A Theory of Sudden Stops, Foreign Reserves, and Rollover Risk in Emerging Economies

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Abstract

Emerging economies, unlike advanced economies, have accumulated large foreign reserves holdings. We argue that this policy is an optimal response to an increase in foreign debt rollover risk. In our model, reserves play a crucial role in reducing debt rollover crises ("sudden stops"), akin to the role of bank reserves in preventing bank runs. An unexpected increase in rollover risk leads to a global rise in sudden stops, prompting emerging economies to update their priors about the risk they face. We show that a global increase in rollover risk explains the outburst of sudden stops in the late 1990s, the subsequent increase in foreign reserves holdings, and the salient absence of sudden stops ever since.

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1 Introduction

As noted by Obstfeld, Shambaugh, and Taylor (2010), the sustained accumulation of massive international reserves in emerging economies constitutes a puzzle. Standard models predict that emerging economies should hold very little reserves or none at all, while in the data these economies hold as much as 50 percent of GDP in reserves. Due to this disconnect between theory and practice, the management of international reserves remains one of the main topics in policy debates on global imbalances.

In this paper, we document an increase of reserve holdings relative to external liabilities in emerging economies, and argue that this policy is an optimal response to an increase in foreign debt rollover risk. We build a model of optimal reserves management in which the external debt of a small open economy is subject to the rational rollover decision of foreign lenders. In this model, reserves play a crucial role in reducing debt rollover crises ("sudden stops"), akin to the role of bank reserves in preventing bank runs.[1] An unexpected increase in rollover risk leads to a global rise in sudden stops, prompting emerging economies to update their priors about the risk they face. We show that a global increase in rollover risk explains the outburst of sudden stops in the late 1990s, the subsequent increase in foreign reserves holdings, and the salient absence of sudden stops ever since.

We consider an environment in which emerging economies borrow from international lenders to finance domestic projects. The governments of these emerging economies hold “reserves” because some foreign lenders who are subject to interim liquidity shocks may refuse to rollover their loans. The domestic project can be partially liquidated to pay back these lenders in the interim, but governments will first pay with their reserve holdings because liquidation can be costly. When reserve levels are inadequate for interim payments, liquidation of the domestic project will inevitably diminish the final returns of the “patient” international lenders. Even so, these lenders choose to optimally rollover as long as the final returns are higher than the world interest rate. However, when the government cannot

[1] A sudden stop is a sudden slowdown or reversal of capital inflows into emerging economies.

guarantee such returns, all lenders refuse to roll over their loans in the interim, causing a "sudden stop". On the one hand, higher reserves allow governments to absorb larger amounts of interim called debt, and decreases the likelihood of a sudden stop. On the other hand, higher reserves imply less capital invested into the domestic project and therefore less final output. Given these trade-offs, the levels of reserves and debt will then be chosen to maximize expected domestic consumption.

We extend this environment to a dynamic framework with N emerging economies in order to quantitatively account for the increase in reserve holdings and the pattern of sudden stop occurrences. Faced with an unexpected change in the liquidity shock process, the countries gradually learn the true process through Bayesian updating. We argue that sudden stops surged as an outcome of two factors: greater international capital mobility in the globalization era and agents' initial underestimation of this increased mobility. This is modeled as an unexpected change in the liquidity shock process. Once agents have fully learned the new regime, governments hold a higher level of reserves and thus sudden stops decrease.

Relation to the Literature

This paper builds on a large body of literature on reserves and sudden stops. For a long time, reserves were seen as an integral part of a country's export promotion strategy: they promote export by slowing appreciation. Dooley, Folkerts-Landau and Garger (2004) recently reiterated this explanation to justify the large foreign reserve holdings of emerging economies, in particular China. As documented by Aizenman and Lee (2007), this export promotion view cannot explain the recent increases in reserves of most countries, including China. In fact, reserves mostly increased long after exports started growing. If reserves mainly served to promote exports, they should have grown during the export growth.

Heller (1966) and Frenkel and Jovanovic (1981) model reserves as a buffer against exogenous stochastic balance-of-payments (BOP) deficits. In Frenkel and Jovanovic (1981), the government seeks to minimize the one-time adjustment costs that are incurred when reserves dry up. Higher reserves increase the distance-to-adjustment because the exogenous adjustment threshold is hit less often. Reserves however have an opportunity cost represented by the forgone interest earnings. This trade-off determines the optimal reserves held by a government. Numerous papers follow this inventory approach to the role of reserves, e.g. Flood and Marion (2001).

More recently, precautionary motives have been explored as a potential key determinant of reserve allocations. Jeanne and Ranciere (2008) and Alfaro and Kanczuk (2007) focus on the consumption smoothing role of foreign reserves against output contractions associated with sudden stops. In Jeanne and Ranciere (2008), a government can purchase a special insurance ("reserves") for which it pays a premium every period until an exogenous event (a "sudden stop") occurs. Using the sovereign default framework of Arellano (2008), Alfaro and Kanczuk (2007) considers the role of reserves in an environment where the government can default on debt. In this setup, reserves serve as a post-default consumption smoothing mechanism since reserves can be used even after a country has defaulted. However, these consumption smoothing models of reserves can neither account for the rise in reserve holdings nor the pattern of sudden stop occurrences. In fact, Alfaro and Kanczuk (2007) suggest that emerging countries should hold no foreign reserves at all. Obstfeld, Shambaugh, and Taylor (2010) also document the predictive failure of the existing "sudden stop" theories of reserves in that they are not able to rationalize the level of reserves accumulated by emerging economies. In contrast to these studies, our model generates time series of reserves and sudden stop occurrences that are consistent with the data. The main difference in our work is that reserves serve more than just to smooth consumption; they also play an active role in preventing sudden stops.

Our work is closely related to Aizenman and Lee (2007) who use a simple Diamond-Dybvig framework with exogenous interest rate, investment scale, and exogenous sudden stop probability to model reserve hoarding. In Aizenman and Lee (2007), countries face exogenous balance of payments (BOP) deficits which must be financed with reserves or by liquidating domestic investments. Reserves hence serve as a cushion against the costly liquidation of productive domestic projects. We also use a Diamond-Dybvig technology specification in which reserves play an essential role in the foreign lenders' decision to roll-over debt, akin to the role of reserves in banks to prevent bank-runs. Our work departs from Aizenman and Lee (2007) by considering the joint decision of borrowing and reserves, and by crucially endogenizing the probability of financial crises. We can account for the rise in reserve holdings and the sudden stop occurrences precisely because reserve holdings do affect sudden stop probabilities.

Our contribution to the literature is twofold. First, we develop a theoretical framework capable of analyzing optimal reserves-to-debt ratios in a model with endogenous sudden stop probabilities and endogenous interest rate premia. In this theory, sudden stops arise when all foreign lenders rationally choose not to roll over the entirety of the country's debt. Despite having full commitment, the government's ability to repay its debt is limited by the resources available in the economy. On the one hand, reserves protect domestic projects from liquidation and make foreign lenders calmer as the country is solvent in more states of the world. On the other hand, foreign reserves reduce the capital used in the productive sector.

Our second contribution is that we can explain and quantitatively account for an important aspect of global imbalances: the outburst of sudden stops in the late 1990s, the subsequent increase in foreign reserves holdings, and the salient absence of sudden stops ever since. We do so by using a dynamic multi-country extension of our model with Bayesian learning and a regime switch in the stochastic liquidity shocks.[2] During the transition when agents are learning about the regime switch, governments under-invest in reserves, leading to an endogenous increase in sudden stop occurrences.

This paper is structured as follows. Section 2 empirically analyzes reserve holdings and

[2]We argue in a different manuscript that the process of worlwide liberalization can rationalize our suggested regime switch in the stochastic liquidity shocks faced by an economy. In particular, in a portfolio allocation problem, asymmetric patterns of liberalization across economies can induce changes akin to the switch in country-specific liquidity shocks.

sudden stops in emerging economies from 1990-2007. Section 3 provides a simple three-period model of reserves allocation that delivers an optimal reserves-to-debt ratio with endogenous interest rates and sudden stop probabilities. Section 4 presents a multi-country dynamic model with learning and regime change. In Section 5, the model is parameterized and we show it can quantitatively account for the observed stylized facts from section 2. Section 6 concludes.

2 Facts on Reserves and Sudden Stops in Emerging Economies

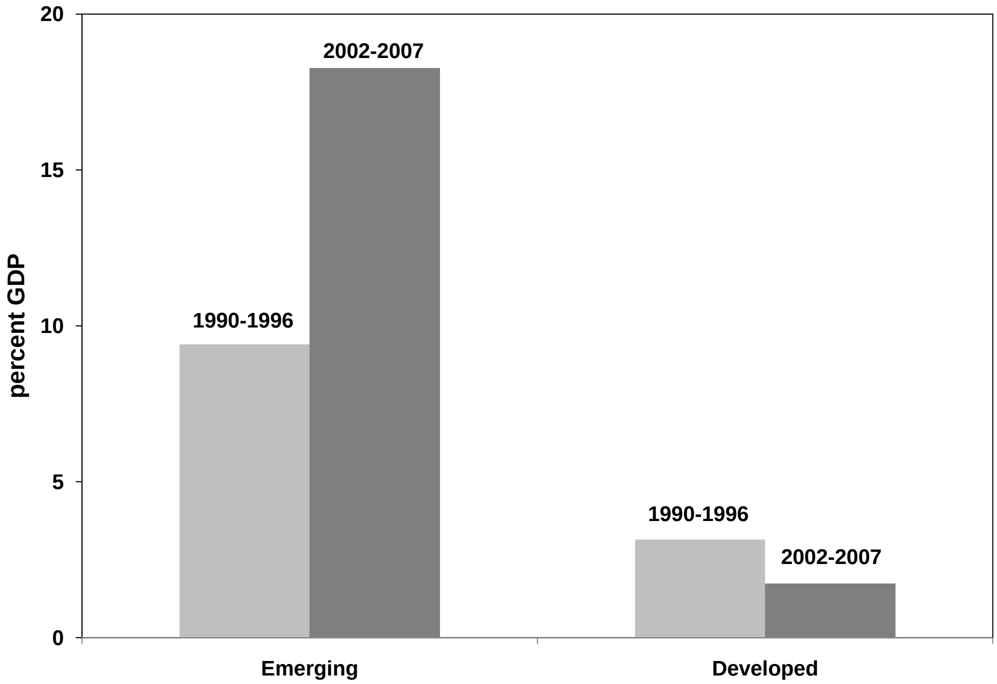
In this section, we document a set of stylized facts regarding foreign reserves, external liabilities, and sudden stops in 24 emerging economies during 1990-2007. We use data on international liquidity from the IMF IFS dataset and the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007). The list of emerging economies includes Argentina, Brazil, Chile, China, Colombia, the Czech Republic, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, South Africa, South Korea, Thailand, and Turkey. This list includes countries appearing in most classifications of emerging countries with the notable exception of Taiwan for which the available data is limited.

2.1 Foreign Reserves

In the IFS dataset, foreign reserves are constructed as Total Reserves minus Gold. This definition includes convertible foreign exchange, SDR holdings, and IMF reserve position. There are two notable facts regarding foreign reserves holdings. The first is that foreign reserves as a percent of GDP in emerging economies are significantly higher than those in developed economies.[3] The second is that these ratios have increased in emerging economies while the opposite holds for developed economies. These facts are summarized in Figure 1.

[3] The developed economies refer to the United States, the United Kingdom, France, and Germany.

Figure 1: Foreign Reserves over GDP



It is worth noting that this phenomenon of increasing reserves is not limited to just a few countries; in fact, they are increasing in all but one of the 24 emerging economies with Chile being the exception.[4] This robust observation can be seen in the first 3 columns of Table 1.

2.2 Foreign Reserves and Gross External Liabilities

Here, we document two facts using the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007) for gross external liabilities. External Liabilities are constructed using "Other Investment Liabilities" and the "Debt Securities" item under "Portfolio Investment Liabilities".

The first fact is that reserves-to-liabilities ratios are much higher in emerging economies than in developed economies. The average of these ratios for emerging economies for 2002-

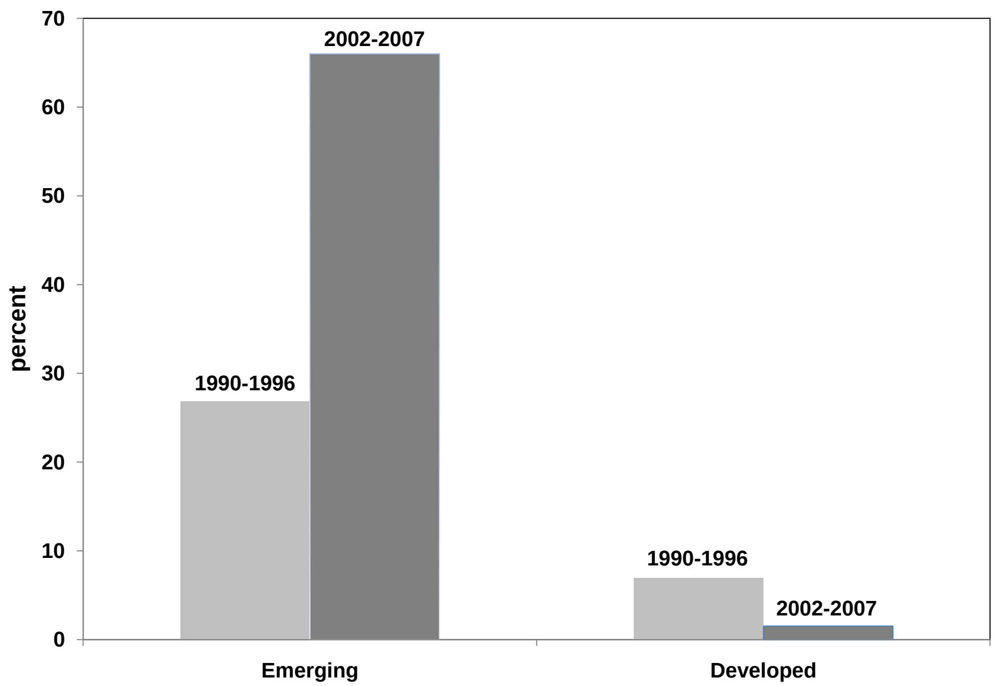
[4]Note, however, that Chile had very large reserve holdings as a percent of GDP during 1990-1996.

Table 1: Detailed Foreign Reserves

	Foreign Reserves/GDP			Foreign Reserves/External Liabilities		
	1990-1996	2002-2007	% Change	1990-1996	2002-2007	% Change
Emerging						
Argentina	4.9%	12.5%	154%	14.3%	21.4%	50%
Brazil	4.8%	8.3%	73%	19.5%	35.3%	81%
Chile	20.1%	15.7%	-22%	50.9%	39.0%	-23%
China	8.1%	32.9%	306%	54.3%	271.3%	400%
Colombia	9.7%	10.5%	8%	35.6%	35.1%	-1%
Czech Republic	18.3%	25.2%	38%	57.2%	77.5%	36%
Egypt	18.8%	19.3%	3%	35.2%	65.5%	86%
Hungary	15.8%	16.7%	5%	26.6%	26.2%	-2%
India	3.4%	18.2%	429%	11.8%	101.0%	754%
Indonesia	6.7%	12.4%	86%	11.7%	25.2%	116%
Korea	5.5%	24.6%	345%	28.0%	97.4%	248%
Malaysia	28.7%	47.0%	64%	77.0%	125.6%	63%
Mexico	4.6%	8.1%	78%	12.2%	40.9%	235%
Morocco	10.4%	28.7%	175%	15.9%	99.0%	522%
Pakistan	1.8%	10.2%	472%	4.3%	28.2%	552%
Peru	11.1%	18.2%	64%	16.7%	49.1%	195%
Philippines	8.1%	17.2%	113%	13.3%	28.4%	113%
Poland	7.0%	14.0%	101%	15.9%	35.8%	125%
Romania	4.0%	18.1%	352%	22.7%	53.9%	138%
Russia	3.2%	22.9%	615%	7.3%	68.5%	836%
Saudi Arabia	7.3%	9.4%	28%	56.2%	87.3%	55%
South Africa	1.0%	7.0%	591%	4.7%	34.9%	646%
Thailand	19.8%	30.6%	54%	41.8%	112.1%	168%
Turkey	3.9%	10.6%	171%	13.0%	25.5%	96%
Developed						
France	2.1%	1.7%	-19%	3.8%	1.3%	-66%
Germany	3.9%	1.8%	-54%	8.7%	1.5%	-83%
United Kingdom	3.6%	1.8%	-49%	2.4%	0.6%	-73%
United States	1.0%	0.5%	-46%	3.2%	0.8%	-74%

2007 is more than 30 times higher than that for developed economies. The second fact is that reserves-to-liabilities ratios have been increasing in emerging economies while they have been decreasing in developed economies. This observation holds for all but three of the 24 emerging economies with Chile, Hungary, and Colombia being the exceptions. These facts can be seen more succinctly in Figure 2 and with more details in the last 3 columns of Table 1.

Figure 2: Foreign Reserves over External Liabilities



2.3 Sudden Stops in Emerging Economies

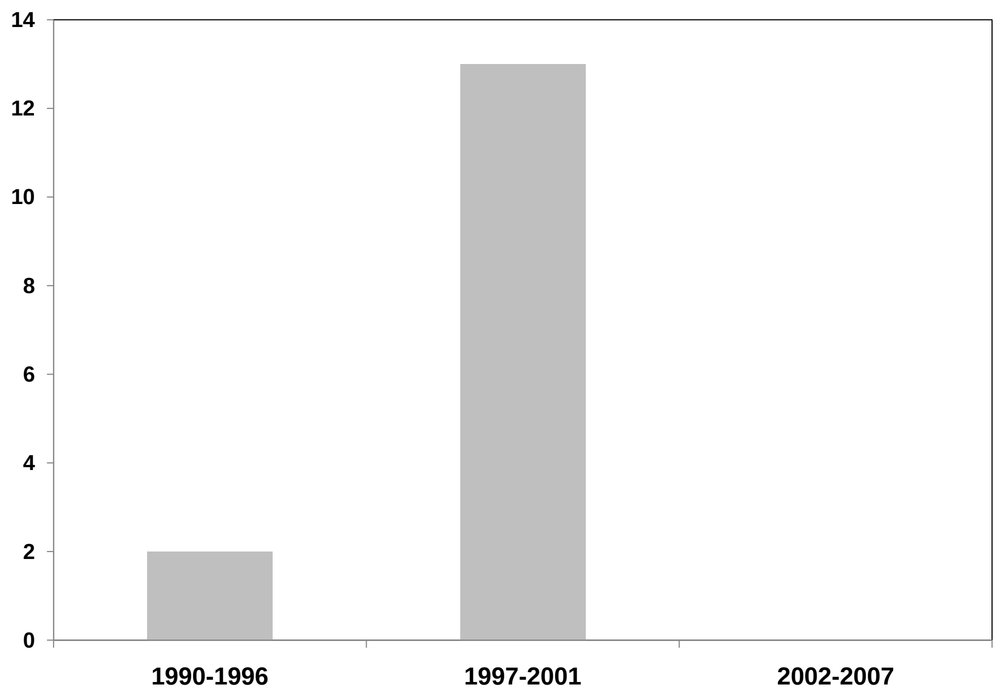
We define a sudden stop to be a sudden slowdown or reversal of capital inflows into emerging economies.[5] Using the sudden stop episodes classified in Durdu, Mendoza, Terrones (2009), there were 15 sudden stop experiences during 1990-2007 across the 24 emerging economies

[5] Calvo et al. (2004) define a sudden stop to be a year-on-year fall in capital flows of at least two standard deviations below the mean.

we are studying.[6] In particular, we divide this time frame into three periods as shown in Figure 3:

- 1990-1996 is a period of low-frequency sudden stops (with two occurrences),
- 1997-2001 is a period of high-frequency sudden stops (with thirteen occurrences),
- and 2002-2007 is a period of low-frequency sudden stops (with no occurrence).

Figure 3: Sudden Stops Occurrences



3 A Three-Period Model of Optimal Reserves Allocation

In this section, we provide a theory of optimal reserves allocation in an environment where governments face rollover risk. In the subsequent section, we extend the basic model to a

[6] Sudden Stop episodes: Argentina, Mexico (1994), Indonesia, Malaysia, Philippines, South Korea, Thailand (1997), Brazil, Chile, Colombia, Pakistan, Peru, Russia (1998), Argentina, Turkey (2001).

dynamic framework to provide a theory for the increase in reserves and the pattern of sudden stops in emerging countries documented in the previous section.

3.1 Environment

We consider a small open economy model with three periods: $t = 1$ (initial), 2 (interim), 3 (final). There is a unit measure of risk neutral foreign lenders who can choose to lend to the domestic country.[7] The domestic country has a representative agent who has preferences over final period consumption, given by $u : R_{++} \to R$, which is twice continuously differentiable, strictly increasing, and concave. The government chooses allocations and debt arrangements that maximizes the expected utility of the domestic agent.

3.2 Technologies

The domestic country has access to two technologies à la Diamond and Dybvig (1983). The first transforms the investment K made in the initial period into $f(K)$ in the final period if production is uninterrupted, where f satisfies the Inada conditions. However, if production is interrupted in the interim through the liquidation of $L \in [0, K]$ units of investment, the technology yields λL in the interim and $f(K - L)$ in the final period. We assume $\lambda < 1$, reflecting the idea that it is costly to divest from the long-term investment in the interim. The second technology transforms a unit of investment into a unit of output in the subsequent period.[8] This technology is referred to as the "reserves" technology. The technologies are represented by:

Technologies	$t = 1$	$t = 2$	$t = 3$
Production and liquidation	$-K$	λL	$f(K - L)$
Initial reserves	-1	1	
Interim reserves		-1	1

[7]For technical reasons, we assume that the foreigners' capital endowment is finite and large enough.
[8]We assume that the reserves do not earn any return. This assumption can be relaxed.

3.3 International Financial Markets

Timeline

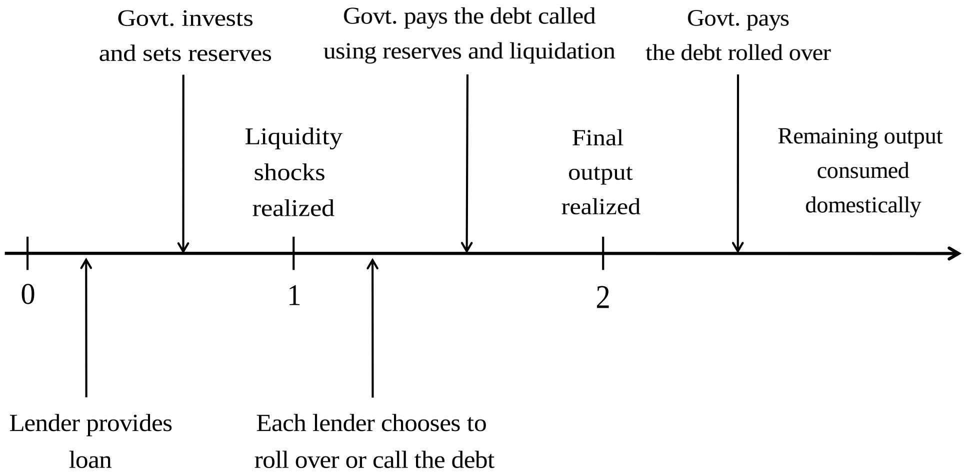
In the initial period, the domestic government borrows D from foreign lenders to finance its initial period investments. The timeline presented in Figure 4 provides an overview of the following sequence of actions taken in this environment. In the interim, a fraction φ of the foreign lenders receive liquidity shocks, denoted by $\varphi_i = 1$, meaning that they must call the loan and be repaid back. The fraction φ is stochastic and has a cumulative distribution function $F: [0,1] \to [0,1]$. We assume that $F(0) < 1$. The remaining fraction $(1-\varphi)$ of lenders with $\varphi_i = 0$ can choose to call or roll over their loans. Let $\psi_i \in \{0,1\}$ denote the rollover decision of lender i:

$$\begin{cases} \psi_i = 1 & \text{if lender } i \text{ chooses to call the loan} \\ \psi_i = 0 & \text{if lender } i \text{ chooses to roll over the loan} \end{cases}$$

We assume that each individual lender i takes the actions of the other lenders as given. In particular, they take as given the call/rollover decisions $\{\psi_j(\varphi, \varphi_j)\}_{j \neq i}$. This implies that once the aggregate liquidity shock is realized, the fraction of lenders calling the loan $\psi(\varphi) \triangleq \int \psi_j(\varphi) dj$ can be inferred by each individual lender. The liquidity shock φ and the induced call/rollover decisions of the foreign lenders represent the rollover risk faced by the domestic government.

We allow the debt contract to be state-contingent. In particular, foreign lenders receive $(1 + r(\varphi))\, D$ if they call their loan in the interim, and $(1 + r(\varphi))^2\, D$ if they roll over the loan. The dependence of payments on the aggregate shock allows the government to partially or, in some cases, fully default if ψ, the fraction of lenders that call the loan in the interim, is sufficiently large.

Figure 4: Timing of actions



Debt contract definitions

A debt contract is defined to be:

- three positive scalars $\{D,\ K,\ R_1\}$, representing the loan amount, the initial capital, and the initial reserves, and
- four state-contingent functions $\{R_2(\varphi),\ L(\varphi),\ r(\varphi),\ \psi_i(\varphi,\varphi_i)\}$, which denote the state-contingent interim reserves, interim liquidation, interest rate, and individual rollover policy, respectively.

While interim reserves, interim liquidation, and the interest rate are functions of the aggregate liquidity shock φ, the rollover policy is a function of both the aggregate state φ and the individual liquidity shock φ_i of an investor.

A debt contract is *resource feasible* if it satisfies the following constraints:

$$
\begin{aligned}
R_1 + K &\leq D \\
R_2(\varphi) + \psi(\varphi)\,(1 + r(\varphi))\,D &\leq R_1 + \lambda L(\varphi) && \forall\varphi \\
0 &\leq R_2\,(\varphi) && \forall\varphi \\
L\,(\varphi) &\leq K && \forall\varphi \\
(1 - \psi(\varphi))\,(1 + r(\varphi))^2\,D &\leq R_2\,(\varphi) + f\,(K - L\,(\varphi)) && \forall\varphi
\end{aligned}
$$

In other words, initial reserves and invested capital cannot exceed the loan amount; and interim reserves and interim payments cannot exceed initial reserves and interim output. Obviously, the main friction here is that the government cannot lend in the interim against the future output from the initial investment. Hence, liquidation and reserves are the only resources available to make interim payments. Similarly, final output and residual reserves are the only resources available to make final payments.

A debt contract is *individually rational* if, for each aggregate liquidity shock φ and individual liquidity shock φ_i, it satisfies $\psi_i^*(\varphi, \varphi_i) \in \arg\max V\,(\cdot \,|\, \varphi, \varphi_i)$ where V, the lender's payoff, is given by

$$
V\,(\psi_i \,|\, \varphi, \varphi_i) = \begin{cases} (1+\hat{r})(1+r(\varphi))D & \text{if } \psi_i = 1 \\ \mathbb{1}_{\varphi i=0} \cdot (1+r(\varphi))^2 D & \text{if } \psi_i = 0 \end{cases}
$$

This condition requires that the interim rollover policy is to roll over the loan if and only if rolling over yields a higher payoff than calling the loan in the interim.

A debt contract satisfies the *participation constraint* if the debt contract yields as much payoff, in expectation, to the lender as what the lender can get by investing at the world interest rate:

$$
\mathbf{E}_\varphi\left[V\,(\psi_i \,|\, \varphi, \varphi_i)\right] \;\geq\; (1+\hat{r})^2 D
$$

An *optimal debt contract* is a tuple $C^* = \{D^*,\, K^*,\, R_1^*,\, R_2^*(\varphi),\, L^*(\varphi),\, \hat{r}^*(\varphi),\, \psi_i^*(\varphi,\varphi_i)\}$ which maximizes the expected utility of the domestic representative agent subject to resource feasibility, individual rationality, and the participation constraint:

$$\max_C \mathbf{E}_\varphi \left[u \left(\underbrace{f(K-L(\varphi)) + R_2(\varphi)}_{\substack{\text{final ouput +} \\ \text{remaining reserves}}} - \underbrace{(1-\psi(\varphi))(1+r(\varphi))^2 D}_{\substack{\text{time 2} \\ \text{payments}}} \right) \right]$$

subject to

$$\begin{array}{rcll}
R_1 + K & \leq & D & \\
R_2(\varphi) + \psi(\varphi)\,(1+r(\varphi))\,D & \leq & R_1 + \lambda L(\varphi) & \forall\varphi \\
0 & \leq & R_2\,(\varphi) & \forall\varphi \\
L\,(\varphi) & \leq & K & \forall\varphi \\
(1-\psi(\varphi))\,(1+r(\varphi))^2\,D & \leq & R_2\,(\varphi) + f\,(K - L\,(\varphi)) & \forall\varphi \\
\psi_i(\varphi,\varphi_i) & \in & \arg\max V\,(\cdot\,|\,\varphi,\varphi_i) & \forall\varphi \\
(1+\hat{r})^2 D & \leq & \mathbf{E}_\varphi\,[V\,(\psi_i\,|\,\varphi,\varphi_i)] &
\end{array}$$

3.4 Optimal Contract Characterization

We now characterize the optimal contract. First, we establish some intermediate lemmas that will help us establish the main results of the model.

Lemma 1. Interim call fraction

$$\psi(\varphi) = \begin{cases} \varphi \\ 1 \end{cases}$$

Proof: By symmetry.

Assumption 1. Liquidiation Technology

$$\lambda(1+\hat{r})^2 < 1$$

Lemma 2. Interest rate

$$(i)\;\; r^*(\varphi) \geq \hat{r} \;\text{ if } \psi(\varphi) = \varphi$$
$$(ii)\;\; r^*(\varphi) \leq \hat{r} \;\text{ otherwise}$$

Proof: See Appendix

Lemma 3. Reserves

Interim payments are paid exclusively with reserves until they are depleted, i.e.

$$\exists \varphi_R \in [0,1] \text{ s.t. } \begin{cases} R_2(\varphi) > 0 & \Leftrightarrow \varphi \in [0, \varphi_R) \\ L(\varphi) = 0 & \Leftrightarrow \varphi \in [0, \varphi_R] \end{cases}$$

Furthermore, $\varphi_R = \dfrac{R_1}{(1+r(\varphi))D}$

Proof: See Appendix

Lemma 4. Sudden stops

For sufficiently large aggregate shocks, all lenders call their loans in the interim, i.e.

$$\exists \varphi_S \in [\varphi_R, 1] \text{ s.t. } \begin{cases} \psi(\varphi) = \varphi & \Leftrightarrow \varphi \in [0, \varphi_S) \\ \psi(\varphi) = 1 & \Leftrightarrow \varphi \in [\varphi_S, 1] \end{cases}$$

Proof: See Appendix

Discussion In Lemma 3, φ_R is the liquidity shock above which reserves are depleted and the government must start liquidating the invested capital to meet the promised payments. Because $\lambda < 1$, the government always uses existing reserves to meet payments before eventually liquidating the invested capital.

φ_{SS} is the liquidity shock at which the government cannot deliver sufficient time 2 payments to persuade the patient lenders to roll over the debt. Any liquidity shock higher than φ_{SS} will cause total exit, and we identify this phenomena as a *sudden stop*.

The cutoffs are illustratively shown in Figures 5 and 6.

Figure 5: Fraction of K_0 liquidated

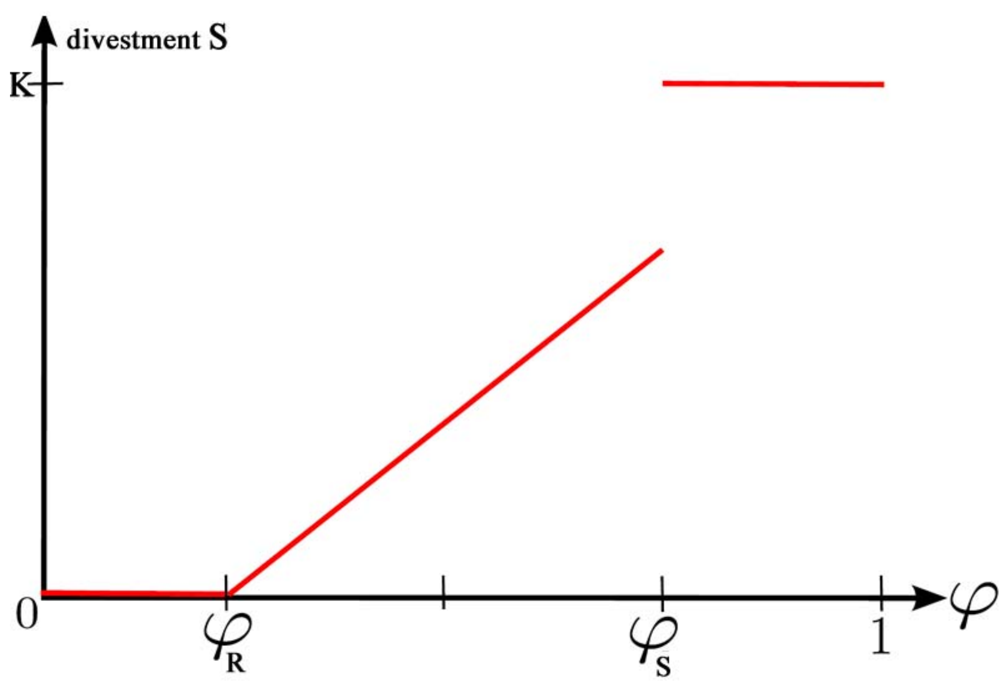
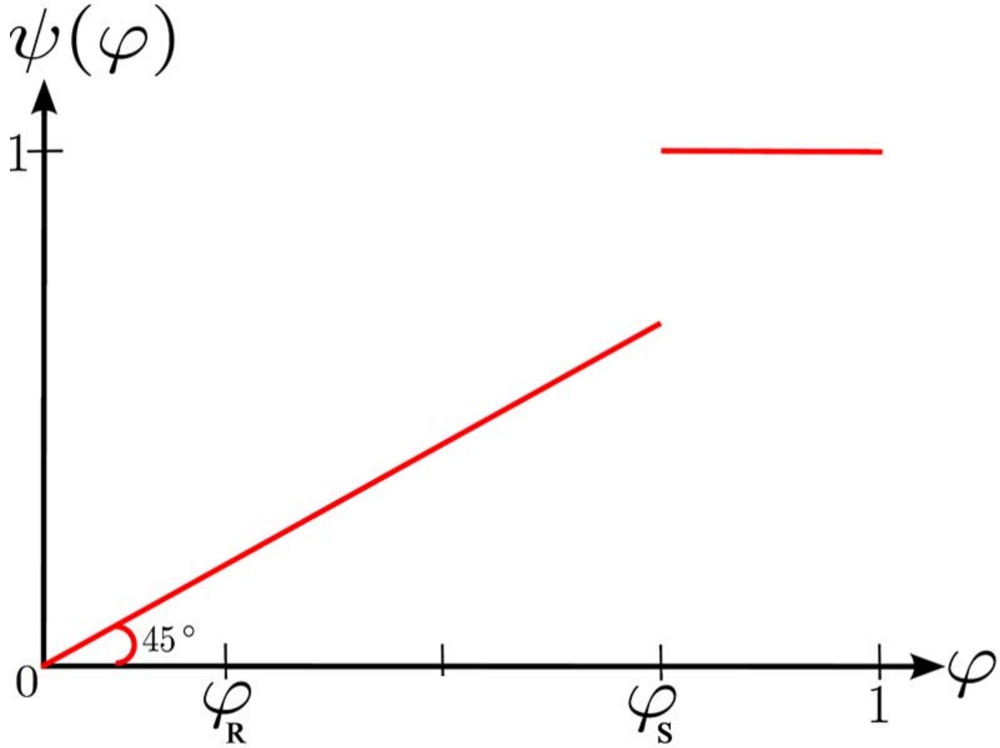


Figure 6: Fraction of lenders who call the debt



Lemmas 2 and 4 imply that the individual rationality constraint can be written as $(r(\varphi) - \hat{r})(\varphi - \varphi_S) \leq 0$. That is, the interest rate is higher than the world interest rate if and only if there is no sudden stop. This is consistent with the empirical observation that emerging countries often pay higher interest rates on their debt during "normal times", but often pay lower rates, by debt restructuring or default, in times of sudden stops.

Lemma 5. Sudden stop interest rate

If $\varphi \geq \varphi_S$, then the interest rate is constant, i.e.

$$\forall \varphi \in [\varphi_S, 1]\ r(\varphi) = r_L$$

Proof:

By lemma 4, $\varphi \in [\varphi_S, 1] \Leftrightarrow \psi(\varphi) = 1$.By lemma 2, $\psi(\varphi) = 1 \Leftrightarrow r(\varphi) < \hat{r}$. Hence consumption is given by $c(\varphi) = f(K - \frac{1}{\lambda}(1 + r(\varphi))D)$. The result immediately follows due to the concavity of u.

These characterizations allows us to formulate the optimal contract problem as follows:

$$\begin{aligned} \max_{R_1,D,K,r,\varphi_S} \quad & \int_0^{\varphi_R} u\left(f(K) + R_2(\varphi) - (1-\varphi)(1+r(\varphi))^2 D\right) dF(\varphi) \\ & + \int_{\varphi_R}^{\varphi_S} u\left(f(K - L(\varphi)) - (1-\varphi)(1+r(\varphi))^2 D\right) dF(\varphi) \\ & + u\left(f(K - L_{SS}\right)\left(1 - F(\varphi_S)\right) \end{aligned}$$

subject to

$$\begin{aligned} R_1 + K &\leq D \\ 0 &\leq R_1 - \varphi(1+r(\varphi))D && \equiv R_2(\varphi) \quad \forall \varphi \in [0, \varphi_R] \\ 0 &\leq \tfrac{1}{\lambda}\left(\varphi(1+r(\varphi))D - R_1\right) && \equiv L(\varphi) \quad \forall \varphi \in [\varphi_R, \varphi_S) \\ 0 &\leq \tfrac{1}{\lambda}\left((1+r_L)D - R_1\right) && \equiv L_{SS}(\varphi) \\ \hat{r} &\leq r(\varphi) \end{aligned}$$

Proposition 1. Existence and uniqueness *The optimal contract problem has a solution C^* and it is unique.*

Proof:

The result follows from the Theorem of the Maximum.

Corollary 1. Endogenous sudden stop probability *The optimal contract C^* induces an ex ante endogenous probability that a sudden stop occurs.*

That is:

$$\Pr(\chi = 1) = 1 - F(\varphi^*_{SS})$$

where $\chi = 1$ if a sudden stop occurs.

Proof:

This follows from Lemma 4 and Proposition 1.

3.5 A Numerical Illustration

We now consider a simplified version of the model to illustrate the optimal contract and how it changes when higher liquidity shocks become more likely. In particular, we assume that, in non-sudden states, the interest rate $r(\varphi)$is constant and equal to an endogenously chosen level $\bar{r}$. This simplication allows us to focus on the main channel in our model, the rollover risk. By restricting the flexbility of the offered interest rate, we mainly limit the amount of consumption smoothing that occurs across non-crisis states. This subsection will be updated in future versions.

4 A Multi-Country Dynamic Extension with Learning and Regime Change

The previous section illustrated the main forces determining the optimal allocation of foreign reserves: the delicate interaction between sudden stop probabilities and productive capital use. We now propose a fully dynamic model with N small (emerging) economies that face an unexpected change in the liquidity shock process and gradually learn the true process through Bayesian updating. This framework can quantitatively account for the dynamics of the foreign reserves holdings, external liabilities, and sudden stops.

We argue that sudden stops surged as a result of greater international capital mobility in the globalization era along with agents' initial underestimation of this increased mobility. This increased mobility is modeled as an unexpected change in the liquidity shock process. The extension we propose formalizes a thought experiment in which the unexpected switch occurs in the late nineties. As predicted by our theory, any underestimation of the true process of liquidity shocks will induce an increase in sudden stop occurrences. Countries gradually update their beliefs on the liquidity shock process using Bayes' rule and the endogenous sudden stop probabilities predicted by our theory.

In this section, we set up the extended environment. We present numerical results in the next section.

4.1 Environment

We consider N identical small economies indexed by $j = 1, \ldots, N$. Time is infinite, discrete and indexed by $t = 0, 1, \ldots, \infty$. Each country is populated by an infinitely-lived representative agent and a welfare-maximizing domestic government. There is a continuum of infinitely lived risk-neutral foreign lenders $i \in [0, 1]$. Agents discount future consumption by the discount factor β.

Each time period t is divided into three stages, $s = 0, 1, 2$. Each period t effectively encapsulates the three stages of the previous section's basic model:

- $s = 0$ is the initial contracting stage
- $s = 1$ is the interim stage during which liquidity shocks are realized, and lenders make call/rollover decisions
- $s = 2$ is the final production and consumption stage.

Within each period t, the technologies available at a stage s are identical to those in the previous section with the addition of an inter-period reserves savings technology. As in the previous section, at $s = 2$, the government can use final output $f(K - L(\varphi))$ and remaining reserves $R_2(\varphi)$ for consumption and final payments $P_2(\varphi)$. In addition, the government may choose to save part of the remaining reserves for next period. The choice of reserves to carry over R_0' is constrained by:

$$R_0'(\varphi) \leq R_2(\varphi) - \max\left\{P_2(\varphi) - f(K - L(\varphi)),\ 0\right\}\ \forall \varphi$$

Shocks and Information Structure The aggregate liquidity shock in country j at time t is denoted by $\varphi_t^j \in [0, 1]$. The N aggregate shocks $\left\{\varphi_t^j\ :\ j = 1 \ldots N\right\}_{t=0}^{\infty}$ are independent and identically distributed across countries and time[9]. These aggregate liquidity shocks follow a common stochastic process with cumulative distribution function F_{σ_t} with higher values of σ_t indicating higher likelihood of large aggregate shocks[10]. As in the basic model, a fraction φ_t^j of foreign lenders lending to country j receive liquidity shocks and must call the debt in the interim.

We assume $\sigma_t \in \{\sigma_L, \sigma_H\}$ with $\sigma_L < \sigma_H$. This regime parameter σ_t is unobserved and unknown to the agents, though agents share a common belief ρ_t at time t:

$$\rho_t \triangleq \Pr\left(\sigma_t = \sigma_L\right)$$

At the end of each period t, agents observe the sudden stop occurrences in the N countries. Using these sudden stop occurrences and the endogenous sudden stop probabilities,[11] agents update their beliefs according to Bayes' rule.[12]

4.2 Optimal Dynamic Stochastic Contracts

An important difference with the basic model is the endogeneity of reserve endowments. In the basic model, the reserve endowment was zero; in the dynamic model, governments will face a consumption/savings decision and will choose the reserve endowments of the following period. As in the previous section, let $C = \{D,\ K,\ R_1,\ R_2(\varphi),\ L(\varphi),\ r(\varphi),\ \psi_i(\varphi, \varphi_i),\ R_0'(\varphi)\}$. For each belief ρ about the prevailing liquidity shock regime, we can characterize the path of the optimal dynamic stochastic contracts $\left\{\mathbb{C}_t^{*j} \mid \rho\right\}_{j,t}$ by solving the following functional

[9] This setup can easily be extended to internationally and/or serially correlated aggregate shocks.

[10] That is: $\sigma_L \le \sigma_H \Rightarrow F_{\sigma_L}(\varphi) \ge F_{\sigma_H}(\varphi) \quad \forall\, \varphi$

[11] See Corollary 1.

[12] If we denote $\chi_t \triangleq \left\{\chi_t^j\right\}_{j=1}^{N} \in \{0,1\}^N$ as the vector of sudden stops where $\chi_t^j = 1$ if a sudden stop occurs in country j at time t, then agents can use the sudden stops vector χ_t, and the endogenous sudden stop probabilities $\Pr\left(\chi_t^j = 1 \mid \sigma\right)$ to update their beliefs according to Bayes' rule

$$\rho_{t+1} = \frac{\rho_t \Pr\left(\chi_t \mid \sigma_L\right)}{\rho_t \Pr\left(\chi_t \mid \sigma_L\right) + (1-\rho_t)\Pr\left(\chi_t \mid \sigma_H\right)}$$

equation:

$$
\begin{aligned}
W\left(R_0;\ \rho\right) = \max_{R_1,D,K,r,\varphi_S} \quad & \int_0^{\varphi_R} u\left(f(K)+R_2(\varphi)-(1-\varphi)(1+r(\varphi))^2 D - R_0'(\varphi)\right) dF(\varphi) \\
& + \int_{\varphi_R}^{\varphi_S} u\left(f\left(K-L(\varphi)\right)-(1-\varphi)(1+r(\varphi))^2 D - R_0'(\varphi)\right) dF(\varphi) \\
& + \int_{\varphi_S}^{1} u\left(f\left(K-L_{SS}\right) - R_0'(\varphi)\right) dF(\varphi) \\
& + \beta \int_0^1 W\left(R_0'(\varphi);\ \rho\right) dF(\varphi)
\end{aligned}
$$

subject to

$$
\begin{aligned}
R_1 + K &\leq D + R_0 \\
0 &\leq R_1 - \varphi(1+r(\varphi))D && \equiv R_2(\varphi) && \forall \varphi \in [0, \varphi_R] \\
0 &\leq \tfrac{1}{\lambda}\left(\varphi(1+r(\varphi))D - R_1\right) && \equiv L(\varphi) && \forall \varphi \in [\varphi_R, \varphi_S) \\
0 &\leq \tfrac{1}{\lambda}\left((1+r_L)D - R_1\right) && \equiv L_{SS}(\varphi) \\
\hat{r} &\leq r(\varphi) \\
0 &\leq R_0'(\varphi) && && \forall \varphi \in [0,1]
\end{aligned}
$$

Recall that at the end of each period t, the common belief of the agents is updated to ρ_{t+1} using the sudden stop occurrences and sudden stop probabilities in the N countries according to Bayes' Rule[13]. Hence, given a sequence of sudden stop occurrences $\{\chi_t\}_t$, an initial belief ρ_0, and initial reserve endowments $\left\{R_-^j\right\}_{j=1}^N$, the realized sequence of optimal contracts $\left\{\mathbb{C}_t^{*j}\right\}_{j,t}$ is well-defined and can be fully characterized using the functional equation solutions and Bayes' rule.

[13] Denoting $\chi_t \in \{0,1\}^N$ as the vector of sudden stops, Bayesian updating is done following:

$$
\rho_{t+1} = \frac{\rho_t \Pr\left(\chi_t \mid \sigma_L\right)}{\rho_t \Pr\left(\chi_t \mid \sigma_L\right) + (1-\rho_t)\Pr\left(\chi_t \mid \sigma_H\right)}
$$

5 Quantitative Analysis

In this section, we discuss the quantitative results of a carefully parametrized model. Our simulations show that our extended model can account for the stylized facts we documented.

In particular, we simulate the following thought experiment. We assume that the period of 1990-1996 was an era of relatively low volatility in international capital movements, i.e. a σ_L regime. By 1997, globalization and widespread financial liberalization allowed less restrictive capital movements but governments and investors underestimated the increase in capital mobility, i.e. there is an unexpected change to a σ_H regime. Based on our theory, this will cause an underinvestment in reserve holdings which increases the probability of sudden stops. Governments and investors, seeing the rise in sudden stops, update their common belief about the prevailing regime. By 2002, agents have fully learned the new regime; as a result, reserves-to-debt is higher and sudden stops decrease.

5.1 Parametrization and Functional Forms

A period in the model is assumed to be a quarter. We choose $N = 24$ as we have 24 emerging economies in our dataset.

The domestic agents are assumed to have a constant relative risk aversion utility function:

$$U(c) = \frac{1}{1-\gamma} c^{1-\gamma}$$

Here, we report results for the log case ($\gamma = 1$) and we present sensitivity analysis in the

with

$$\begin{cases} \Pr(\chi_t \mid \sigma_L) & \triangleq \prod_{j=1}^{N} \left\{ \left[1 - F_{\sigma_L}\left(\underline{\varphi^j_{ss,t}}\right)\right] \cdot \chi^j_t + F_{\sigma_L}\left(\underline{\varphi^j_{ss,t}}\right) \cdot \left(1 - \chi^j_t\right) \right\} \\ \Pr(\chi_t \mid \sigma_H) & \triangleq \prod_{j=1}^{N} \left\{ \left[1 - F_{\sigma_H}\left(\overline{\varphi^j_{ss,t}}\right)\right] \cdot \chi^j_t + F_{\sigma_H}\left(\overline{\varphi^j_{ss,t}}\right) \cdot \left(1 - \chi^j_t\right) \right\} \end{cases}$$

where $\underline{\varphi^j_{SS,t}}$ refers to the sudden stop cutoff induced by the solution to $[\mathcal{RE}]$ at $(R, \rho_t) = \left(R^j_{2,t-1}, \rho_t = 1\right)$ and $\overline{\varphi^j_{SS,t}}$ refers to the sudden stop cutoff induced by the solution to $[\mathcal{RE}]$ at $(R, \rho_t) = \left(R^j_{2,t-1}, \rho_t = 0\right)$. The agents take these endogenous cutoffs as given.

Table 2: Parameter values

Name	Symbol	Value
Discount factor	β	0.98
World interest rate	r	0.01
Low liquidity shock parameter	σ_L	0.1
High liquidity shock parameter	σ_H	0.32
Divestment parameter	λ	0.7
Production exponent	θ	0.85
Production parameter	A	1.5

appendix.

The liquidity shock process is an important element of the model. We assume the aggregate liquidity shock distributions $(F_{\sigma_L}, F_{\sigma_H})$ belong to the class of Generalized Bounded Pareto distributions on $[0, 1]$:

$$F_\sigma(\varphi) \triangleq 1 - (1-\varphi)^{\frac{1}{\sigma}}$$

An increase in σ shifts the cumulative distribution function F_σ to the left. The switch from σ_L to σ_H therefore reflects the increase in capital mobility. We choose this kind of power law distribution to reflect the idea that high aggregate liquidity shocks are very unlikely.

The parameters β, r, σ_L, and σ_H are then set to match some facts regarding international liquidity. In particular, we set β to match average interest rates of 2% in emerging economies over 1990-2007, r to match the risk-free rate of 1%, while σ_L and σ_H are set to match average reserves-to-debt ratios in the emerging economies for the periods of 1990-1996 and 2002-2007 respectively. We follow Ennis and Keister (2003) to set the divestment cost $1-\lambda$ to be 30%. We follow Atkeson and Kehoe (2005) to set θ to 0.85 and we assign an arbitrary value of A to 1.5.[14] The parameters are summarized in Table 2.

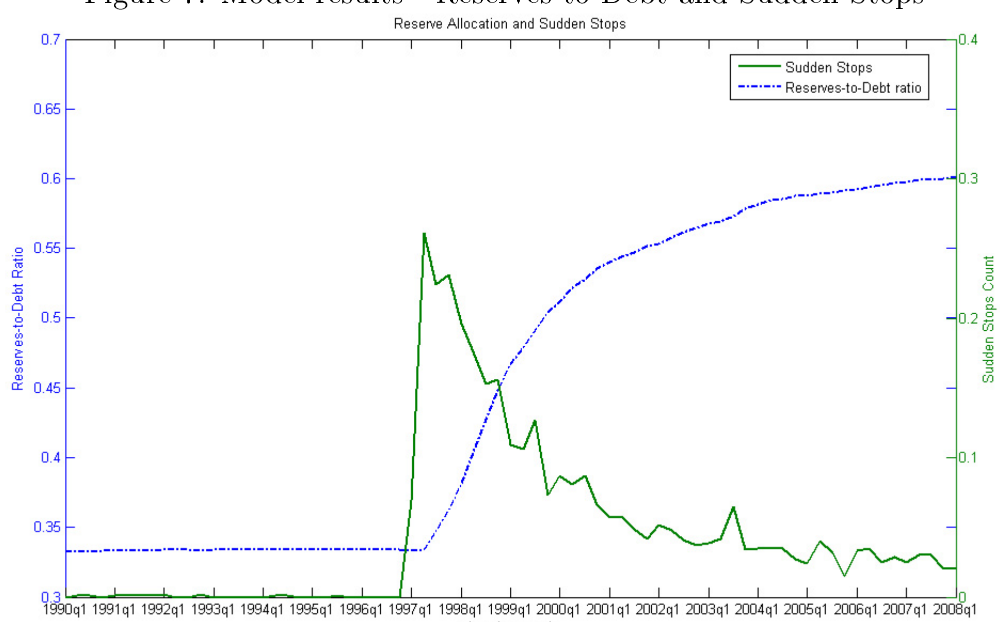
[14]The quantitative results are not sensitive to changes in A. We plan to provide a detailed sensitivity analysis in future versions of this paper.

5.2 Quantitative Results

We consider $N = 24$ identical economies starting with different initial foreign reserves[15]. As the N economies experience different aggregate liquidity paths, their reserves holdings and sudden stops paths also evolve differently. The results shown are the average across a large number of simulated paths for the N countries.

As can be seen in Figure 10, our model is able to replicate the pattern of low frequency sudden stops during 1990-1996, high frequency sudden stops in the transition (1997-2001), and low frequency after the transition (2002-2007). During the transition, governments are under-investing in reserves, thereby increasing the probability of sudden stops. Once the governments have learned of the regime change to higher liquidity shocks, they choose to hold a higher level of reserves, thus returning sudden stop probabilities to lower levels.

Figure 7: Model results - Reserves-to-Debt and Sudden Stops



In our theory, misaligned beliefs beget "abnormal" sudden stop occurrences. Our ex-

[15]The initial levels of foreign reserves were generated by simulating, for a few periods, the model with initial zero foreign reserves.

Table 3: Summary of Numerical Results

	1990-1996	1997-2001	2002-2007
Data			
Reserves-to Debt Ratio	27%	36%	65%
Sudden Stops	2	13	0
Model			
Reserves-to Debt Ratio	27%	51%	65%
Sudden Stops	0.03	5.84	0.94

tended model is able to replicate the surge in sudden stops and the subsequent stabilization because our theory of optimal reserves allocation endogenizes the sudden stops probabilities. As sudden stops occurrences increase, reserves dry up more often but governments keep building up their foreign holdings as a result of updated beliefs. In this model, model reserves do not serve as a *post*-sudden stop insurance. Instead, in contrast to most consumption smoothing theories of reserves allocation, reserves play an active role of preventing sudden stop occurrences and they do not help increase consumption after sudden stops.

Table 3 summarizes our key results. One drawback of the results is that the speed at which agents learn the true process is quite fast: this leads to governments increasing reserves faster and sudden stops ceasing sooner than 2002 as seen in the data. Also, since $\sigma_L < \sigma_H$, the post-crisis era is characterized by slightly more sudden stops than the pre-crisis era. Of course, both periods feature much less sudden stops than the crisis/adjustment era. The extended model also predicts a rise in country-specific interest rate premium during the surge of sudden stops. However, in the absence of risk aversion in the agents' preferences, the premia we generate are very modest and we do not report these results here.

6 Conclusion

In this paper, we have studied empirically and theoretically the joint dynamics of external liabilities, foreign reserves, and sudden stops in emerging economies. Using international liquidity data for 24 emerging economies, we document that reserves holdings as a percent of GDP and reserves-to-external liability ratios have dramatically increased in emerging countries from 1990 to 2007. We also present the time series of sudden stop occurrences: there were virtually no sudden stops in these emerging economies except during 1997-2001.

We then develop a small open economy model where reserves endogenously affect the probability of sudden stops. In our model, foreign lenders choose to roll over their loans as long as their returns are not undermined by the divestment made to repay lenders calling in the interim. Sudden stops occur when all foreign lenders choose to call the loans. On the one hand, reserves protect domestic projects from liquidation and make foreign lenders calmer as the country is solvent in more states of the world. On the other hand, foreign reserves incur opportunity costs by reducing the capital used in the productive sector. Consequently, the model yields an endogenous probability of sudden stop and optimal reserves-to-debt ratios.

Furthermore, we explore the empirical validity of this channel. In particular, the model implies that any underestimation of the true process of liquidity shocks will induce a higher likelihood of sudden stops. We propose a dynamic multi-country model with Bayesian learning and a regime switch in the stochastic liquidity shocks. With the gradual learning of the true regime, we obtain a transition path during which sudden stops surge. The optimal reserves-to-debt ratios are higher at the end of the transition as seen in the data. The calibrated model generates levels that are consistent with the three stylized facts we documented.

This paper therefore provides a useful theory of optimal reserves allocation and sudden stops. Our model however is highly stylized. It does not include many relevant features that can affect our results such as a domestic sector or multiple debt maturities. For instance, Cole and Kehoe (2000) suggest that the composition of a country's debt portfolio matters for sudden stop occurrences. We leave these questions for future research.

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7 Appendix

7.1 Proofs

In this section, we present the sketches of the proofs. Complete formal proofs will be updated shortly.

Lemma 2. Interest rate

$$(i)\ \ r^*(\varphi) \geq \hat{r}\ \text{ if } \psi(\varphi) = \varphi$$
$$(ii)\ \ r^*(\varphi) \leq \hat{r}\ \text{ otherwise}$$

Proof: (i) Note that $\psi(\varphi) = \varphi \Leftrightarrow \psi_i^*(\varphi, \varphi_i = 0) = 0$, i.e. the aggregate call fraction, is equal to the aggregate liquidity shock if and only if lenders with $\varphi_i = 0$ roll over their loans. Suppose there exists $\varphi \in [0,1]$ such that $\psi_i^*(\varphi, \varphi_i = 0) = 0$ and $r(\varphi) < \hat{r}$. This contradicts individual rationality.

(ii) The claim that $r(\varphi) \leq \hat{r}$ if $\psi(\varphi) \neq \varphi$follows from (i) and the participation constraint holding with equality.

Lemma 3. Reserves

Interim payments are paid exclusively with reserves until they are depleted, i.e.

$$\exists \varphi_R \in [0,1] \text{ s.t. } \begin{cases} R_2(\varphi) > 0 & \Leftrightarrow \varphi \in [0, \varphi_R) \\ L(\varphi) = 0 & \Leftrightarrow \varphi \in [0, \varphi_R] \end{cases}$$

Furthermore, $\varphi_R = \frac{R_1}{(1+r(\varphi))D}$

Proof:

The marginal cost of paying with liquidation technology is given by, $\frac{1}{\lambda} f'(K)$. This represents forgone final period consumption. It remains to show that this exceeds the marginal cost of paying with reserves, which is 1. *Proof incomplete.*

Lemma 4. Sudden stops

For sufficiently large aggregate shocks, all lenders call their loans in the interim, i.e.

$$\exists \varphi_S \in [\varphi_R, 1] \text{ s.t. } \begin{cases} \psi(\varphi) = \varphi & \Leftrightarrow \varphi \in [0, \varphi_S) \\ \psi(\varphi) = 1 & \Leftrightarrow \varphi \in [\varphi_S, 1] \end{cases}$$

Proof:

By lemma 1, $\psi(\varphi) = \begin{cases} \varphi \\ 1 \end{cases}$. For $\varphi \in [0, \varphi_R)$, it is clear that $\psi(\varphi) = \varphi$, since the marginal cost of paying with liquidation technology exceeds the marginal cost of paying with reserves, as shown in lemma 3. For $\varphi \in [\varphi_R, 1]$, consumption given by $c(\varphi) = f(K - \frac{1}{\lambda}\psi(\varphi)(1 + r(\varphi))D) - (1 - \psi(\varphi))(1 + r(\varphi))^2 D$. Consumption is decreasing in ψ if $f'(K - \frac{1}{\lambda}\psi(\varphi)(1 + r(\varphi))D) > \lambda(1 + \hat{r})$. *Proof incomplete.*

7.2 Computational Methods

This section will be updated in future versions,

7.3 Sensitivity Analysis

This section will be updated in future versions.