inequality follows from  $\pi_0^A = \pi_1^B$ ,  $\pi_1^A = \pi_0^B$ ,  $\pi_0^A < \pi_1^A$ , and  $\delta < 1$ . But  $\Pi^A > \Pi^B$  cannot hold in an equilibrium, a contradiction. *Q.E.D.*

*Proof of (ii)*

Recall that  $S(z, x^a, \theta) = 0$  is the equilibrium condition for suppliers and that the first-order condition  $A(z, x^a, \theta)$  is the equilibrium condition for assemblers. Here it is convenient to write these as functions of  $\theta$ , holding the remaining parameters constant. Define the point  $(z^\circ, c^{a^\circ}, \theta^\circ) = (0, \frac{1}{2}, 0)$ . It is immediate that

$$\begin{aligned} A(z^\circ, x^{a^\circ}, \theta^\circ) &= 0 \\ S(z^\circ, x^{a^\circ}, \theta^\circ) &= 0. \end{aligned}$$

Now consider what happens for small positive  $\theta$ . Straightforward differentiation of  $S$  and  $A$  shows that the following are true:

$$\begin{aligned} \frac{\partial A}{\partial z} &= \frac{\partial S}{\partial z} > 0 \\ \frac{\partial A}{\partial c} &< \frac{\partial S}{\partial c} < 0 \end{aligned} \tag{19}$$

$$\frac{\partial A}{\partial \theta} > 0 \text{ and } \frac{\partial S}{\partial \theta} < 0.$$

A standard application of the implicit function theorem shows that for small  $\theta$  there exist unique functions  $z^*(\theta)$  and  $x^{a^*}(\theta)$  satisfying the necessary conditions

$$\begin{aligned} A(z^*(\theta), x^{a^*}(\theta), \theta) &= 0 \\ S(z^*(\theta), x^{a^*}(\theta), \theta) &= 0 \end{aligned}$$

for an equilibrium and satisfying

$$\frac{dz^*(\theta)}{d\theta} > 0 \text{ and } \frac{dx^{a^*}(\theta)}{d\theta} > 0.$$

It is straightforward to show that for small  $\theta$ ,  $z^*(\theta)$  and  $x^{a^*}(\theta)$  satisfy the sufficient conditions for an equilibrium. I claim that for small  $\theta$  this is the unique equilibrium. Suppose not. Then for each  $n$ , there exists a  $\theta_n < \frac{1}{n}$ , with at least one step-by-step equilibrium  $(z_n, x_n^a)$  besides  $(z^*(\theta_n), x^{a^*}(\theta_n))$ . Straightforward arguments then show in the limit at  $\theta = 0$  there must exist another equilibrium where  $z > 0$  but this contradicts Part (i).

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Table 1  
 Number of Large Automobile Plants in Michigan-Alabama Corridor  
 By State and Year

State	1974	1982	1992	2000
Michigan	21	18	14	13
Northern Ohio, Northern Indiana	7	6	5	6
Southern Ohio, Southern Indiana	2	6	7	7
Kentucky	2	3	4	5
Tennessee	1	1	3	3
Alabama	0	0	0	1

Source: Authors calculations with County Business Patterns data from the U.S. Census. A large automobile plant is defined as a plant with 500 or more employees in SIC 3711 (1974-1992) and NAICS 3361 (2000). The border between North and South in Ohio and Indiana is latitude equal to 40.5.

Figure 1  
Reaction Functions with Moderate  $\theta$

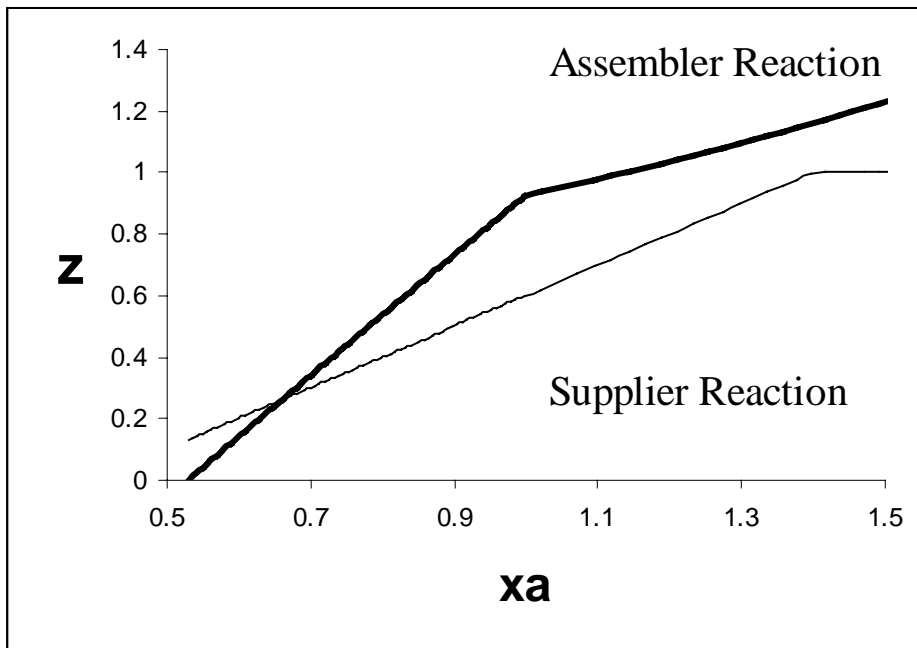


Figure 2  
Reaction Functions with High  $\theta$

