Contributions of the Older German Schools to the Development of Utility Theory*

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Introduction

The contributions of German economists to utility theory in the 19th century are virtually unknown outside Germany, and today hardly even known inside it.[1] Yet Streissler (1990), in a pioneering article, has shown convincingly that they played an enormous role in the later development of the "Austrian school"; he has also explained the reasons for their neglect. The present paper, stimulated entirely by Streissler's, tries to probe a bit more deeply into one aspect of this development: the utility theory that was handed down to the Austrians.

My conclusion is that the major figure in all this development was Karl Heinrich Rau, whose treatise (which went into eight editions) had an extraordinary influence. Rau introduced in the second edition (1833) a dichotemy between two types of "use value": "species value" (*Gattungswerth*) and "concrete value", the first being an objective quantity, independent of the quantity consumed of the goods in a given group ("species") of goods, and the second a function of these quantities. One of Rau's principal concerns was the measurement of national wealth, and he was aware that the use of current market prices in such an index would not correctly indicate the direction of movement of national welfare; some fixed prices needed to be used, and this function could be served by the "species values", except

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[1]Since this paper was completed I came across a fine study by Baloglou (1995) and an interesting collection of articles edited by Priddat (1997) (including one of his own on Rau's *Gattungswerth*), showing that a revival of interest in this subject is well underway.

that these could not be directly observed, but only estimated on the basis of past prices. On the other hand, a different concept of use value was needed to explain market prices, and this was the "concrete value". He (at first) adopted the heroic postulate that under the assumption of satiable wants (an assumption that was accepted by almost all the German writers, and was fully retained by their Austrian successors), within the bounds of satiety ("requirement") of a commodity, the species and concrete use values would agree; beyond these bounds the concrete use value would be equal to be zero, or to the "exchange value" (price) if higher. This discontinuity was clearly a source of discomfort to Rau who, in successive editions of his work, kept finding ways to smooth it over; he finally (1868) allowed the use value to decline continuously below the species value beyond the "requirement" level, as a function of the quantity consumed, thus in effect dispensing with the assumption of satiable preferences. Already in the earlier editions he allowed for insatiable preferences, and for these he introduced in the fifth edition (1847) a clear statement of the principle of diminishing marginal utility (that is, concrete use value as a decreasing function of the quantity consumed), successively clarified in later editions.

Rau's principal followers were the three founders of the older historical school (Hildebrand, Knies, and Roscher), who developed Rau's theory in a particular way: under an assumption, deeply influenced (via Hildebrand) by Proudhon, of unitary price-elasticity of demand. Hildebrand (1848) identified a commodity's constant expenditure share under unitary price elasticity with Rau's species value. With this, Hildebrand introduced a formulation of the law of diminishing marginal utility and a demonstration that concrete use value and exchange value remained proportional to each other; this was reiterated by Knies (1855), who then tried to use Hildebrand's model to explain the data brought to light by Roscher (1854), showing that increases in grain output were accompanied by declines in the market value of the total output. Since these data (exhibiting inelastic demand) were inconsistent with Hildebrand's assumptions, already a great difficulty in the theory had to be faced; Knies's attempted explanation required adjusting the premise of constant expenditure shares in an *ad hoc* manner. Roscher (1857) used the Hildebrand-Knies model to provide an explanation of Smith's paradox of value, but his brief demonstration was difficult for others to follow, and he finally withdrew it from his treatise.

In section 1, I survey the contributions that appeared prior to the publication of the first edition of Rau's treatise (1826), with attention to their focus on use value and exchange value. It appears that, without exception, these writers considered "use value"—a concept which of course descends from Smith's "value in use"—to be a quantity that was independent of the amount consumed. While Storch regarded it as quite variable, being subject to the whims of human evaluation, nevertheless he along with the others made no connection between use value and quantity consumed. This tradition was broken in 1833 by Rau's introduction of the "concrete" or "quantitative" use value, which he juxtaposed with the older concept, which he called "species value" and later, following Riedel (1838), "abstract value". He made explicit the relative constancy of species value, allowing it to differ according to climatic and other conditions, and he stressed its dependence on physical attributes, relating, for example, the species value of different kinds of grain to their nutritive power. As further developed by Friedländer (1852), Knies (1855), and Michaelis (1863), this foreshadowed the theory of "characteristics" introduced by Lancaster (1971).

In the fourth edition of his work (1841a) and shortly thereafter in a Belgian journal (1841b), Rau introduced intersecting supply and demand curves independently of Cournot (1838); he also noted that an individual's demand-price for a quantity of a good was equal to its "concrete use value" (marginal utility) divided by the marginal utility of income, which he took to be the reciprocal of income. I also show that the idea of a hierarchy of wants, usually attributed to Menger, had been fully developed by, among others, Soden, Lotz, Hermann, Rau, Riedel, Friedländer, and Knies, and that Rau and Knies pioneered in the development of a theory of substitutable and complementary commodities.

In the subsequent sections I examine the contributions author by author, except that I have dealt with those of Hermann and Riedel within the other sections. The question obviously arises: where does Gossen fit into this development? I have therefore devoted a section to Gossen, showing among other things that (1) his subjective and even utilitarian approach was fully in the tradition of German economics; (2) the greatest influence on his development was that of Rau; and (3) a special assumption he introduced to simplify his theory made it equivalent to Rau's theory as reinterpreted by Hildebrand, Knies, and Roscher.

I conclude with two sections discussing the reactions of outside commentators (mostly two Ukrainians) on the German economics of this period, and those of later German and Austrian writers, including in particular Knies's two students, Wieser and Böhm-Bawerk.

1 Contributions prior to Rau

The study of the theory of value and utility began in earnest in Germany with the translation by Garve of Adam Smith's *Wealth of Nations* in 1794, and the expositions and developments of Smith's work by Sartorius (1796) and Lueder (1800–1802). There followed a steady development up to the 1880s. In this section I treat the writings that preceded Rau (1826), whose work started a whole new train of thought.

1.1 Kraus

One early response to Smith's work, by a student and later colleague of Kant's, C. J. Kraus—whose work did not see publication until after his premature death—provides an illustration of the excitement generated by Smith's work (cf. Kraus, 1808, §16, pp. 102–3):

> The unit or measure of exchange value devised by Smith is as important for political economy [*Staatswirthschaft*] as is the unit of velocity devised by Galileo in physics. And the notion of regarding labor as the absolute value and the basic standard of value of all things is—to the customary notion that regards money as the absolute value and basic standard of value of all things, including labor—what the Copernican or Newtonian astronomy is to the common view that the earth is at the center of the universe and the sun and stars rotate around it.

As we shall see, however, later German writers—with the partial exception at least of Michaelis (1863) and Schäffle (1870b)—were to replace the notion of measuring the value or utility of a thing (or of an amount of a thing) by the amount of effort one was willing to

devote to obtain it, by the notion of the subjective esteem, or utility, that one attached to the thing, or to an amount of it. This was to go hand in hand with the notion of measuring the utility of a *bundle* of things—"wealth" in Smith's conception—by a weighted average of its commodity components, the weights being appropriate indicators of the commodities' "value".

1.2 Schlözer

An early exposition by Schlözer (1805, §§50–52, pp. 39–41) follows Smith quite closely, but also departs from Smith in an interesting way:

> As long as someone does not contemplate *exchanging* a good which he possesses, but consumes it himself, solely its *inner*, or *absolute*, or *use value* comes into consideration. As soon, however, as he has a mind to sell it, he must also take its *relative* or *exchange value* into consideration. Such a person is one who values the utility of a good in relation to other goods as one who is in a position to obtain for himself a greater or smaller quantity of other goods through exchange.
>
> Both values, *use value* and *exchange value*, are completely independent of one another. The evidence shows this. For there are goods which possess a high *use value* but scarcely any *exchange value*. For others the reverse is true. Nevertheless there are many [goods] that also possess both values at the same time.
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> *Water* has a high *use value* and *no exchange value at all*. A *precious stone* possesses a *high exchange value* and hardly *any use value*. Finally, *metals*, *grain*, *cattle*, *wine*, etc., have both values at the same time.
>
> No-one, for example, would be willing to give up a *pearl* for a pail of *water*, notwithstanding the latter possesses a far *higher use value*.

Now Schlözer proceeds to fill the gaps in Smith's argument and explain this paradox (§52, p. 41):

> That the *exchange value* of a thing stands in relation to the *quantity of goods that other people are inclined to give for it*, is subject to no doubt. This *disposition to give up more or less for a thing* is again none other than the action of the *greater or less desire* to place oneself in possession of a thing. If we now find this degree of desire for various things to be entirely independent of their inner value or of the utility which they promise in use ..., then it is clear that there must be another basis upon which this desire for a thing, or the relative desire that one assigns to it—its exchange value—depends.

What is this "other basis"? It is the allocation of people's productive powers (§53, p. 41):

> We find this basis in the natural *indolence* of each and every man, and in his *self-interest*. Accordingly, man's endeavor is directed towards *enjoying* without *exerting* himself proportionately, and thus each is concerned with gaining possession of the fruit of a third party's *productive powers*.

In a very detailed analysis of the motivations of two contracting parties in using their "productive powers", he comes to a precise explanation of the paradox of value as follows (§53, pp. 44–45):

> ...nobody would be inclined to give up a *pearl*, which is completely devoid of inner value, for a quantity of *water* with all its inner value; for it costs an insignificant expenditure of energy [*Kraftaufwand*] to gain possession of the latter [water] and a very great one to gain possession of the former.

Thus Schlözer here essentially follows Smith in arguing that a pearl will require a greater effort to obtain than water. But if pearls have no "inner value", why will people want to acquire them? Schlözer at least had the merit of recognizing that this was a problem, but chose to postpone the answer to a future occasion (1805, p. 45):

> ...as concerns the *causes* which enable men to direct the expenditure of their energy to the acquisition of a good which, like a pearl ..., possesses *no inner value* whatsoever and for this reason appears to designate a completely worthless good, this is another question, and its resolution does not yet belong here.

According to what criterion does Schlözer assign higher "inner value" to one good than to another? He says (§53, p. 45) that "if the *inner* or *use value* were to serve as the standard, then the exchange value of iron would have to exceed that of copper", suggesting, in part at least, that certain physical attributes of iron make it more useful to people than copper; likewise with water and pearls.

1.3 Fulda

Fulda (1805) was perhaps the first of the German economists (and one of the few early ones) who recognized that human need was more fundamental than labor in determining value, since it was the motivating force for the exertion of labor in order to satisfy the need. The following is his nice statement to this effect (§16, p. 11):

> Indisputably, need lies deeper than labor, for the former first gives rise to the latter. If, therefore, practically everything which sheer existence, in addition to living well, requires can only be obtained through labor, we are then not entitled to regard labor as the first price which is paid for all things, and hence to regard it as the true measure of the exchange value of all goods; rather, life, and living well itself, is what it is that induces us to estimate these values.

1.4 Soden

Like the three previous contributions, Soden's (1805)[2] had found its starting-point in Smith (1776); but there the resemblance ends. In the preface to his work (1805, p. III), Soden

[2]Count (later Imperial Count) Soden had an unusual and colorful career (cf. Schmidt (1893, 1901), Schanz (1911, pp. 50–52)). At age 14 he wrote the libretto for an operetta that was subsequently set to music and performed on several stages; he later became a theater director and dramaturge, made the acquaintance of Mozart in Vienna, and in later life built theaters in Bamberg and Würzburg. In 1803 he submitted one of his plays to Goethe for his consideration; Goethe responded (cf. Rothkirch 1999, p. 15) by devoting the fifth

points out that it originated in a request he received from a critical journal to review Smith's work as translated by Garve in 1794, and that he realized that he needed much more space to expound his views than was permitted by a journal devoted to book reviews. But his judgment of Smith's work was hardly flattering; after stating (p. IV) that it "undoubtedly possesses great merits", he went on to complain of its "lack of logical order, of a correct view of the whole, of a solid coherence, and of a systematic plan." Thus, "one can regard Smith's work only as praiseworthy fragments." For such judgments Soden was severely rebuked by Hermann (1832, p. 47) and Roscher (1874, p. 677). Soden also characterized Smith's "political economy" [*Staatswirthschaft*], with its principle of the "highest welfare of the state domestically", as one that "places nations in a state of mutual antagonism, and would have to dissolve the bonds of world citizenship" (p. VII). He therefore replaced this by his proposed science of "national economics" [*Nazional-Ökonomie*], which would embrace "the entire science of humanity"—as if Smith would have disagreed with such an objective.[3]

What is particularly interesting about Soden's work is its strong (pre-Benthamist) streak of utilitarianism, possibly stemming from Quesnay (1767, pp. 391–398; Meek, 1962, pp. 211–213) or Le Mercier de la Rivière (1767, Ch. IV: I, p. 45; Ch. XXIX: II, p. 85), but undoubtedly with a firm basis in Stoic philosophy (particularly that of Marcus Aurelius Antoninus with its emphasis on world citizenship).[4] Thus, the "principle of national economics" is "the

stanza of Part 1 of his "Des neuen Alcinous" (Goethe 1803: 1986, p. 63) to "Der Direktor *Graf von Soden*". Soden held a number of high government posts; his political career, stopped in its tracks by the Napoleonic invasions, is well recounted by Rothkirch (1999) (I am indebted to Prof. Scheer for this reference). And (cf. Schanz 1911) he engaged in some quixotic business ventures that cost him most of his fortune. He did not turn to economics until his fifties; for the circumstances, see footnote 4 below.

[3]By contrast, and much more accurately, List (1827) criticized Smith for his cosmopolitanism and failure to appreciate the importance of the nation state. Soden continued with his distinctions in Vol. IV of his work (1810, §9, p. 5) replacing the terminology *Staatswirthschaft* (literally "state economics") by *Staats-Haushaltung* (literally "state housekeeping"), which he identified with the French *économie politique*, yet defined formally (§15. p. 8) as "the aggregate of rules according to which the state administration determines the rights and duties of citizens in their social relationships". In an apparent dig at J.-B. Say's translator Jakob (cf. Say 1807) he stated (§11, p. 6n): "*Staats-Haushaltung* is what the French call *économie politique*. National economics (*économie nationale*) as a science of its own is still foreign to them." However, Jakob had rendered Say's *économie politique* as *National-Oekonomie*. Ironically, the term *économie politique* in the title of the work by Baden-Durlach (1772) so admired by Soden (see footnote 4 below) was translated by Saß as *Staatshaushaltung* (Baden-Durlach 1782). Later (1812) Soden devoted an entire monograph to *Staats-Haushaltung* (which, logically, he could not include in his 9-volume work); while it lists (p. 14) Smith (1776) and Say (1803, 1807) as examples of the subject, as well as Jakob (1809) (though with the erroneous title *Grundzätze der National-Wirthschaft*!), the book consists largely of definitions of terms and does not cover any of the substantive material dealt with by Smith and Say (or Jakob). On Soden's subsequent controversy with Jakob see footnote 9 below.

[4]Lenz (1924, pp. 9–11 ([413]–[415]) has argued the case that Soden, while in Würzburg writing his *philosophischer Versuch* (the title of one of Schelling's works) in his fifties (see Groß, 1892), fell under the influence of Schelling (1799, 1803), twenty-one years his junior and at that time professor of philosophy at Würzburg. His argument is that Schelling's concept of productive power or force (*Produktionskraft*) (e.g., Schelling, 1799: 1858, pp. 171–2) is the source of Soden's concept; but we have seen that this concept was also in Schlözer (1805)—indeed it is found in the Introduction and Book I, Chapter I, of Adam Smith (1776) (see also Sommer's comments in List (1930, p. 509)). Even if this is so, however, it cannot account for Soden's utilitarianism. Schanz (1911, p. 51) recounts that on a visit to Karlsruhe and Durlach in 1789, Soden made the acquaintance of the physiocrat Carl Friedrich, Margrave of Baden-Durlach, whose essay (1772, p. 3) had proclaimed that "the welfare of the human race consists in the multiplicity of its enjoyments",

generation of prosperity distributed amongst the greatest number of people" (Soden 1805, §88, p. 106); that principle is "human welfare, and this consists in the sum-total of pleasures" (§104, p. 140); "the goal of production is pleasure, whether this pleasure is only sensual and animal, or moral and spiritual" (§108, p. 147); "the principle of national economics does not intend that man *merely exist*, but rather that he *enjoy*, and exist *in order to enjoy*" (§159, p. 224); "well-being consists in pleasure" (1806, §294, p. 173); "the principle of national economics is that of world citizenship [*ist Weltbürgerlich*]. National economics can have no other goal than to secure lasting well-being [*Wohlstand*] for the greatest possible number of citizens" (1806, §301, p. 188). "First of all, national economics is concerned with prosperity [*Wohlstand*], with the *pleasure* of the greatest possible number of people ..." (1808, §450, p. 10); "The principle of national economics, namely the greatest possible welfare for the greatest number of national citizens, entails *freedom* of productive power" (1810, §323, p. 243); "The greatest possible welfare of the nation is the law of national economics" (1810, §340, p. 263); "The principle of national economics is the greatest possible *serene [heiterer] life-pleasure*" (1811, §180, p. 130); "The national-economic principle, which intends that the greatest possible number of people live in well-being [*Wohlstand*], is then determined and delimited by the human organism itself" (1816, §206, p. 277).

In order to remedy what he regarded as Smith's "lack of exhaustive definitions and properly formulated complete concepts" (1805, p. V), Soden laid out a classification of different types of value, as follows (1805, Ch. III, §§43–44, pp. 39–42; 1810, §§53–62, pp. 23–26): "positive value" is defined as the capacity of a commodity to produce "enjoyability", as evidenced by the "urge for its possession"; in particular (§44), scarcity or abundance "are foreign to the concept of *positive* value", and "whether a good is scarce or abundant ... has nothing to do with its enjoyability, with the *urge for its possession*"; thus, the degree of positive value has nothing to do with the scarcity or abundance of the commodity, nor the difficulty of procuring it, whereas these are properties of what Soden calls "comparative value". Thus, "positive value" seems to be just another name for an interpretation of Smith's "value in use", and "comparative value" for Smith's "value in exchange".[5] These

and who had carried out an extensive correspondence with Mirabeau (1769–1789) and Du Pont (1771–1806) (cf. Knies 1892). (As pointed out by Mirabeau in a letter of 14 June 1772 to Carl Friedrich (Knies 1892, I, p. 59) referring to the latter's 1772 article, Du Pont de Nemours (editor of the *Éphémérides*) had edited and cut the essay substantially, reducing it "to a lapidary style".) Schanz quotes from Soden's unpublished autobiography (in which he referred to himself in the third person) as follows: "they devoted a number of unforgettable evenings to informative conversations primarily on national economics .... And it is these conversations which above all caused Soden to dedicate himself thereafter to this field of literature." We know from a letter of 1770 from Carl Friedrich to Mirabeau (Knies 1892, I, p. 18) that the former had arranged to have Du Pont's (1768) exposition of physiocratic doctrine—which was particularly that of Le Mercier (1767)—translated into German (Du Pont 1779). Given the fact that Soden had already published a prize-winning treatise on criminal law (1782–1783, second edition 1792), one might be tempted to conjecture that he might have been influenced in his utilitarianism by Beccaria's work (1764), especially given that according to Liebel (1965, p. 18) it had made a strong impression on the Margrave. But there is no trace of such influence in this work. The utilitarianism first appeared in his 1805 work.

[5]Hufeland (1807, I, §33, p. 130) spoke of Soden's "piled-up new terminologies"; and Hermann (1832, p. 47) stated: "Count Soden changes nothing substantial [in Smith's theory], [but] only replaces comprehensible names by incomprehensible ones." List, who had earlier described Soden, who had contributed to a weekly journal that List edited, as "the most celebrated German author in political economy" (cf. List 1827: 1931, pp. 111, 380), later said of him (1841, p. LI; 1930, p. 30): "Count Soden, whom I knew quite well, was, in contrast [to Lotz], incomparably more learned in his social intercourse than in his writings, and he was

concepts are best understood by seeing how Soden uses them in the following illustration (§46, pp. 44–45):

> Water, however, has positive value for humanity under all conditions, because it contains enjoyability, in other words urge to possession. Absence of scarcity does not exclude the concept of positive value. We see that in dry areas water is sold or apportioned to the inhabitants; and in the deserts of Africa water it is of inestimable value. Through the absence of scarcity, the surplus amount of a good, though not the good itself, can thus lose value.

We see here that Soden is rather loose in the use of his own terminology: as Hildebrand and Knies (and of course Menger) were later to stress, each unit of a good must (owing to arbitrage) have the same value—unless one has in mind two quite different concepts of "value", as is apparently the case here: the value of a good abstractly considered ("positive value"), and the value of a particular quantity of the good ("comparative value"). Thus, we may understand Soden to mean that water never loses its "positive value" but may lose its "comparative value". This is confirmed by (at least) two other passages: First: (1805, §46, p. 45):

> If a diamond, as in *Eldorado*, rolled on the streets, then it would surely lose the greatest part of its comparative value, though indeed not *all* of its positive value. It would have the value of gleaming pebbles. But scarcity increases the degree of its comparative value; for this reason the Crown of Portugal has all diamonds from Brazil which exceed a certain weight and a certain size deposited in the royal treasury.
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> If this treasury were opened, the diamonds would indeed not lose their positive value, but a major part of their comparative value.

At first Soden equivocates, allowing the diamond to lose *some* of its positive value, but he later corrects himself. Second: In Vol. 2 (1806), he states (§355, p. 327) that there was no international agreement concerning the relative values of gold and silver, but that "no doubt there was agreement that gold should have a significantly higher positive value [than silver]. This lay in its greater metallurgical nobility. There was also agreement that the degree of

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exceedingly liberal in the face of doubt or contradiction. What is new in these writings consists primarily of method and terminology. Unfortunately, however, the latter is far more grandiloquent than the former, and would lead the science even further into the mud of scholasticism than that of Smith or Say."

Stein (1858, pp. 42, 46), who himself rivals Soden in obscure terminology, expressed bewilderment over Soden's concept "*Ktemometer*" (cf. Soden, 1806, §338, pp. 299-300, and §339, p. 308n), which was presumably a misspelling of *Ktenometer* (ctenometer), which he used to express a measure of wealth in integral multiples of a smallest discernable unit, by analogy presumably to the teeth of a comb (from the Greek κτενός). Soden later (1825, p. 3) replaced this by the more sensible terminology *wealth-meter* [*Vermögens-Messer*]. Although expressed only in the vaguest terms, this idea may be conceived as foreshadowing the use by Weber (1834, 1846) and Fechner (1860) of the concept of "just-noticeable differences" to obtain a measure of sensation. As we know, this idea was championed by Edgeworth (1881, p. 7) as a basis for measuring utility. In particular it led to what Fechner (1860, I, pp. 64–5; 1966, p. 54) called "Weber's Law", according to which sensation is a logarithmic function of the stimulus (measured in natural units). This law was compared by Fechner (1860, I, pp. 236–8; 1966, pp. 197–8) to that of Bernoulli's (1738) utility function adopted by Laplace and Poisson (see footnote 103 below); see also Wundt (1874, pp. 304–7, 432–4; 1880, I, pp. 356–361, 468–9). Of course, this all went far beyond Soden!

its comparative value was considerably higher; this lay in its scarcity." This suggests that "positive value" depends (at least in part) on certain physical attributes of the good that make it potentially useful to people.

1.5 Jakob

Jakob was one of a number of German economists who at some points in their careers took up teaching positions in Russia, which then included the Ukraine and the Baltic States, and who according to Roscher (1870) formed a "German-Russian school".[6] Jakob, who taught at Halle, and had to move to Kharkov during the Napoleonic invasions, produced the first German translation of Henry Thornton's *Paper Credit of Great Britain* in 1803 and of J. B. Say's *Traité* in 1807.[7]

The idea of a distinction between a type of value that is independent of quantity and one that is dependent upon quantity is suggested by the following passage from Jakob (1805, 1809, §29, p. 16):

> The value of a thing ...comes to be determined either through the type of need and degree of urgency for the thing, or through the quantity and type of useful things which are received for it. The first may be called the *need value* [*Bedürfnißwerth*], the second the *exchange value*.

A revised version of this statement appeared in the third edition of Jakob's text (1825, §40, p. 36):

> The principle of exchange is ... *value*, i.e., the *magnitude or degree of utility* of a thing. However, one can consider the utility of a thing from a dual standpoint: 1) its fitness for the satisfaction of needs; 2) its fitness to be exchanged for other means of satisfaction. The first kind of value may be called *need value*, the other the *exchange value*.

In the version from the first two editions, quantity is mentioned only with respect to exchange value, which suggests that "need value" is considered to be independent of quantity. The main reason for drawing this inference, however, comes from a remarkably explicit statement by one of Jakob's students, the French-born naturalized British economist Daniel Boileau (1811, pp. 6–7):[8]

[6] Roscher discussed the work of Schlözer, Storch, and Georg Cancrin (Kankrin), and mentioned Jakob in a footnote. He did not include Friedländer, who taught in Estonia. He later (1874, p. 1041) added Bernhardi (1849). The concept of a "German-Russian school" was subsequently challenged by Seraphim (1924).

[7] Jakob began his career as a philosopher and published numerous philosophical works including one on natural law (1795a). These advanced (and according to Prantl 1881, plagiarized) Kantian philosophy. He founded and edited the *Annalen der Philosophie* in 1795–97, which featured anonymous book reviews and signed short notes. One of the former—a scathing review (Anonymous 1795) of Schelling's first work (1795), which described it as "a satire on the altogether newly woven and yet-to-be woven spiderweb of overly refined and fruitless speculations ... in the face of which the most subtle of scholastics must go into hiding"—was undoubtedly by Jakob himself (see footnote 9 below). For a good discussion of Jakob's later economic contributions see Pototzky (1905).

[8] This work is described in the fine study by Kaulla (1906, pp. 206–7) as a "reworking of Jakob's book". Boileau was self-effacing enough to state (pp. v–vi): "I lay no claim to either originality or invention. My work is merely a compilation, founded partly upon Professor Jacob's text book for German universities [Jakob (1805)], and partly upon additional notes of my own."

> The value of a commodity is determined either by the kind of want which it supplies, and the degree in which it is necessary, or by the number and sort of useful things which may be obtained for it in exchange. One is its intrinsic, real value, value in use; the other its exchangeable value. Or in other words, one is its absolute value; the other its comparative or relative value. One depends entirely on the nature of the commodity itself, and is uninfluenced either by its quantity and the demand for it, or by the quantity of, and demand for any other commodities. Relative or exchangeable value supposes that an operation of barter renders it necessary to compare one commodity or a portion of it, with another, or the portion of another commodity; and this comparison always depends on the proportion between the supply of and the demand for both commodities.

The above-quoted passage from the third edition of Jakob was followed by the following Remark (Jakob, 1825, §40, p. 36):

> From the stipulation that the expression *value* designates the magnitude or degree of goodness or utility of a thing, it would appear that this concept is by itself so obvious and clear that it retains its clarity in all of its applications. The distinctions among *positive*, *comparative subjective* and *objective* value etc. appear to me to amount more in the way of scholastic subtleties and fruitless torment than to advance the science.

This barb, directed at Soden, has an interesting history which is worth reporting; I relegate it to a footnote.[9]

[9] In a postscript (*Nachschrift*) to Vol. I of his work (Soden 1805, p. 328), dated March 1805, Soden had stated: "During the printing of this volume, Professor Jakob ... was good enough to share with me his just-published compendium on national economics .... It is certainly flattering to me that this worthy scholar has come upon the idea of recognizing national economics as a *science of its own* at the same time as I. ..." (This reappeared as a Notice (*Nachricht*) in the 1815 reprint of the first four volumes of his work (Soden 1815, I, p. XIV).) However, in Vol. IV of his work, Soden (1810, p. 10n) remarked: "At the same time that the first volume of my work on national economics was published, so was the textbook by the worthy Herr Professor Jacob on national economics, which he understands to be *Staats-Haushaltung*. The content demonstrates, however, that we have in common only the name, though not the concept, of the science." (See footnote 3 above.) Jakob protested (1819, §269, pp. 159–161) that "I cannot let the small honor be taken from me of having perceived and undertaken the exposition of the necessity or usefulness of separating out a science which includes a self-contained theory of national wealth ...."

The origins of the squabble are to be found in a letter from Jakob to Soden dated 9 May 1805 and published in Schanz (1911, letter 5, pp. 67–8). I wish to thank Professor Scheer for tracking this reference down; the letter had been cited—and to a degree misquoted—by Lenz (1924, p. 7), who did not supply the source. This letter had been preceded by one of 13 April 1805 in which Jakob asked Soden whether there was any chance of his being able to occupy a vacant chair at the University of Würzburg (Schanz 1911, letter 4, pp. 66–7). We may assume that Soden's reply was not encouraging. In the subsequent letter of May 9th Jakob stated (p. 68): "You have withstood an arduous struggle with the concepts of value and price. In reading my book I think you will perhaps find that I too have envisioned all the difficulties these concepts give rise to." He continued with a criticism of Soden (1805): "The introduction to your work seems obscure to me here and there, although the gist of it emerges clearly enough. For a work that is to gain acceptance by statesmen I would want absolute clarity, and freedom from scholastic terminology, which practical men have a horror of, particularly when it is drawn from the very latest—or even just the more recent—philosophy; and furthermore, for what does a man like you need aids of this kind? Nonetheless, the air of Würzburg

1.6 Sartorius

Despite having been the earliest expositor in German of Smith's work, Sartorius (1806) was by no means uncritical of his master, starting out his text with the statement concerning Smith's theory of value (1806, p. 1): "Adam Smith's theory appears to be partly obscure and partly defective." Like Schlözer before him, Sartorius proceeded to try to fill the gaps left untreated by Smith. Thus, for example (1806, Ch. I, p. 10):

> Water has in most regions either no or very little exchange value; it is produced by nature in such quantities that, as a result, as a rule nearly all the demand is satisfied. But where this is not the case, as in most regions of the orient, the wells there are true treasures and have an exchange value through the need for and scarcity of the thing.

This was followed by a substantial elaboration and a number of illustrations, but the analysis did not go as deep as Schlözer's.

1.7 Hufeland

The most noteworthy aspect of Hufeland's approach was his emphasis on the role of knowledge in the estimation of value. A good has value because it is a means for a human purpose (1803, §. 255, p. 96). Thus, referring to Sismondi (1803, Vol. I, p. 30), he stated (1807, Vol. I, §3, p. 21): "the inhabitants of New Holland, for example, are poor not primarily on account of the fact that they do not work, as Simonde would have it according to Smith's principles, but rather because they don't know any purposes for the things that nature offers them in such abundance (which other better instructed people know to value as means, and thus as goods)."

With regard to the theory of value, however, Hufeland adheres closely to Smith. Thus he says (1807, §30, p. 121): "If each person merely had purposes and goods for himself alone, then no significant amount of goods, and no significant wealth, is imaginable, either for the individual or for a large number of people." His treatment of the paradox of value does not go significantly beyond that of Lauderdale (1804), which he cited. He did introduce some terminological innovations of his own, though based on Sismondi's (1803, I, p. 283) "intrinsic" and "relative" price, namely the "inner price" set by a supplier, and the "outer price" set by a demander (Hufeland, 1807, I, pp. 132–136), but these simply correspond to what Marshall (1890) later called the "supply price" (Book IV, Ch. I, p. 189) and "demand price" (Book IV, Ch. VII, p. 298) respectively.

1.8 Lotz

Lotz's work shows the strong influence of Soden and Hufeland. He states (1811, I, §4, pp. 14–15) that Soden "refers to the value which the human mind assigns to a good on

---

seems to have rendered this evil endemic, so that even your work has been unable to remain entirely free of it." The allusion is evidently to Schelling (see footnotes 4 and 7 above). In later years, however, the two men appear to have acquired some respect for each other. Soden (1817, p. 26) referred to Jakob (1805) as "one of the most thoughtful [*denkensten*] recent economic writers", while Jakob (1821, p. 20) for his part referred to Soden's 1811 work as "an original and noteworthy system of public finance."

the whole and independently of the value of other goods as the *positive* value of that good; the value, in contrast, which it assigns to a good in comparison with another is called its *comparative value*" (Lotz's emphasis); and that the degrees of positive value "are determined according to the degrees of fitness of a thing as a means for human purposes" (1811, I, §5, p. 17). However, without apparently realizing it, Lotz departs from Soden in defining comparative value simply as the ratio of two positive values; thus, the comparative value is also independent of quantity. These concepts are thus not intended to provide explanations of market prices. The "actual price" was explained by the interaction of supply and demand (1811, §33, pp. 112–115), without reference to the concepts of positive and comparative value. Particularly significant in relation to Rau's subsequent work is the following passage from Lotz (1811, I, §8, p. 30): "For each species, characteristic features must be sought out by which it can be recognized precisely and reliably. The characteristic feature of *use value*, however, can be none other than: *fitness of a good as means for the one or several particular personal purposes of a particular individual* who either possesses, or at least wishes (*desires*) to possess that good".

Lotz (1811, I, §6, p. 21) apparently used the term "absolute" in a different sense than Soden: "It lies in the nature of the case that each determination of the degree of positive value of a good can never be anything other than *absolute. The fitness* of a good as a means for human purposes is in this case the only principle which can guide the determination of degree, and this principle is one and the same for all goods." We see here the remarkable fact that instead of ranking *bundles of goods* (as economists do today), Lotz sought to find a rank-ordering of *individual goods* (or at least, individual species of goods), independently of the quantity of them available to the consumer. This foreshadowed the idea of a hierarchy of wants developed by Hermann (1832, pp. 349–350; 1870, pp. 616–617), Riedel (1838, §§18–22, pp. 15–18), Friedländer (1852), and Knies (1855); that this is the proper way to interpret Lotz's concept of positive value is confirmed by a remark in his later work (1837, p. 25, note **) that, similarly to his own ideas, "an idea entirely of his own" had been developed by Louis Say (1827, p. 29), in a passage which I quote: "one can obtain quite well a general idea of the utility of each object of consumption by examining those of which one would decide to deprive oneself successively with successive declines in one's income."

Lotz was apparently untroubled by the paradox of value (1811, I, §7, pp. 24–25):

> One of the most important factors which must always be scrupulously taken into consideration, both in the determination of the positive value of goods as well as in the ascertainment of their relative value, however, is *the fitness of a thing for the direct satisfaction of human purposes*, and *its fitness for their mere indirect realization*. Goods of the first sort deserve beyond any doubt one of the highest places in the rank-ordering of human goods. Their value is truly the highest which goods can generally have; leaving aside the fact that in human trading activities they do not always appear in this position, but rather, in the rank-ordering of the individual things of value, very frequently many of these goods have been assigned an exceedingly low position, no matter how undeniable it may be that according to their use value they deserves a far higher place. Water and air, two of the most indispensable needs in human life, and indisputably goods of highest value, are conspicuous examples of this.

> Both are goods having direct fitness for human purposes, and nonetheless in human trading activity they frequently are second to goods of the most indirect fitness.—

No attempt was made to explain this divergence between his theory and the observed facts. Later (1821, p. 42; 1837, p. 41) he made the claim (contrary to all evidence!) that "a thing to which we ascribe a high value *generally* tends to have a high price in trade."

The same material that is developed in §§3–10 of Lotz's early work (1811, pp. 9–36) is covered in §§8–20 of his two later works (1821, pp. 17–74; 1837, pp. 18–73) but with more elaboration. In these Lotz makes the intriguing observation that the concepts "positive value" and "comparative value" are to be found in a work of John Locke,[10] but I have not been able to verify this in any of Locke's writings.

1.9 Storch

The last of the German economists prior to Rau that I consider is Storch (1815). Storch's work is considerably influenced by J.-B. Say (1803), especially in its treatment of *utilité* (utility, or usefulness). It is contained mostly in Chapters III and IV of Vol. I of his Treatise. He states (III, p. 57):

> It is not enough for a thing to exist or to be useful that it have value; this utility must also be recognized. From this we draw the important conclusion that value springs not from causes that make things exist, but from the judgment of people who wish to make them useful for their needs.

Thus (p. 60), "value is not an inherent quality of things, but it springs from our judgment" (also IV, p. 64). It is "just an *opinion*" (IV, p. 62). "Value has no other source than opinion" (p. 64). It follows that "there are very few things to which one can attribute an absolute value" (p. 64), only essential things that have no substitutes (p. 65). This accords with Soden's idea of "absolute value".

Thus, value is very much subject to opinion, climate, and many other things; but one thing Storch does not mention is quantity. Hence Storch must be counted among the pre-Rau German economists who regarded use value as independent of the quantity consumed.

[10] Cf. Lotz (1821, I, §10, p. 23, note **; 1837, I, §10, p. 25, note **), where he states: "With regard to what I have said here about the dependence of the comparative value of goods on their positive value, [see] Locke, Essay of Government ch. IV, §. 13." I know of no work of Locke's with this title, and there is no §13 in Chapter IV of Locke's *Second Treatise of Government* (1698) (which deals with slavery) or in any other of his political works that I have consulted. My best guess is that Lotz was referring to Chapter V (Of Property) of the *Second Treatise* where Locke states (in §37: 1988, p. 294): "...before the desire of having more than Men needed, had altered the intrinsick value [Lotz's 'positive value'?] of things, which depend only on their usefulness to the Life of Man; ...Yet this could not be much ...where the same plenty was still left, to those who would use the same Industry. ...For the provisions serving to the support of humane life, produced by one acre of inclosed and cultivated land, are ...ten times more [Lotz's 'comparative value'?], than those which are yielded by an acre of Land, of equal richnesse, lyeing wast in common." Hildebrand (1848, p. 18n), who knew Lotz's work, quoted a similar passage from §40 of Locke (but without reference to Lotz).

2 Rau

Karl Heinrich Rau was a towering figure in 19th-century German economics.[11] His three-part text went through five editions (1826–1865), and the first part through three more (1855–1869); it was continued by Wagner (1876) who dubbed his work the "ninth edition" of Rau's, which itself went through several editions. Marshall (1890) is said to have modelled his text on that of Roscher, who in turn followed that of Rau (cf. Streissler, 1990, pp. 33, 51), although as we know he failed to complete it.

In the second edition of his treatise, Rau (1833, I, §57a, p. 57) introduced a distinction that was to play an important role in the subsequent development of utility and value theory in German economics: that between two kinds of "use value": "species value"—an abstract value associated with a class (species) of commodities unrelated to their quantity—and "concrete value" (or "quantitative value", a function of the quantity)—a concept which may be regarded as a forerunner of marginal utility:

> Upon closer examination of value, two ways appear in which to conceive of it:
>
> 1) with reference to *a particular species of goods*, in that one considers their capability to advance human purposes in general. This estimation may be called *species value*, for example, that of a hundredweight of wheat, iron, etc.
>
> 2) with reference to a certain amount of a good in its relation to the supplies of it which a particular person possesses and to his requirements of it: *concrete* or *quantitative value*. The satisfaction of needs requires in most cases a more or less precisely delimited quantity of goods of a particular type, so that a supply which exceeds this appears as superfluous. The concrete value of the excess must therefore completely disappear for the owner, notwithstanding its perhaps very high species value, and the estimation of those quantities of goods which no longer serve for personal use will only be able to take place according to their price. Within these limits of the requirement, on the other hand, the concrete value coincides with the species value.

If one interprets the last sentence literally, and if the "concrete use value" is identified with marginal utility, then this implies a utility function of the form

$$U(x_1, x_2, \ldots, x_n) = \text{constant} + \sum_{j=1}^{n} c_j \min(x_j, b_j) \quad \text{for } x_j \geqq 0, \tag{1}$$

where the "limits of the requirement" for commodity j are $0 \leqq x_j \leqq b_j$. This implies that the marginal utility of commodity j is equal to its "species value" c_j for $0 \leqq x_j \leqq b_j$ and zero for $x_j > b_j$. Appended to the last sentence in the above quotation, however, Rau added an important qualification in an endnote (c) (p. 57):

[11] Rau was a professor at Heidelberg. Streissler (1990, p. 52) has justly described him as "one of the most underrated pioneers in the history of economic thought." Rau's papers have been deposited at the University of Michigan, and a good account of his life has been given by Dickinson (1958). Rau's writings are analyzed in detail in Karl Neumann (1927).

> Further gradations can be imagined here, in that, for example, one keeps on hand a certain supply over and above the requirement, for comfort or as a precaution, the concrete value of which, however, is indeed smaller.

This appears to allow for a more smoothly diminishing marginal utility.[12]

To further reveal what he meant by "species value" Rau added the following observations (1833, §58, 1, pp. 57–8):[13] First (and this was fully in accord with Lotz's rank-ordering of species by their "positive value"), "the means of satisfaction of the urgent needs have the highest value" (p. 58). Secondly, if there are several goods that serve as means of satisfaction of the same purpose, "then the value of any single one of [them] *compared with the other* is determined according to the degree of its suitability for the achievement of its purpose, e.g. according to the strength, duration, certainty, etc., of its effects. This relative value of several means [of satisfaction] in comparison with one another is easily determined in many cases where it is based simply on physical chacteristics" (1833, §58, 2, p. 58). This is followed in an endnote by an interesting illustration (note (*b*)):[14]

> Thus, for example, the [species] value of several different foods, kinds of wool, textiles for clothing, and materials for illumination can be conveniently expressed numerically in terms of one another. 1 bushel of wheat is worth approximately as much as $1\frac{1}{2}$ bushels of rye, or 2 bushels of barley, or 3 bushels of oats, etc.

Thus, if the nutritive power of grain could be measured by a single physical attribute (say calories, or carbohydrates, per gram), and no other characteristic (such as taste) were relevant to the consumer, then these grains could be considered as perfect substitutes in consumption, hence their market prices relative to that of wheat would vary inversely with the numbers given by Rau (cf. formula (11) below). As Rau subsequently made explicit in the fifth edition (1847, §61, 2, note (*a*), p. 80): "In this regard it is not out of the question that the types [*Sorten*] of different goods belonging to the same species [*Art*] also have

[12] In the final edition of his treatise Rau proposed the following *continuous* form for the concrete value of good j (1868, §62, note (*c*), p. 95):

$$u_j'(x_j) = \begin{cases} c_j & \text{for } x_j \leqq b_j, \\ c_j b_j / x_j & \text{for } x_j > b_j \end{cases} .$$

This is a continuous function (though nondifferentiable at b_j), and it integrates to the subutility function

$$u_j(x_j) = \begin{cases} c_j x_j & \text{for } x_j \leqq b_j \\ c_j b_j [1 + \log x_j - \log b_j] & \text{for } x_j > b_j \end{cases} .$$

This defines a total utility function $U(x) = \text{constant} + \sum_{j=1}^{n} u_j(x_j)$ which is strictly increasing, concave, continuous, and once-differentiable; thus the satiable preferences have become insatiable. This was actually developed as an elaboration of a discussion begun in the second edition (1833, §57a, note (*b*), p. 57) and continued in §57a of the 3rd (1837, note (*a*), p. 61) and 4th (1841a, note (*b*), p. 63) editions and in §61, note (*c*), of the 5th–7th editions (1847, p. 81; 1855 and 1963, p. 77).

[13] See also the 3rd edition (1837, §58, p. 61), 4th (1841a, §58, p. 64), 5th (1847, §59, a, p. 76), 6th (1855) and 7th (1863), §59, a, pp. 72–3, and 8th (1868, §67, pp. 104–5.)

[14] For the corresponding passages in subsequent editions see the 3rd (1837, §58, note (*c*), p. 62), 4th (1841a, §58, note (*c*), pp. 64–5), 5th (1855, §59, note (*c*), p. 77):"1 volumetric unit of wheat is worth approximately as much as $1\frac{1}{3}$ units of rye", 6th (1855, §59, note (*c*), p. 73): "100 volumetric units of wheat are worth approximately as much as 133 of rye or 166 of barley", 7th (1863, *ibid.*), 8th (1868, §59, note (*c*), p. 91).

unequal species value."[15] Even in the absence of such a unique characteristic, the goods comprising a "species" could be expected to be substitutes, and goods from different species could be expected to be complements. This idea was to be taken up by Knies (1855, 1873), as we shall see.

In the third edition of his treatise Rau restated his theory in wording that would continue to undergo slight modifications in later editions. The following passage from the third edition (1837, §57a, pp. 59–60), accompanied by footnotes I have added indicating changing in wording from the third edition to the fourth (1841a, §57a, pp. 62–3),[16] shows the continuing and in some cases significant evolution of his ideas:

> A more precise examination of value reveals two approaches that can be adopted to understand it.
>
> 1) If in general one devotes one's attention to the potential of a certain class of goods to promote human purposes, in other words to bring about utility [*Nutzen*] or pleasure [*Vergnügen*], then one finds[17] the *species value*, of, for example, grain or iron. In order to make a comparison[18] of the species values of several goods, one must assume certain[19] quantities of them, for example one pound of wood and one pound of coal.[20]
> 2) The value of a single particular quantity of a good for a certain subject—for example, a single bushel of grain for a person, to whom it is being offered, or who is offering it, for sale—may be referred to as *concrete* or *quantitative value*. It does not conform solely to the species value which the person accords to the good, but also takes into account the supply on hand that he possesses in addition to the particular quantity, and the amount of his requirement.[21] In most cases the satisfaction of a need requires only a certain more-or-less precisely limited quantity of a good, and the superfluous supply in excess of this amount, since it does not come into use, cannot express its species value for that particular person.[22] Therefore, such a superfluous or indeed easily-dispensed-with portion of the supply of a good commonly has either no or only slight concrete value for its possessor, and its estimation depends in this case solely upon the price[23] which can be received upon exchange with other persons. In the case of goods for which no limit of requirement can be specified—luxury

[15] In the 6th (1855) and 7th (1863) editions this was strengthened to the unequivocal "The types belonging to the same species [*Art*] of goods have unequal species value" (§61, 2, note (*a*), p. 76). Note that Rau here used the term *Art* (type, or species) instead of *Gattung* (species), but the meaning is clearly that of "species".

[16] See also the comparable statements found in the subsequent editions: 5th (1847, §61, pp. 79–80); 6th and 7th: (1855 and 1863, §61, pp. 75–6); 8th (1868, §62, pp. 94–6).

[17] In the 4th edition, "finds" was replaced by "arrives at".

[18] In the 4th edition, "a comparison" was replaced by "a precise comparison".

[19] In the 4th edition, "certain" was replaced by "equal".

[20] In the 4th edition Rau added: "and investigate the benefits arising from their respective uses for the intentions and needs of people."

[21] The 4th edition adds "as well".

[22] The 4th edition adds "effectively".

[23] In the 4th edition, the clause starting "its estimation ..." was replaced by "even though the species

> articles, for example—at least the concrete value of a particular quantity is typically less, the smaller is its portion of the entire supply. ...

The last sentence in this passage provides a peculiar early version of the principle of diminishing marginal utility. What is peculiar about it is that instead of saying that the larger is the total *quantity* of an object that an individual consumes, the smaller is the "concrete value" (marginal utility) of the last unit consumed, Rau states that the smaller the *proportion* of the last unit consumed to the "entire supply" (presumably the entire amount in the possession of the individual),[24] the smaller is its "concrete value".[25] As we shall see presently, however (see the passages quoted before and after footnote 39 below), Rau improved upon it considerably in the later editions, removing the proportionality feature altogether.

The above-quoted passage from the third and fourth editions continues with some conceptions that are much closer to the idea of marginal utility (1837, §57a, p. 60–61):

> The sacrifice that one is inclined to make for the acquisition of a thing, when, to be specific, it is provided for one's own personal use, depends on the concrete value that one accords it, as does one's degree of reluctance to sell it. It has, therefore, a tremendous influence on the price.

Rau's distinction between species and concrete value was soon adopted by Riedel[26] (1838, I, §52, pp. 38–40):

> Each economizing subject apprehends, of course, the value of objects only in their direct or indirect relationship to his own needs, i.e., as *concrete* value, which can be based on both use value and exchange value, instead of estimating the objects only according to their relationship to human needs on the whole and according to their significance for the achievement of the highest goals extant in the nature and destiny of man in general, or according to entirely abstract value. Such an *abstract* valuation of material objects lies, however, partially outside of the boundaries of bare economic reasoning and consequently of economic theory as well, in that it presupposes an examination of all relationships of material

value in this case persists unchanged, it nonetheless exerts no influence on the possessor's actions; rather, these are determined solely according to the price ...".

[24] The word translated as "supply" in the above is *Vorrath*, which (in contrast to the word *Angebot*—which means supply in the sense of the amount offered) means supply in the sense of a stock of holdings ("the supply on hand that he possesses", as indicated in the second sentence of the quoted passage), so in fact it can be interpreted as the total amount demanded, as contrasted with a particular amount (e.g., one unit), of this demanded quantity.

[25] If individual i's total demand for commodity j is x_{ij}, and if the "particular quantity" of this is, say, 1 unit, then Rau is saying that the "concrete value" of 1 unit of good j to individual i varies as $1/x_{ij}$. This passage may be considered as forming the basis of Hildebrand's subsequent formulation (see section 3 below).

[26] Riedel (1809–1872) was a historian who held the title Professor Extraordinary of Political Science at the University of Berlin, where he graduated in philosophy. He became director of the Prussian Archives and historiographer of the early history of Brandenburg and the history of the House of Hohenzollern. He edited a 41-volume *Codex diplomaticus Brandenburgensis* in 1869. His 3-volume treatise on economics contains a superb 80-page bibliography towards the end of Vol. III. In his last years he published a treatise on *The Care of the Fruit Tree in Brandenburg* (1871). For details see Holtze (1889).

> objects according to their various influences on the highest human interests and aspirations and a general explanation of all the interactions which exist between the physical and the mental; in part the abstract value coincides, in the majority of cases, with the concrete value as soon as one considers an entire nation as economizing subject, in that in this case the value of most things must be measured according to universal principles valid for all mankind and, by taking into consideration the differences of peoples, comparatively only rarely need a valuation occur which deviates from it.

In the fourth edition, in an endnote to paragraph 1) of the passage quoted above,[27] Rau stated that "Riedel ... took up the distinction between these two kinds of value and refers to the species value by the name *abstract value*." Starting with the fifth edition (1847, §61, 1, pp. 79–80) Rau adopted Riedel's terminology in introducing his concept by saying, "This *abstract* or *species value* expresses the relationship of a type of thing to human purposes ...." Likewise (p. 80), he dropped the terminology "quantitative value" (unfortunately, in my opinion).

As we have seen in the previous section, Rau's concept of abstract or species value (as opposed to the terminology) was not essentially new, being basically equivalent to Schlözer's "inner" or "absolute" value, Jakob's "need value", Soden's and Lotz's "positive value", etc. Indeed, these were all interpretations of Smith's "value in use", and Louis Say (1822, pp. 47–50) (whose work was cited by Rau), provided a similar interpretation with his "effective intrinsic value" [*valeur intrinsique effective*] or simply "intrinsic value" (1836, pp. 32–4).[28] But these interpretations were vaguely defined, whereas Rau made the concept precise. Rau's more novel concept was that of concrete or quantitative value; prior to him, it is difficult to find a work in the German literature that explicitly notes the dependence of use value on quantity, and even those that noted that the scarcity of a good would lead to its having a high value, either did not attempt to provide an explanation, or else attributed this to the extra labor required to procure the good, overlooking the fact that this disutility would be willingly expended only because of the high marginal utility of the scarce good.

Rau apparently had two distinct (but related) purposes in view in introducing his concepts of species versus quantitative, or abstract versus concrete, value. One was evidently the need to provide a solution to Smith's "paradox of value";[29] some concept had to be formulated to correspond to Smith's "value in use"; and this presumably was Rau's "species",

[27] Rau (1841a, §57a, note (*a*), p. 63), (1847, §61, 2, note (*b*), pp. 80–81), (1855 and 1863, §61, 2, note (*b*), p. 77), (1868, §62, note (*b*), p. 95).

[28] See also L. Say (1837, p. 31): "The *utility* of a sack of wheat, as a means of satisfying our needs, as an *object of consumption*, varies very little; but the *utility* of a sack of wheat, as a commodity or *means of acquisition*, is extremely variable." Rau described Louis Say as "the older brother" of J.-B. Say; cf. §45 of Rau (1826, p. 82; 1833, p. 41; 1837, p. 43; 1841a, p. 45; 1847, p. 55; 1855, p. 53; 1863, p. 53; 1868, p. 62). However, following J.-B.'s death in 1832, L. Say (1836, p. II) wrote that he was "very young" when the first edition of J.-B. Say (1803) was published; and indeed, Lutfalla (1979, p. 392) confirms that Louis was the youngest of the three brothers, seven years younger than Jean-Baptiste, having been born in 1774. It is curious that the *Nouveau dictionnaire* (1893–1892, 1900) coedited by J.-B.'s grandson Léon did not list his great-uncle Louis among its entries, or even mention him, although it listed Léon's father, Horace-Émile (1892, 1900, Vol. II, p. 791), who is not known for any significant contributions to economic theory.

[29] Contained in the well-known passage (1776, Book I, Ch. IV, p. 34): "The word *value*, it is to be observed, has two different meanings, and sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called

or Riedel's "abstract" value. Smith's paradox—previously stated by Locke (1668),[30] Montanari (1683),[31] and Law (1705)[32]—involved comparison of incommensurables: diamonds and water. On the other hand, Lauderdale (1804)—and much later, Proudhon (1846)—was concerned with the prices of identical objects, or the market values of collections of identical objects, at different times. Lauderdale noted that if the demand for a particular grain were inelastic, then (with unchanged money supply) the market value of a bumper crop of this grain would be less than the market value of a smaller normal crop. But obviously the utility of the bumper crop would be larger than that of the normal crop. Therefore, measuring utility by value—as had been proposed by J. B. Say (1803–1826)—made no sense. This leads us to Rau's second apparent purpose.

It is important to note that both Rau and Riedel were concerned less with explaining market prices (Rau left this to a chapter on prices, and the famous appendix to the fourth and subsequent editions with supply and demand curves) than with carrying out Adam Smith's program of inquiring into the nature and causes of the wealth of nations. The title of Rau's chapter was "The valuation of the national wealth" (or "national riches" to use an older English term which is not restricted to stocks as opposed to flows). This title brings to mind Hicks's researches in the 1940s into the "valuation of social income", and Samuelson's 1950 article on the "evaluation of real national income". The question which concerned Hicks (1940) and Samuelson (1950) was whether one could use national income data to measure changes in a country's welfare. This was also the concern of Lauderdale (1804) and Rau (1826–1868). Already in the first edition Rau stated (1826, §61, 2, p. 44):

'value in use;' the other, 'value in exchange.' The things which have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce any thing; scarce any thing can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it." As will be apparent in comparison with the statements of Locke, Montanari, and Law cited below, Smith's innovation consisted in the introduction of the concepts "value in use" and "value in exchange", although even these had been introduced earlier by Quesnay (1767, Note on the Same Maxim (XVIII), pp. 161–162; Meek, 1962, p. 257) under the names *valeur usuelle* and *valeur vénale*.

[30] "The being of any good and usefull quallity in any thing neither increaseth its price nor indeed makes it have any price at all, but onely as it Lessens its quantity or increases its vent, each of these in proportion to one another. for what more usefull or necessary things are there to the being or well being of men then Aire and water, and yet these have generally noe price at all, nor yeild any mony, because their quantity is immensely greater then their vent in most places of the world, but as soone as ever water (for aire still offers its selfe every where without restraint or inclosure and therefore is noewhere of any value) comes any where to be reduced into any proportion to its Consumption, it begins presently to have a price, and is sometimes sold dearer than Wine, and hence it is that the best and most usefull things are Commonly the Cheapest, because though their consumption be greate yet their production is large and suitable to it."—Locke (1668: 1991, I, p. 187). This was substantially repeated in Locke (1692, p. 187).

[31] "Thus scarcity makes any commodity dear, as everybody can observe in the case of jewelry and the [precious] metals themselves, and abundance makes them cheap. Water, which is an element of so much importance to human life, is worth nothing because it abounds almost throughout the world ..."—Montanari (1683, Ch. III: 1804, pp. 60–1; 1913, p. 265).

[32] "Goods have a value from the Uses they are apply'd to; And their Value is Greater or Lesser, not so much for their more or less valuable, or necessary Uses: As from the greater or lesser Quantity of them in proportion to the Demand for them. *Example.* Water is of great use, yet of little Value; Because the Quantity of Water is much greater than the Demand for it. Diamonds are of little use, yet of great Value, because the Demand for Diamonds is much greater, than the Quantity of them."—Law (1705, p. 4).

> From the increase or decrease of a price total [*Summe von Preisen*][33] a similar change in the amount of goods to which these prices pertain cannot always be inferred, and conversely. Through cost savings the prices of a portion of goods can decline without, for this reason, the influence this has upon the national wealth having to decline in the least relative to people's condition. Further, if the supply of a type of goods decreases, for example with a failed harvest, and consequently total wealth declines, the price of this good can rise so much that in drawing conclusions on the basis of the price total, one would still have to regard the national wealth as equally large or even increased.

Thus, Rau was well aware that it made no sense to measure a country's welfare (or "wealth" in Adam Smith's sense of a vector of quantities of goods consumed) by the sum of quantities multiplied by current prices ("concrete values"); some invariant measure of value had to be substituted for current prices, and this was presumably the "species value". Thus, "species value" could be measured by *base-year prices*. Rau did not actually make this suggestion, but came close to it in his suggestion that an average of prices over a series of years be used as an empirical estimate of species value (Rau, 1841a, §67, pp. 73–4):[34]

> Although according to the principles [set forth] so far, the mere money prices of goods are little suited to form the basis of the (national) economic valuation of the goods—rather, a preponderance of weight must be given to value and its sharp distinction from price .... For statistical use one is equally compelled to stay predominantly with price data, though one must seek, in order thereby to render it more meaningful for the conditions of wealth of a people, to investigate at the same time:
>
> 1) the price in the selected measure (money) at which the kinds of goods of highest value are found, from which one is then to infer the extent of utility and enjoyment that a certain pecuniary amount is able to provide;
> 2) in what quantities the useful goods are contained in the national wealth;
>
> In addition, one must use as a basis not price at a single point in time, but rather an average over a period of time.

Regarding the "quantitative value", Rau supplied a few hints regarding the possible decreasing or increasing marginal utility of certain objects (1837, §57a, notes (*a*) and (*b*), p. 61; 1841a, §57a, notes (*b*) and (*c*), p. 63; 1847, §61, note (*c*), p. 81; 1855 and 1863, §61, note (*c*), p. 77; 1868, §62 (*c*), p. 95):[35]

[33] Here, Rau speaks of "sums of prices", but the meaning is clearly the sum of prices times quantities. The proper English expression would normally be "sums of values", but Rau in this chapter is intent on distinguishing "prices" from "values".

[34] See also §67 of the 5th (1847, p. 90), 6th and 7th (1855, 1863, p. 85), and 8th (1868, pp. 104–5) editions.

[35] One finds an unmistakable allusion to this passage in Gossen (1854, pp. 47–8; 1983, pp. 55–6): "It was believed that even this fact [that only a distinct quantity of any object has value] was subject to one exception, namely art and other types of collections. It was thought that in these cases, the limitations of the quantity [that is, the decrease of value due to increasing quantity] did not hold—that, on the contrary,

> The second copy of a most prized book, copper engraving, etc., is almost entirely without concrete value to its owner.
>
> There are, however, cases in which the requirement is infinite, and no satiation of wants appears, for instance in collections for scientific purposes or as a hobby, in which cases the value of each individual piece increases with the completeness of the whole.

Rau also noted—quite likely influenced in this regard by Louis Say (1818–1837)[36]—the phenomenon of diminishing marginal utility of income (1833, §63a, p. 63; 1837, §63a, p. 67; 1841a, §63a, p. 69):

> The more well off someone is, the more abundantly he is in the habit of being provided with everything necessary, [and] the more trivial the items he is capable of purchasing. Therefore, one and the same amount of money for the more wealthy, which counts only as a token of an easily dispensable good, serving, for example, as a luxury, is of less concrete value than for the less well-off, and the value of one and the same sum of money for several owners is, accordingly, roughly in inverse proportion to their entire wealth.

This passage, considerably rephrased, reappeared in the fifth edition (1847, §64, 2, p. 85) as follows, with footnotes indicating slight changes in the sixth and seventh editions (1855 and 1863, §64, p. 80):

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with increasing completeness of the collection, the value of the still missing items would actually increase." He went on to argue: "As long as the new addition is different from the already existing pieces, it represents no increase in the existing quantity, but something novel." As was his habit, Gossen did not refer to Rau. It is of interest to note that the passage about the increasing marginal utility of components of a collection was withdrawn from the 5th and later editions of of Rau (1847, §61a, p. 81; 1855 and 1863, §61a, p. 77; 1868, §62 (*c*), p. 95).

[36]1818, p. 38: "... if one deprives a very rich man of a sum of 300 francs, one deprives him of an amount of utility much smaller than if one carries off the same sum from one who is badly off." 1836, p. 40: "One who has an income of a hundred thousand francs can devote a hundred francs to the purchase of something of very little utility, since he will sacrifice only a thousandth of his income; while another, for the same sum, will sacrifice one tenth of his income, and he could devote such a large portion only for things of extremely high utility, and not for trifles." See also L. Say (1827, Ch. XIV, esp. pp. 139–145).

A later and more precise statement is that of Bentham (1831): "The effect of wealth in the production of happiness goes on diminishing, as the quantity by which the wealth of one man exceeds that of another goes on increasing: in other words, the quantity of happiness produced by a particle of wealth (each particle being of the same magnitude) will be less and less at every particle; the second will produce less than the first, the third than the second, and so on." This was not published until posthumously (1843, III, §5, I, 4, p. 229). Professor Scheer has drawn my attention to the early work by Guicciardini (1526–1529) who states, regarding the burden of an income tax, that (1932, p. 198) "the equality of a burden does not consist in each paying the same rate (*rata*), but that the payment should inconvenience one as much as another", and goes on to explain that the burden of a tax payment will be greater for low-income than for high-income people. For later precise statements by mathematicians see footnote 103 below.

In fact, the principle of diminishing marginal utility of income can be traced back at least to Cicero (46 B.C.) in explaining his sixth paradox to the effect that "the wise man alone is rich" (1982, pp. 294–303): "For whom are we to understand as being rich, or to what person are we to apply this term? I suppose to the person who owns so much property that for the purpose of living liberally he is easily contented, the person who looks for and aims at and desires nothing further." Unfortunately, Gossen's teacher, Peter Kaufmann (1830, §§32–5, pp. 43–7), who tried to interpret Cicero's paradox, failed to grasp this point.

> The greater the sums of money of a particular amount someone has at his disposal, i.e., the more well-off he is, the more insignificant and easily dispensable things he may permit himself to acquire. If one thus investigates[37] the value of such an amount not just at a single point of time, but for a person's economic situation as a whole, then what results is that the amount[38] has a lower concrete value the smaller a portion it constitutes of the entire available quantity of goods; it is worth, for the wealthy, little, for the needy, much.

The precise inverse proportionality has been removed, leaving just the diminishing marginal utility of income.

In the fifth edition of his Treatise—one year before Hildebrand, and seven years before Gossen—Rau replaced the statement from the third and fourth editions referred to above as a "peculiar early version of the principle of diminishing marginal utility" (see footnote 25 above) by the following precise statement of the principle of diminishing marginal utility (for the case of goods of insatiable wants) (1847, §61a, p. 81):

> With things of which one does not just at the moment require a particular amount, such as with a number of luxury articles, at least the concrete value of an individual unit or quantity customarily becomes that much smaller, the higher an owner's entire supply rises.

As far as I know this is only the third independent statement to be found in the 19th-century literature of the principle of diminishing marginal utility of goods-consumption.[39]

The wording was still further improved in the sixth (also the seventh) edition as follows (1855 and 1863, §61a, p. 77):

> If one does not just at the moment require a particular amount of some goods, such as with a number of luxury articles, at least the concrete value of an individual unit or quantity commonly becomes smaller, the higher an owner's entire supply rises.

In 1841 Rau introduced intersecting demand and supply curves in an appendix to the fourth edition of his *Grundsätze* (1841a, Appendix to §. 154, pp. 525–527), which was

[37] In the 6th and 7th editions this word is changed to "examines".

[38] In the 5th edition, "the amount" was "it", while in the 6th and 7th it was "the latter". I have thought that "the amount" is a clearer translation for both.

[39] The first is generally acknowledged to be that of Lloyd (1834, pp. 11–12). A hint of the principle is certainly to be found in Schön (1835, p. 41): "Through speculative purchases a grain profiteer increases the use value of grain for certain people without making the grain more useful." Senior, joint dedicatee with the Belgian statistician Adolphe Quételet of the 5th–8th editions of Rau's *Volkswirthschaftslehre*, provided a second statement (1836, p. 133; 1850, pp. 11–12): "Not only are there limits to the pleasure which commodities of any given class can afford, but the pleasure diminishes in a rapidly increasing ratio before those limits are reached. Two articles of the same kind will seldom afford twice the pleasure of one, ..." A still earlier but vaguer hint may be found in Malthus (1820, Book I, Ch. VI, p. 340; 1836, p. 301): "The most useful commodity, in respect of its qualities, if it be absolutely in excess, not only loses its exchangeable value, but its power of supplying the wants of the society to the extent of its quantity, and part of it therefore loses its quality of wealth." The entire lengthy passage in which this sentence appeared (pp. 337–340) was reproduced by Friedländer (1852, pp. 9–10).

followed shortly thereafter by an article published in a Belgian journal (1841b) (together with a short essay on location theory); the Appendix was continued in the subsequent editions;[40] this analysis was subsequently further developed by Mangoldt (1863), as well as later and independently by Jenkin (1870). Rau's curves appeared only three years after (and independently of) the first appearance of a demand curve in Cournot (1838).[41] An English translation of the essay on location theory in Rau (1841b)—but not the one on intersecting demand and supply curves—was published by Baumol and Goldfeld (1968, pp. 181, 183) with the following comments:

> The presence of a piece by Rau is rather surprising. He is remembered primarily as the author of a rather pedestrian textbook. Rau is not generally considered to be an original thinker, let alone a pioneer mathematical economist.
> ...
> [The second extract describes the elementary geometry of supply-demand curves.]

The editors apparently thought that intersecting demand and supply curves were already well known to the profession in 1841.

In his discussion of market price, Rau (1841a–1863, §147) clearly shows the relationship of the consumer's demand price for a commodity (which he calls "its value to the buyer" [*Werth desselben für den Käufer*]—(1841a, p. 155; 1847, p. 183; 1855–1863, p. 166; 1868, p. 197)) to both the marginal utility (concrete use value) of the commodity and the marginal utility of the consumer's income. First of all he stresses that "the price is determined by the *concrete*, not the *species value*" (1841a, §147, note (*a*), p. 154).[42] He then compares the demand prices for a commodity of a rich and a poor man as follows (1841a, §147, note (*c*), pp. 154–5; 1847, §147, note (*d*), p. 184; 1855–1863, §147, note (*d*), p. 167; 1868, §147, note (*c*), p. 198):

[40] The 8th edition (1868, pp. 368–372) reproduces not only the appendix with supply and demand curves, but also an abridged version of the article on location theory, as well as a third geometrical note on prices and rents in spatially separated markets.

[41] For an excellent survey of the history of supply-and-demand curve analysis see Humphrey (1992). Humphrey points out (p. 5) that in analyzing the effect of an excise tax, Cournot superimposed onto his demand curve $F(p)$ (Fig. 1, §24, p. 57) two supply curves $\Omega(p)$ and $\Omega(p-u)$ (Fig. 6, §51, p. 103), one of them including a tax, so as to assess the effect of an excise tax on the price. In this respect, then, he preceded Rau. But unlike Rau after him, he did not consider the dynamic stability of equilibrium. Humphrey also mentions an unpublished paper by Hennings (1979) which pointed out that "neither Rau's library nor that of his university possessed Cournot's book." Even if this is true, however, Rau did list Cournot (1838) in §45, note (*e*), of the subsequent editions (1847, p. 56; 1855 and 1863, p. 53; 1868, p. 63), and discuss his work in §63, note (*b*), of these editions (1847, p. 84; 1855 and 1863, p. 79; 1868, p. 98). There he spoke approvingly of Cournot's discussion (1838, §§2–3) of the fact that when a publisher destroys two-thirds of its inventory of books to gain a larger revenue, this is regarded according to the criterion of "exchange value" as "a veritable creation of wealth in the commercial sense of the term" (pp. 6–7). However, there is no reason to suppose that Rau had already read Cournot in 1841 (the formulations are too different), but it seems likely that by 1847 he had read beyond this first chapter, though we have no evidence of it. On a comparison of Cournot's and Rau's analyses see the interesting discussion by Schefold et al. (1997).

[42] In the 5th to 8th editions one finds instead in a different section (§154, note (*e*): 1847, 1855, and 1863, p. 174; 1868, p. 205) the weaker statement: "in supply as in demand the *concrete value* forms the main driving force."

> If, for example, A estimates a good $1\frac{1}{2}$ times as highly as B, and has 3 times as much to consume as the latter, so that he places the value of a certain quantity of money at only $\frac{1}{3}$ that of B, then he will be inclined to pay $4\frac{1}{2}$ times as much for the thing as B.

Assuming Rau to mean that A estimates *a certain quantity* of a good $1\frac{1}{2}$ times as highly as B, and identifying the "amount to consume" with income, Y, where $Y_A = 3$ and $Y_B = 1$, then from the above-quoted dictum (Rau, 1841a, §63a, p. 69) that "the value of one and the same sum of money for several owners is ... roughly in inverse proportion to their entire wealth" (see also formula (9) below) we have, for $x_A = x_B$:

$$\frac{\partial U_A}{\partial x_A} = 1.5\frac{\partial U_B}{\partial x_B} \quad \text{and} \quad \frac{\partial V_A}{\partial Y_A} = \frac{1}{3}\frac{\partial V_B}{\partial Y_B},$$

where $\partial V_i/\partial Y_i$ is individual i's marginal (indirect) utility of income; hence, denoting individual i's demand price for x_i units of the commodity by p_i $(i = A, B)$, we have for the "value to the buyer" and for the same quantity $x_A = x_B$

$$p_A \equiv \frac{\partial U_A/\partial x_A}{\partial V_A/\partial Y_A} = 4.5\frac{\partial U_B/\partial x_B}{\partial V_B/\partial Y_B} \equiv 4.5p_B. \tag{2}$$

Rau gave a detailed explanation of the process of adjustment to equilibrium in his diagram, stressing that "one cannot speak of the amount of the supply and demand as such, but rather only under the assumption of a particular price bid or asked" (1841a, Appendix to §154, p. 527; 1847, p. 580; 1855, p. 583; 1863, p. 588; also 1863, p. 174; 1868, p. 205).

3 Hildebrand

Hildebrand's sole book, *The Economics of the Present and Future* (1848), published in the same year as the *Communist Manifesto* of Marx and Engels, constitutes a landmark in the history of economic thought. In its breadth of analysis, originality, and respectfulness and fairness towards those whose doctrines he attacks, it is a gem of a work. While his critique encompasses Adam Smith (whom he describes as "the Kant of national economics" (p. 285)), Adam Müller, Friedrich List, and others, what is of particular interest to us here is his critique of the then leading socialist theorists (whom he describes as "social theorists"), Engels and Proudhon.

In a very sarcastic contribution, Engels (1844) had criticized the paradox of value in economic theory as follows (p. 93):

> The economist, who lives off antitheses, naturally has a double *value* too: abstract or real value, and exchange value. There was a lengthy dispute between the English writers, who defined the cost of production as the expression of real value, and the Frenchman Say, who purported to measure this value by the utility [*Brauchbarkeit*] of a thing. This dispute has been pending since the beginning of this century, and has died down, undecided. Economists cannot decide anything.

He also pointed out the inconsistency of the Ricardians in relying on Say's utility to explain why a product whose cost of production exceeded its price could not be sold, and of the followers of Say in relying on Ricardo's cost of production to explain why necessities are cheaper than luxuries. He concluded that the only way to solve these contradictions between "real" value and observed market value was to abolish private property.

Hildebrand (1848, §36, p. 166) replied to these criticisms by, first of all, pointing out that Smith did not distinguish between "real value" and exchange value, but rather between "use value" and exchange value, as had already been done by Aristotle.[43] He went on to point out that the German economists (of whom Engels was apparently ignorant)—and here he singled out Hermann (1832), Rau (1841a), Lotz (1811), and Thomas (1841)[44]—had carried the Smithian system much further, hence "Engels was entirely in error when he had the national economists seeking out a real or abstract value of all things as opposed to exchange value. Their endeavor was rather the discovery of a fixed, invariable standard and the true basis of determination of exchange value, in other words the value that occurs in exchange itself" (pp. 167–8). Foreshadowing Gossen, he concluded (p. 168): "Engels overlooks the fact that a pure absolute value of goods under all circumstances does not exist, and that value is always a relationship of a thing to a person and to human society, and depends upon human estimation." He also pointed out (p. 169) that even in the absence of private property, "the products of labor and the amounts of consumables are necessarily subject to estimation" and that "some kind of definite formula would have to apply to the turning over of individual contributions of labor to the common good, opposite the share of benefits which fall to the individuals" (pp. 169–170), and even suggested some slogans from Saint-Simon and Fröbel (1847, II, p. 329) anticipating those of the *Communist Manifesto*.[45]

Hildebrand's critique of Proudhon was his most interesting and important. Proudhon's own critique of economics went as follows (1846, Ch. II, p. 37; c1867, p. 63; 1888, p. 78):

> If the crop of wheat is double throughout the whole country, twenty sacks will sell for less than ten would have sold for if it had been half as great; so, under similar circumstances, fifty yards of linen will be worth less than twenty-five: so that value decreases as the production of utility increases, and a producer may arrive at poverty by continually enriching himself.

Proudhon followed this by the explanation: "In the preceding examples the use value exceeds the exchange value".[46] Thus we are to interpret "production of utility" in the above

[43]Cf. Aristotle (c322 B.C., Book I, III, 11; 1932, pp. 38–41): "with every article of property there is a double way of using it; both uses are related to the article itself, but not ... in the same manner—one is peculiar to the thing and the other ... not .... Take for example a shoe—there is its wear as a shoe and there is its use as an article of exchange; for both are ways of using a shoe, inasmuch as even he that barters a shoe for money or food with the customer that wants a shoe uses it as a shoe, though not for the use proper to a shoe, since shoes have not come into existence for the purpose of barter."

[44]Hildebrand (p. 167) described Thomas's work as "a very astute treatise". Its most quoted passage is the statement (§. 2, p. 11): "It is not possible to think of the concept of valuation without thinking at the same time of a subject who evaluates and an object which is being evaluated." This theme is found repeated in many of Hildebrand's phrases. Diehl (1908, I, p. 49) went so far as to speak of Thomas and Gossen as the "two precursors" of the subjective value theory of the Austrians Menger, Wieser, and Böhm-Bawerk.

[45]In fact, Schefold (1998, pp. 40–41) has made the interesting suggestion that Hildebrand may already have been acquainted with the Communist Manifesto when he wrote these words.

[46]Tucker's English translation renders Proudhon's *valeur utile* and *valeur échangeable* more literally as

passage—which presumably means the total utility of the produced output—as corresponding to Proudhon's "use value".

The phenomenon illustrated in the above-quoted passage, resulting from inelastic demand, had of course already been noted by Lauderdale (1804, 1819).[47] Proudhon later shifted his assumptions, and commented that use value and exchange value "are inversely proportional to one another" (1846, p. 38; c1867, p. 64; 1888, p. 79), where again by "use value" (*valeur utile*) he apparently meant the quantity produced and consumed (or the total utility of this quantity);[48] this would then imply unitary price elasticity of demand. He subsequently introduced an example to this effect, invoking the mythical god Promethius[49] (1846, p. 60; c1867, pp. 80–81; 1888, p. 100):

> Suppose, then, that suddenly ... Prometheus finds a way of producing in one day as much of a certain object as he formerly produced in ten: what will follow? The product['s ...] relative value will be proportionately diminished, and, instead of being quoted at one hundred, it will thereafter be quoted at only ten. But this value will still and always be none the less accurately determined, and it will still be labor alone which will fix the degree of its importance. Thus value varies, and the law of value is unchangeable ....

This "law of value" Proudhon enunciates in terms of an interesting chemical analogy (Ch. II, §II; 1846, p. 51; c1867, p. 74; 1888, p. 91):

> Let us picture wealth, then, as a mass held by a chemical force in a permanent state of composition, in which new elements, continually entering, combine in different proportions, but according to a certain law: value is the proportional relation (the measure) in which each of these elements forms a part of the whole.

This appears to make the implicit assumption of a unitary price-elasticity of demand, resulting in the value of each commodity always being a constant share of national income.

Hildebrand is known, as we shall see in the next section, particularly for his criticism of Proudhon; what is not so well known is the deep debt his own analysis owes to Proudhon. This we shall now see.

Hildebrand elaborates on Proudhon's example of unitary price-elasticity of demand as follows (1848, §56, p. 287):

> Use value and exchange value stand in inverse proportion to one another. If all spinners double their labor and each of them spins 100 instead of 50 yards, then the spinners do not receive twice the price for their labor, but rather the products of their labor become cheaper by half, because they are available in twice the amount; and if in lean years the farmer produces half as much grain

"useful value" and "exchangeable value".

[47] Hildebrand had cited Lauderdale earlier in his work (p. 21, note 3; p. 72, note 2) but curiously made no mention of Lauderdale's similar illustrations in his discussion of Proudhon.

[48] His definition of "*valeur d'utilité*" was: "the capacity possessed by all products, whether natural or the result of labor, of serving to maintain man" (1846, p. 34; c1867, p. 61; 1888, p. 75).

[49] "Prometheus, according to the fable, is the symbol of human activity. ... Then let us call society Prometheus." (Ch. II, §II; 1846, p. 56; c1867, p. 77; 1888, p. 96).

> as previously, then the exchange value of the grain increases, although its use value remains entirely the same.

Here Hildebrand seems to be paraphrasing Proudhon: in the first illustration he follows Proudhon's definitions, since the exchange value is halved and the output (and therefore consumption) doubles, and this is the measure of Proudhon's "use value"; in the second illustration, however, he uses "use value" in quite a different sense, namely the *value* of the entire output of the good, relative to the total value of all goods—which turns out to be the sense adopted by Proudhon in his above chemical analogy, and, as we shall see, the sense subsequently adopted by Hildebrand himself as his interpretation of Rau's "species value".

Adhering to the hypothesis of unitary price-elasticity of demand, Hildebrand summarizes his position as follows (1848, §57, p. 290):

> Regarding the variability of value, an unchangeable law predominates, which simultaneously eliminates the contradiction between use value and exchange value: the law, namely, *that each product becomes steadily more needful*[50] *the less does its production cost in labor, and the smaller, therefore becomes the degree to which it can contribute to the formation of total wealth; or in other words, that the development of society and its wealth consists in a progressive diminution in production costs and in a progressive growth in the needfulness of all products.*

This passage is not easy to interpret, but it seems to assert a "law of conservation of value shares" which would hold under unitary price-elasticity of demand. But this is just Proudhon's "law of value" enunciated in his Promethean example and his chemical analogy! This analogy is no doubt what provided Hildebrand with his interpretation of Rau's species value of a commodity as the proportion of total expenditure devoted to it.[51] Hildebrand adds (1848, §57, p. 289), in perfect agreement with Proudhon: "The force which generates the elements and determines their respective proportions in total wealth is the *labor* already celebrated with great eloquence by Adam Smith, and the *proportion* itself, or the *amount in which each of these elements contributes to form the whole, is value.*"

Hildebrand finally comes to his highly significant critique of Proudhon's antinomy (1848, §64, p. 318):

> *The more the quantity of a useful item is increased, the more, in a state of unchanged need,*[52] *the use value [Nutzwerth] of each individual unit declines.*

[50]The German word used is *nothwendiger*, which could be translated as "more necessary" or "more essential", but here the meaning seems to simply be "in greater demand"; this has therefore been rendered here as "more needful".

[51]Cf. Hildebrand (1848, pp. 288–9). This was also Turgot's definition of "esteem value"; see footnotes 56, 57, and 70 below.

[52]The term used by Hildebrand was *unverändertem Bedürfniß*. Without an independent definition of "need", one might legitimately ob ject that the "need" for something might depend on its price (as indeed is suggested by the above-quoted passage concerning "the progressive growth of needfulness"), and therefore that Hildebrand might be committing the fallacy of implicitly defining "unchanged need" as that hypothesis that is needed for his conclusion to be true. Below I shall argue that this is not the case; but "unchanged need" does need a precise definition. For now we may interpret the assumption to mean "unchanged tastes" in the sense that a person's indifference map remains unchanged. Below I shall argue that it does have a more precise meaning, namely unchanged tastes plus constant unitary price-elasticity of demand.

> For, since use value is always a relation of a thing to a person, accordingly the measure of use value for each species of goods is found in the sum and ranking of the human needs that it satisfies; and where no person and no needs exist, nor is there any use value. The sum total of the use value which is possessed by each species of goods—so long as the needs of human society do not change—thus remains constant, and is distributed over the individual units of the species according to their quantity. The more the sum total of the units increases, the smaller is the proportion of the species's use value that is accorded to each unit; conversely, the less the abundance, so much greater is each unit's proportion of the species' use value.

Here, Hildebrand alters Rau's concept of species value by postulating it to be the constant share of consumers' incomes spent on each species of goods. Suppose that in a society of m individuals it is assumed that a fixed proportion, θ_{ij}, of individual i's nominal income, Y_i, is spent on commodity j, for $j = 1, 2, \ldots, n$, i.e., $p_j x_{ij} = \theta_{ij} Y_i$, where $0 < \theta_{ij} < 1$ and $\sum_{j=1}^{n} \theta_{ij} = 1$, and x_{ij} is individual i's demand for good j. Assuming (as is implictitly done by Hildebrand) that all consumers have the same tastes, we have $\theta_{ij} = \theta_j$ for $i = 1, 2, \ldots, m$. Accordingly, defining aggregate income by $Y = \sum_{i=1}^{m} Y_i$ and aggregate demand for good j by $x_j = \sum_{i=1}^{m} x_{ij}$, we have $p_j x_j = \theta_j Y$, i.e., the community's total expenditure on good j is the same share θ_j of national income Y.

Then, supposing nominal national income to be held constant (say by monetary policy) we may make the identifications[53]

$$
\begin{aligned}
\theta_j Y &= \text{“abstract” or “species” use value of commodity } j; \\
\frac{\theta_j Y}{x_j} &= \text{“concrete” or “quantitative” use value of commodity } j.
\end{aligned} \tag{3}
$$

(Alternatively, we might wish to interpret the species and concrete value as simply θ_j and θ_j / x_j respectively, but for a fixed nominal aggregate income, Y, there is no practical difference.)

The aggregate demand function for commodity j is

$$
x_j = \frac{\theta_j Y}{p_j}, \tag{4}
$$

which is generated by maximizing the aggregate log-linear utility function

$$
U(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} \theta_j \log x_j \tag{5}
$$

[53] I have taken the liberty of employing Rau's term “concrete value” for what Hildebrand describes above as “the proportion of the species's use value that is accorded to each unit.” The only use of Rau's term “concrete” that I have found in Hildebrand is contained in a passage criticizing Engels in which he states (1848, pp. 168–9): “The utility of concrete quantities of goods as well as entire species of goods remains constantly fluctuating, depending upon need, moral characteristics, and the capability of people to utilize them, and is always relative.” The justification for our appellation is to be found in Knies's 1855 treatment to be discussed in section 6 below; Knies owned that “we are admittedly not using the latter word [‘concrete’] in quite the same sense as Rau” (Knies 1855, p. 440).

subject to the budget constraint $\sum_{k=1}^{n} p_k x_k = Y$. The marginal utility of commodity j is

$$\frac{\partial U}{\partial x_j} = \frac{\theta_j}{x_j}, \tag{6}$$

which decreases with x_j. Substituting (4) into (5) we obtain the aggregate indirect utility function

$$V(p_1, p_2, \ldots, p_n, Y) = \log Y + \sum_{j=1}^{n} \theta_j \log\left(\frac{\theta_j}{p_j}\right), \tag{7}$$

whence the marginal utility of income is

$$\frac{\partial V}{\partial Y} = \frac{1}{Y}. \tag{8}$$

(Note that for individual i, writing this formula as

$$\frac{\partial V_i}{\partial Y_i} = \frac{1}{Y_i}, \tag{9}$$

it expresses in an exact way Rau's 1833–1841 dictum—quoted above—that "the value of one and the same sum of money for several owners is ... roughly in inverse proportion to their entire wealth.")

Dividing (6) by (8) we obtain the demand-price for commodity j, or what Knies was later to identify with Rau's "concrete (use) value" of commodity j (see section 6 below):

$$\frac{\partial U/\partial x_j}{\partial V/\partial Y} = \frac{\theta_j Y}{x_j} \tag{10}$$

—a special case of Rau's "value to the buyer" (2), which, in view of (4), is equal to the "exchange value" p_j of commodity j.

If commodity j is wheat, a rise in wheat output, x_j, will lead to a proportionate fall in its price, p_j, in accordance with the formula $p_j = \theta_j Y/x_j$. The "sum total of the use value" possessed by the species "wheat" is $\theta_j Y$, and the "amount accorded to each unit" is $\theta_j Y/x_j = p_j$. Accordingly, "the more the sum total of the units" x_j, "increases, the smaller the proportion of the species' use value", $\theta_j Y/x_j = p_j$, "that is accorded to each unit." Thus Hildebrand in 1848 was able, albeit for the special case of loglinear or Cobb-Douglas utility functions—but the same special case that had been employed by Torrens (1819) in his dispute with Sismondi, and by Mill (1852, p. 155) in his enunciation of the "law of international value" (cf. Chipman, 1965, pp. 484, 711–2)—to enunciate the principle of diminishing marginal utility in explaining (and refuting) Proudhon's antinomy. That this passage of Hildebrand's—in particular, the first (emphasized) sentence which was, strangely enough, omitted from Menger's quotation (1871, p. 109n; 1950, p. 297)—contained the essence of the principle of diminishing marginal utility, was brought to our attention by Streissler (1990, p. 44).

The above quotation from Hildebrand is continued by the following passage which, it seems, few subsequent writers have been able to interpret, because of the special implicit hypotheses imposed by Hildebrand and his related puzzling terminology; it will, I think, be worth while to present a precise explanation.

> If the total need [*Gesammtbedürfniß*] of a nation is taken to be 100, then it would be possible to express the use value of each species of goods as a certain percentage, and if the use value of iron were, for example, assumed to be 5%, then under the assumption that the national need [*Nationalbedürfniß*] does not change in the instant in which the production of iron increases, a new distribution of the 5% use value over the individual quantities of iron produced takes place as well. Whereas with the production of 500 centners, each centner of iron would possess 1/100% of the combined national use value, it follows that with the production of 5,000; 50,000; 500,000—and so forth—centners, the use value of each centner would decline to .001; .0001; .00001—and so forth—percent of the national total of all use value.

We may interpret the "total need of a nation" as the amount its inhabitants have available to spend, namely national income, $Y = 100$; the species use value of commodity j—iron—is $\theta_j Y = 0.05Y = 5$. This defines the "national need" for iron, $\theta_j Y$; thus, Hildebrand's concept of "need" for a commodity is precisely the same as his notion of its "species value" in Rau's terminology. The "distribution of the 5% [species] use value over the individual quantities of iron produced" is $\theta_j Y/x_j$ where x_j is the quantity of iron consumed, equal to the quantity produced, giving the "concrete value" $\theta_j Y/x_j$. Note that in the first sentence in the above quotation, "use value" refers to the *species* use value, whereas in the second sentence it refers to the *concrete* use value; this is because in his formulation Hildebrand interprets the species value as the total, and the concrete value as the average, use value.[54] We can now see that by "unchanged need" Hildebrand means not only "unchanged tastes" but also constancy of each θ_j, i.e., unitary price-elasticity of demand (see footnote 52 above).

It is interesting to note that twelve years later, no less a figure than Léon Walras wrote his first full-length book which was devoted to a refutation of Proudhon's doctrines. Here is what Walras (1860, pp. 9–10) had to say about Proudhon's antinomy:

> It is plainly evident that the general fact of exchange value has its source in the limitation in quantity of the utilities which makes them scarce. ... One could analyze utility and consider it successively in its *intensity*, according to which it is more or less great or middling; in its *extension*, according to which it is more or less widespread or restricted; in its *direction*, according to which it is more or less mediate or immediate. One would then find that from some of these points of view, utility influences the scarcity, and therefore the value, of things. It would remain no less established that exchange value has its origin in the quantitative limitation of utilities, ... in a word, in the scarcity of things.

It is quite clear that Hildebrand in 1848 was far ahead of Walras in 1860 in his understanding of the principle of diminishing marginal utility!

[54] This is presumably what led Knies (1855) (see section 6 and the quotation following footnote 67 below) to stress this aspect of the concrete value rather than the interpretation given in the important (emphasized) first sentence of Hildebrand's above-quoted critique of Proudhon's antinomy.

4 Friedländer

Friedländer's monograph (1852), prepared for a Jubilee celebration of his university in Estonia (then part of Russia), is concerned with the gap in Smithian economics between use value and exchange value: "Drawing attention to this gap is the purpose of this treatment, which is intended to bring about a more multifaceted and more deeply incisive consideration of use value" (p. 5); he describes his proposed concept as "economic objective use value", by which he appears to mean essentially the same thing as Rau's "species value" (he had apparently not become aware of Hildebrand's work). He affirms (p. 6) that "the true goal of economics is less the pursuit of wealth than that of the general welfare".

The monograph is divided into two parts. The first, preceded by a foreward and an introduction, is a lengthy critical survey of the literature (pp. 9–43), occupying more than half the work. The authors covered are Malthus, Smith, J.-B. Say, McCulloch (whom he had apparently conversed with in London), Turgot, Louis Say, Sismondi, Rossi ("a man of great distinction in the science", p. 24), Lotz, Hermann, Rau (whom he calls "the outstanding master of the science", p. 33), and Riedel (he refers to the last three as "my learned friends" (p. 19)). In each of these he surveys the author's work through paraphrases or outright reproductions of scattered passages,[55] followed by his own critical response to the author's views; a reader of of this monograph must therefore be very careful to distinguish the paraphrased passages from Friedländer's own. The second part of Friedländer's monograph, entitled "On the Theory of Value", is divided into a short introductory paragraph, a section I on subjective and objective aspects of use value (p. 44), a section II on the economic significance of use value, exchange value, and price (p. 52), and a section III on the possibility of a universal expression for use value (p. 59), followed by an epilogue (p. 68). It is section III that contains the most original development.

In his discussion of J.-B. Say, Friedländer rejects Say's idea of consumers' sovereignty (p. 14), retorting (p. 16): "was the Chinese government wrong in its efforts to limit the excessive use of opium?" And with regard to Say's view that exchange value is the only correct indicator of wealth, he cites (pp. 14n–15n) Lauderdale's observation (1804, pp. 50–51) that according to Gregory King's rule, a poor harvest would have a higher total value than a normal one. He then argues that "the goal of the economy should be not the production of exchange value, but rather of economic welfare" (p. 16).

Friedländer's discussion of Turgot is of particular interest to us. Turgot had supplied an objective criterion for ranking human needs by his concept of "esteem value" (*valeur estimative*) of an object for a man in isolation.[56] Friedländer (p. 21n) quotes the following

[55] As he says in his discussion of J.-B. Say (p. 13), "I summarize the abbreviated comments, which are scattered over various places, and for our purposes worth taking into consideration, as far as possible in his own words."

[56] In deriving his concept of "esteem value", Turgot (1769; 1844, I, pp. 83–84; 1919, III, p. 88; 1977, p. 140) acknowledged (albeit critically) the influence of Graslin (1767, Part I, Ch. II, pp. 24–58). Indeed Graslin (pp. 26–30) made very explicit his postulate that to each need there corresponds a fixed total share of expenditures, and used this to explain how increased scarcity of a good would lead to a higher price: "all the individual parts of a single thing, in whatever quantity, always having as their object just a single need, together have the same value. This value of the object, or of the thing taken solely in its relation to the species, and independently of the quantities of the individual parts, cannot change, so long as the needs stand in the same proportion; but the partial value of this thing must absolutely diminish in proportion to

passage (1769; 1844, I, p. 83; 1919, III, p. 88; 1977, p. 140):[57]

> [What then is his measure of value in this case? What is his scale of comparison? It is clear that he has none other than his own resources.] The sum total of these resources is the only *unit* of this scale [,the only fixed point of departure].* It follows from this that the *esteem value* (*valeur estimative*) of an object, for the man in isolation, is precisely that portion of his total resources which corresponds to the desire that he has for this object, or what he is willing to use to satisfy this desire. It might be said, in other words, that it is the ratio of this proportionate part to the total of the man's resources, a ratio expressed as a fraction whose [numerator is this unit and whose]* denominator is the number of values or equal proportionate parts that the man's resources contains.

Friedländer comments (1852, p. 21) that "it is a deep idea that the estimation of value is based on the totality of people's needs, which is compared with their faculties and powers, almost as if they were equivalent." Turgot's formula calls to mind Proudhon's chemical analogy and "law of value" adopted by Hildebrand in his interpretation of "species value" as the ratio of one's expenditure on the species (in order to satisfy one's corresponding needs), to one's total resources ("powers and faculties") measured by income. Unfortunately Friedländer had not been exposed to this literature, however. He ends up rejecting Turgot's measure on the following grounds (p. 21):

> ... indeed if the fraction could be determined, then it would actually be an expression of the price which a person surrenders for the valuable [*werthvollen*] object—not the value itself, whose estimation must have preceded this in order to cause the person to decide to sacrifice exactly *this* proportionate share of his powers for exactly *this* part of his needs.

This identification of the share of a person's total expenditure devoted to a particular object with the price of that object is of course incorrect. Had Friedländer had a better appreciation of the proportionality between concrete use value (marginal utility) and exchange value (price) from a reading of Hildebrand's work (or indeed from his reading of Rau), he might have been more receptive to Turgot's suggestion; on the other hand, his objection may have been based more on the idea that equilibrium market prices need not correspond to what they might be in a "just society".

With regard to Lotz, Friedländer justly complains (p. 29) that "the piling-up of various distinctions does not promote clearer understanding." He discusses (pp. 30–31) Hermann's rank-ordering of categories of goods: necessities $\succ$ luxuries $\succ$ education $\succ$ lustre (1832, p. 68), as well as Hermann's criteria of "general" versus "particular" use value according to

an increase in the number of its parts" (p. 27). He went on (pp. 36–40) to rank-order "relative species" by the relative expenditure shares corresponding to the respective needs.

[57]The first three (bracketed) sentences were omitted by Friedländer. The starred bracketed passages were omitted (by Du Pont de Nemours) from the Daire edition (1844) cited by Friedländer, and are from the original edition published in Schelle (1919); thus they could not have been known to Friedländer. In the quotation I have followed Groenewegen's translation except that I have substituted "ratio" in place of "relation" for Turgot's *rapport*, and made a few other minor changes. Groenewegem's "resources" corresponds to Turgot's *facultés*, and was translated by Friedländer as *Anlagen*.

the greater or less dissemination of the need, and "comprehensive" versus "limited" use value according to the number of needs served (1832, p. 69), etc.

In his discussion of Rau's work (pp. 33–38) Friedländer paraphrases (p. 36) the following passage from Rau (1847, §79, p. 104):[58]

> One can more easily form a notion of the state of a nation's wealth if one compares it with that of others. In this, one may imagine all the nations being compared with one another as being situated at the same educational level, or in a similar process of development, their needs as equally great—thus leave these entirely out of consideration, and just keep to the average amount of income available for people's well-being. Accordingly a nation is richer than another if there accrues to each of its people a greater amount of goods annually.

Against this Friedländer makes the following objection (p. 38):

> The ascertainment of the quantities of goods under the assumption of equal types of needs, cannot thereby suffice. Rather, [the comparison] should depend primarily upon the investigation of the actual amount of needs as they have developed among various peoples in different social classes and in the different stages of national development, in order to arrive at the definition of the average unit of need, tested according to the definition of a reasonable life, which can then become the standard by which individual needs and the use value of things corresponding to them can be measured.

This stance forms the basis of Friedländer's approach in section III of the second part of the monograph, which must be considered the heart of the work.

In section I of Part 2 of the work, Friedländer defends an "objective" conception of use value (p. 46): "The goals which man sets for himself can be evaluated in their objective significance only when one views them as the means for fulfilling the highest destiny of humanity." Thus he defends an unabashedly paternalistic interpretation of economic welfare (p. 47):

> Those distinguished by their capability to render a sound judgment cannot form a majority, for the very reason that they are distinguished, because relative to the masses they are only a few. The subjective judgment of the multitude thus cannot be definitive as to the significance of the secondary human purposes for human needs and the valuation of external things emerging from them, which from the standpoint of the nation and the state can be gained only on the basis of ethics. ... If, therefore, external goods are to be ordered according to their economic significance, it is then insufficient to examine how ... the majority of

[58] Rau adds (1847, §81, pp. 105–6): "The degree to which a nation's income contributes to its economic well-being depends not only on its magnitude but also ... on the manner of its distribution. A nation could have a very large income but such that a small number of people live in wealth bordering on excess, while the majority do not even have quite enough to live on." This is also paraphrased in Friedländer (p. 36). These passages of Rau's are also found with slight variations in the other editions: (1826, pp. 56–7; 1833, pp. 77–8; 1837, pp. 82–4; 1841a, pp. 84–6; 1855 and 1863, pp. 96–8; 1868, pp. 117–9.)

people rank them;[59] rather, science must rank-order the economic significance of the purposes pursued by people according to ethical principles.

Friedländer then provides his definitions of (use) value and utility (p. 48):

> The capability of things to serve a pursued purpose is their utility. ... When a purpose is recognized as worthy of pursuit, and therefore a need is felt, and the capability of the thing to satisfy that need—its utility—is realized, then value is ascribed to the thing. Value is thus the relationship, recognized in human judgment, according to which the thing can be a means for the fulfillment of a purpose worthy of pursuit.

These definitions are hardly very precise! However, it later seems that Friedländer's objective is to obtain an estimate of Rau's *species value* (p. 51):

> For the ranking of commodities[60] by species value, an ethical foundation is indispensable, because only through it can the significance of the various purposes be valuated. ... By relating value to one people, the national-economic value emerges from the indefinite generality of species value, and takes on a concrete character.

Here, Friedländer appears to be using the word "concrete" in a different sense from Rau's. Thus we might interpret one of Friedländer's goals in this monograph as that of estimating the appropriate weights to be attached to commodities in lieu of current prices in order to construct, in modern terminology, an index of "real national income".

In section II of Part 2, Friedländer seeks to show (among other things) that exchange value, which J.-B. Say had upheld as the criterion for measuring wealth, is greatly dependent on use value and bargaining power.[61] This section contains a very interesting analysis of bilateral monopoly, perhaps stimulated by that of Turgot (1769); it goes as follows (p. 53):

> ... exchange is always to be regarded as a struggle between the parties to it, in that [each of] the two parties to the exchange wishes to give up as little reciprocally transferred use value in the exchange as possible, and is anxious to receive as much of it as possible. This self-seeking desire then causes each of the two traders to estimate the objects of the exchange with regard to their use value both for himself and for his counterpart in the exchange; and if we imagine the simplest case, in which two traders, each appearing opposite the other with

[59] Here Friedländer refers in a footnote to his previous discussion (p. 30) of Hermann's rank-ordering of categories of goods, which Hermann (1832, p. 68) had based on the majority view.

[60] Friedländer uses the word *Brauchlichkeit* coined by Zachariä (1832, p. 1) as the German equivalent of the English word "commodity" (Friedländer had used this word in his translation (1852, pp. 9–10) of the long passage from Malthus (1820, pp. 337–340)—see footnote 39), although he does not refer to Zachariä. Zachariä ruled out the word *Waare* as a translation of "commodity" on the ground that (p. 2): "A ware is a commodity insofar as it is a commodity for its owner owing purely to its exchange value."

[61] He had already stated, in his discussion of J.-B. Say (p. 16) that: "Say's assertion that exchange value is the yardstick of utility is ... entirely untenable. Say refuses to consider use value, because it can only be subject to arbitrary estimation; but ..., as will be shown below, this arbitrary estimation is not eliminated in exchange value, ... so that one is deceiving oneself to believe that in the numerical estimation of price one has found an absolute quantity as a basis for scientific consideration ...."

> one object of exchange, then each, even if not clearly conscious of doing so, ... makes four judgments; of the use value for himself of the thing to be given up and of the thing to be received in exchange, and those use values which they have, in his opinion, for his counterpart in the exchange. The eight judgments which the two parties to the exchange make must then be balanced out among one another if the exchange is to take place.

He even considers the case of unequal numbers of traders (p. 55):

> Let us even imagine the case in which the exchange value is at its point of equilibrium—... then ... the bargaining power is weaker on the side where the number of competitors is greater. To be precise, owing to the fact that each of the competitors wishes to realize a purchase or sale, his bargaining power declines. Ten farmers, each of whom brings 10 bushels of grain to the market, will in the case of otherwise equal exchange conditions be in a less favorable position opposite the purchasers of the grain than would two, who each brought fifty, or one who brought a hundred to the market.

We finally consider the most original part of Friedländer's monograph, section III of Part 2.

Friedländer's proposal is to determine the amounts of needs that must be satisfied to secure subsistence. This could be interpreted as constructing a subsistence-level indifference surface (or alternatively, the boundary of a "consumption set") in commodity space, given information on how specific needs are satisfied by specific commodities.

While Friedländer rank-orders the basic means of satisfying needs (food $\succ$ clothing $\succ$ shelter $\succ$ heating $\succ$ defense) (p. 61), he nevertheless considers each of them essential, and seeks a measure (or measures) of the needs themselves that are required for subsistence. In the case of food, he breaks these needs down into nutritiousness, digestibility, fortification, wholesomeness, and flavorfulness, and seems to suggest that to some extent these are substitutes, that is, that some nutritiousness could be sacrificed in favor of some digestibility. He cites the detailed data provided by Rau (1847, §191, note (*c*), pp. 240–242)[62] concerning diets of German and other agricultural field workers, and those obtained from British poor-relief reports, as well as data from several other contemporary authorities on nutrition and hygiene. As with Rau (see footnotes 14 and 15 above), Friedländer appears to regard foodstuffs which can be characterized by their "nutritive value" (calories?) as in effect perfect substitutes, e.g.: "The nutritive value [*Nährwerthe*] of 100 weight-units of rye is equal to [that of] 84 units of wheat, 107 of barley, 110 of oats, [and] 90 of peas" (p. 63). Thus (p. 63):

> Where ... experience shows that definite quantities of foodstuffs have affected the preservation of human life in the same way, then their objective values should be equated, otherwise their values [should be] set proportional to the relative quantities which have contributed to the nourishment. These relative quantities must be found through experience in the determination of practical

[62] This goes back to the 4th edition of Rau (1841a, §191, note (*c*), pp. 207–208), and is continued in the 6th and 7th (1855, 1863, §191, note (*a*), pp. 223–224), and 8th (1868, §190, note (*a*), pp. 258–260) editions.

> equivalents. The comparison of these equivalents with the particular units of need [*Bedürfnißeinheiten*] of diverse peoples will help to determine the relationship of the objective value of the diverse food items consumed among diverse peoples under diverse conditions of need [*Bedürfnißverhältisse*], and facilitate knowledge of the relatively favorable or unfavorable economic conditions.

Friedländer does not suggest any corresponding measures of "units of need" for clothing, but he does so for shelter, asserting (but without citing references) that "according to practical experiences which have been gathered in prisons in England, with continuous occupancy in the same space for a grown man, a minimum of 750 cubic feet has been recognized as indispensable for the preservation of human health, and even then provision for ventilation as still a prerequisite" (p. 64). He then continues:

> This spatial extent remains desirable as the lower bound of space limitations even if the person is not continuously present in the living space, instead, for example, passing time there only for the purposes of sleeping and eating. Where the severity of the climate would necessarily cause death without shelter, then shelter should be estimated with the same proportion [*Quote*] of the unit of need as food and clothing; where this is not the case, the degree of endangerment to life which looms determines the proportional amount.

This seems to be the only place where Friedländer indicates explicitly how he estimates the "objective use value" of one object relative to another. But although he ranks food and clothing above shelter, he necessarily equates the minimum cubic capacity of shelter needed for subsistence with the corresponding "need unit" of food (calories or whatever) required for subsistence. In this respect, therefore, he treats his major categories (food, clothing, shelter, etc.) as perfect complements.

A similar analysis is applied to fuels in their thermal capacity for heating (p. 64). And for defense he provides the following interesting analysis (p. 65):

> Weapons are necessary for the preservation of life where state institutions have not successfully taken on responsibility for protection of security. Their objective value depends upon the frequency and degree of the menacing danger; since the preservation of life in the face of an enemy's attack is just as urgent as that in the face of hunger, the value of the weapon may be expressed by a fraction which results from multiplication of the fraction which expresses the probability of fending off the danger by the fraction which designates the relative justification of the protection provided by weapons as compared with other the necessities of life.

Unfortunately he does not provide further hints as to how he would estimate the second fraction!

Friedländer's contribution is of special interest in the light of later contributions. The study of "need units" characteristic of commodities foreshadowed Stigler's pioneering paper on the cost of subsistence (1945), in which he provided data analogous to those relied on by Friedländer. Stigler's objective was different: to calculate the minimum-cost diet at given market prices; it was also one of the first applications of linear programming in economics.

Then, of course, Friedländer's "need units" are none other than the "characteristics" introduced by Lancaster (1971) in his "new approach" to consumer demand!

5 Roscher, 1

In the first edition of his *Principles*, Roscher (1854) showed the influence of both Rau (1847) and Hildebrand (1848). With respect to the latter, Roscher (1854, §5, note 3, p. 6) provided data from Cordier (1823) showing that while the wheat harvest in France steadily increased from 1817 to 1819, its value steadily declined, and he commented (note 5, p. 7) that: "The contradiction which Proudhon ... attempts to artfully work into the antithesis between use value and exchange value is well put to rest by Hildebrand." However, in his text (1854, §5, p. 6) Roscher had already stated, by way of showing that use value and exchange value were not identical, that "the exchange value of a good can also increase while the use value has decreased, and conversely"; this was to lead to a criticism by Knies (1855, p. 454).

The influence of Rau was shown in a short section on abstract and concrete value. The first paragraph of his statement is worth quoting in full (1854, §6, pp. 7–8):

> The distinction between *abstract* and *concrete value* was first made by Rau (*Lehrbuch* I, §§61ff.). The abstract or species value is based on the relationship which exists between an entire type of goods and the needs of people in general. Hence, e.g., the beech tree has a higher species value than the pine. The concrete or quantitative value, in contrast, is the value that a particular quantity of a type of goods has for a particular person under particular circumstances; thus it depends on the relationship between the requirement and the supply, etc. Thus, e.g., the gold found by Robinson had no concrete value to him whatsoever; duplicates in a private library, etc., also come to mind.

The comment about the beech and the pine as alternative fuels was to have a considerable influence on Knies.

6 Knies

The writings of Rau (1847), Hildebrand (1848), and Roscher (1854) stimulated a brilliant article by Knies (1855).

The main contribution, apart from expanding on the doctrines of his three predecessors, may be said to be his use of the concept of species value to define a hierarchy of wants, the species of wants (*Bedürfnisgattungen*) being set into one-to-one correspondence with the species of goods (*Gütergattungen*), and the goods in each species being considered as substitutes, and (by implication) those in different species being considered as complements. He started out this analysis by distinguishing (a) the intensity of a need which a good satisfies, and (b) the intensity with which a good satisfies a need (1855, p. 429).

(a) The first of these is purely subjective; in his words (p. 430):[63]

[63]Note the remarkable resemblance to the passage from Louis Say quoted in section 1.8 above. Knies did not refer to either L. Say or Lotz in this article, though he may have consulted their writings. It is doubtful that he knew of Graslin (see footnote 56 above), at least at that time.

> ... we arrange the species of goods together in accordance with the species of human needs. We can best estimate which rung on the scale[64] of use value the individual goods-species occupy when we pose ourselves the question: if they were shorn of the possession of all goods, in what order would people supply themselves with the goods-species for consumption and living[?] Or: in what order, conversely, would they abstain from further use, if they were in possession of all imagined goods[?]

To today's readers, the question posed seems to be a puzzling one in the absence of information on market prices; but Menger (1871, Ch. II, §. 3a, item 3, p. 52; 1950, Ch. II, 3A, item (3), p. 95) posed the problem in exactly the same way! Menger (1871, Ch. I, §. 1, p. 3; 1950, p. 52) also adopted Knies's concept of "goods-character" (Knies, 1955, p. 424). It turns out, as we shall see, that Knies solved the problem by making use of Hildebrand's idea of identifying the "species value" with the "need", both of them being defined as the (assumed constant) share of people's income devoted to the commodity group, or "species", in question.

(b) The second intensity can be ranked objectively, and this is the degree to which a good can satisfy a particular need. The examples are those given by Rau and Roscher respectively of the relative nutritive powers of different subspecies of grain, and the relative thermal efficiencies of different subspecies of wood to be used as fuel. Knies referred to this intensity as the good's "subspecies value" (*Specieswerth*).

To compare the species values of two different species, one needs to take account of both (a) and (b). Thus (1855, p. 440):

> The species value of firewood is far higher for a people in colder climatic regions than for a people in greater proximity to the equator; the same holds true for heat-producing fuels and for wool. The subspecies value of wheat is much smaller for a country which cultivates rice than for one that does not.

In both cases the species value will influence the price. The second case is an example of substitutes. This was later made explicit by Knies in *Das Geld* (1873, p. 126; 1885, p. 163):

> Here it is instructive to consider first and foremost the *"surrogates"*, which are to replace the use of other goods. I then indicate goods that I wish to refer to as *substitutable* [*stellvertretende*], in that, for example clothes of cotton, linen, wool, silk, etc., can equally meet the need for clothing; wood, bituminous coal, lignite, anthracite coal, etc., can equally meet the requirement for fuel, the various livestock animals the requirement for meat, etc.

This substitutability was the essential property characterizing *fungibility* (1873, pp. 115–116; 1885, p. 151):

> The *fungible goods (res fungibiles)*—and *with regard to these*, money has been referred to since ancient times as the most fungible of all goods—have the special property that one quantitative unit of it can function as equal in value to every second, third quantitative unit of equal size.

[64] The mixed metaphor is Knies's!

As Block (1890, II, p. 78; 1897, II, p. 77) has remarked, "Mr. Knies does not sparkle with clarity"; however, my interpretation of this passage is that it states that the marginal utility of money is essentially constant, and that the more slowly a species of goods' marginal utility decreases with increasing quantities, the more fungible the species is. Of course this concept can only be *relative*; thus, two goods are *relatively fungible* if and only if the elasticity of substitution between them remains high, and a fungible good is presumably one with a high elasticity of substitution with respect to all other goods.

Knies provided an illustration of the nature of this substitutability by means of an example of two subspecies of grain: wheat and rye (1855, pp. 450–451). Let ν_j be the nutritive power (e.g., the number of calories per million bushels) of grain j, x_j the consumption of grain j on the market in millions of bushels, and p_j the exchange value (price) of grain j in guilders per million bushels. Then the total quantity of nutritive power produced [*Nährkraftsquantität*] by grain j (e.g., the total output of calories from grain j) is the product $\nu_j x_j$. Now (and this part of the reasoning was not included by Knies), one must have

$$p_{\text{wheat}}/p_{\text{rye}} = \nu_{\text{wheat}}/\nu_{\text{rye}}; \tag{11}$$

for the total expenditure on grain is also the total expenditure on grain-calories, and may be decomposed as

$$p_{\text{wheat}}x_{\text{wheat}} + p_{\text{rye}}x_{\text{rye}} = \frac{p_{\text{wheat}}}{\nu_{\text{wheat}}} \cdot \nu_{\text{wheat}}x_{\text{wheat}} + \frac{p_{\text{rye}}}{\nu_{\text{rye}}} \cdot \nu_{\text{rye}}x_{\text{rye}}, \tag{12}$$

where p_j/ν_j is the price per calorie of grain j, so if wheat is a cheaper source of calories than rye (i.e., $p_{\text{wheat}}/\nu_{\text{wheat}} < p_{\text{rye}}/\nu_{\text{rye}}$), the rational consumer will purchase only wheat, and likewise if rye is the cheaper source, all expenditure will be devoted to rye. Since both must be consumed in equilibrium, the equality (11) must hold. Then we may define

$$r_{\text{grain}} = \frac{p_{\text{wheat}}}{\nu_{\text{wheat}}} = \frac{p_{\text{rye}}}{\nu_{\text{rye}}} \tag{13}$$

as the price of a unit of nutritive power of grain. The total expenditure on grain may therefore be expressed as the total expenditure on units of nutritive power of grain:

$$r_{\text{grain}}(\nu_{\text{wheat}}x_{\text{wheat}} + \nu_{\text{rye}}x_{\text{rye}}) = \theta_{\text{grain}}Y, \tag{14}$$

where θ_{grain} is the species value of grain. Knies defines ν_{wheat} and ν_{rye} as the *subspecies values* of wheat and rye. Note that Knies's subspecies value is defined according to quite different principles from that of the species value. We may note further that Knies's concept of subspecies value agrees completely with Rau's concept of "species value" as applied to subspecies (see footnote 15 above).

Thus, Knies generalized the Rau-Hildebrand model to one in which the aggregate consumer preferences may be expressed by a utility function of the form

$$U(x_{1\cdot}, x_{2\cdot}, \ldots, x_{s\cdot}) = \sum_{j=1}^{s} \theta_j \log\left(\sum_{k=1}^{n_j} \nu_{jk}x_{jk}\right), \tag{15}$$

where $x_{j\cdot} = (x_{j1}, x_{j2}, \ldots, x_{jn_j})$ is the bundle of quantities x_{jk} of the n_j subspecies of species j (e.g., the number of bushels of wheat, rye, barley, etc., in the species "grain"), ν_{jk} is the

subspecies value (e.g., nutritive power) of the kth subspecies of species j, and θ_j the species value or "need" of species j, there being s species and $n = \sum_{j=1}^{s} n_j$ subspecies (commodities) altogether. The elasticity of substitution is infinite between any two subspecies of the same species, and unitary between subspecies of different species.

Knies (1855, p. 441) next expanded on Hildebrand's theory of value, expressing it in Rauian terminology (see footnote 53 above):

> [Goods] receive their species value for use through the distinguishing features which characterize the type of need that they satisfy; [they receive] their concrete value, however, through the *proportion* in which ... their available amount stands to the *extent of the quantity* by which need is expressed. ...the concrete *use value* of each type of goods which a people consumes [is] found only in this *proportional relationship* of the quantity in which they are available and the extent and intensity of the need which is to be satisfied by way of this quantity.

Hildebrand had already defined the "need" for good j as the share $\theta_j Y$ of income devoted to it. In the first sentence of the above passage, we may identify the "available amount" of good j with x_j; likewise the "quantity in which they are available" in the second. The phrase "the *extent of the quantity* by which need is expressed" is less clear, but the second sentence's "the extent and the intensity of the need which is to be satisfied by this quantity" permits us to identify this amount with $\theta_j Y$. Accordingly—despite the equivocation as to what goes in the numerator and what goes in the denominator—the sentence may be interpreted as defining the "concrete use value" of commodity j by the ratio $\theta_j Y/x_j$ as in (10).

In other passages Knies muddies the waters with some perplexing statements. Thus he states (1855, p. 442): "the concrete value is determined *by the ratio between the quantity of goods that happen to be available and the quantity of goods that happen to be desired.*" To be consistent with his previous definition, "the quantity of goods that happen to be available" would have to refer to x_j, and "the quantity of goods that happen to be desired" to the "need", $\theta_j Y$. Since constant expenditure shares imply $p_j x_j = \theta_j Y$, this would also identify "the quantity of goods that happen to be desired" with *the amount spent* on these goods, $p_j x_j$, which may indeed be an indicator of "need", but can hardly be described as a "quantity of goods". An alternative possible interpretation of the passage is that the quantity that "happens to be desired" is one unit, hence the concrete value varies inversely with the available quantity. Another statement by Knies (1955, p. 455) seems to confirm the correct interpretation: "The concrete use value of goods, that is to say, the *degree* of their usefulness [*Brauchbarkeit*] for a people's consumption, is based ... not upon *the quantity* in which they are available, but rather upon the ratio of *this* quantity to the quantity of needs which they must satisfy"—except that this inverts the ratio. The phrase "degree of their usefulness" (which could also be translated as "degree of their utility") certainly provides a hint of marginal utility.

Knies's article contains an interesting discussion of the "paradox of value" in which he states (p. 443): "... it is entirely meaningless to conclude that because the species use value of bread is higher than that of fine linen lace, and the use value of the entire supply of bread is higher than that of the available entire quantity of fine linen lace, therefore the use value of a pound of bread is higher than that of a pound of lace." In fact, as is clear from the formula $p_j x_j = \theta_j Y$, the statement is correct if the first unqualified "use

value" is interpreted as the concrete, and the second as the species, value. Knies then proceeds with a numerical example in which the total quantities of bread-grain and lace are $x_{\text{bread}} = 24{,}000{,}000$ and $x_{\text{lace}} = 24$ hundredweight respectively, and their (concrete) use values satisfy $p_{\text{lace}} = 1000p_{\text{bread}}$. Unfortunately he stopped short of showing (as could easily have been done) that this implied that the species use values must satisfy $\theta_{\text{bread}} = 1000\theta_{\text{lace}}$. The gap was soon filled by Roscher (1857), as we shall see in section 7.

Finally I consider Knies's treatment (1855, pp. 451–2) of what he described as the two "stumbling blocks for the recognition of the harmony between use value and exchange value". The first of these was Proudhon's antinomy, which had been answered by Hildebrand. The second was the problem of explaining Cordier's data as cited by Roscher, showing that a bigger harvest can have smaller total value than a smaller one. With respect to the first, Knies's explanation followed Hildebrand's, but it is nevertheless worth quoting in full:[65]

> The first [stumbling block] is that it has been perceived *how, through a increase in the quantities of goods which possess exchange value, the exchange values of just such quantities is diminished.* When a malter of wheat had cost 40 guilders in a year of failed harvests, and then climatic conditions and the farmer's labor in the ensuing years of ample harvest placed at the disposal of the hungry human race a rich abundance of wheat with its evident use value, then the exchange value of the malter fell to perhaps 20 guilders! That appeared not only as an admonition against the significance which economic theory assigns to the exchange value of goods, but rather particularly also as an insoluble contradiction between exchange value and use value. However, this is far from being the case, and present here is rather one of the most beautiful proofs of the harmony between the two values. The error is based on the fact that it has been overlooked how, for example, the proportionate use value [of bread-grain], which is depicted by the ratio to its quantity of the total available quantity, is always assigned to the *single* malter of bread-grain. The total quantity of bread-grains, or of a single species, must satisfy the total need for it. If this need remains the same, and in one year it is satisfied by 1000 malters, in the other by 2000 malters, then in the first case, the use value of the single malter is $\frac{1}{1000}$th of the same total, in the second $\frac{1}{2000}$th—for it would be simply a contradiction were the exchange value to vary otherwise than in the manner decried [by Proudhon].

Here, by "proportionate use value" (*Quotegebrauchswerth*) of bread-grain Knies apparently means the same thing as Hildebrand's "proportion of the species's use value that is accorded to each unit", and in turn as Knies's interpretation of Rau's "concrete (use) value" given by (10) above, which declines in proportion to the harvest, x_j.[66] In accordance with Hildebrand's interpretation, if bread-grain is commodity j, the "total need" for it is $\theta_j Y$,

[65] A similar but much briefer explanation was supplied in *Das Geld* (1873, p. 125; 1885, p. 162), where Rau's "concrete use value" was replaced by Marx's "social use value". In that work, Knies made this theorem concerning the proportionality of use value and exchange value an important foundation of his doctrine that money has its own use value; cf. Knies (1873, p. 139; 1885, p. 185).

[66] Knies's statement is not free from ambiguity, however. The expression "proportionate use value" appears to be used just in the above-quoted passage and nowhere else in Knies's article. Assuming "its quantity"

which for given income is constant and independent of prices (this solves the problem of rank-ordering wants independently of prices). The total quantity of bread-grain, i.e., the output x_j, which, to clear the market, must be equal to the consumption of it, "must satisfy the total need for it"; as suggested above, this can only mean that the amount spent on it, $p_j x_j$, must be equal to the "total need" for it, $\theta_j Y$. The average cost to the consumer—the "proportionate use value"—is obtained by dividing both sides of this equality by x_j to get $p_j = \theta_j Y / x_j$. Thus, Knies's solution to Proudhon's paradox is not essentially different from, though possibly more persuasive than, Hildebrand's. Because of the implicit assumption of Mill-Cobb-Douglas preferences, it turns out that this "proportionate" or *average* use value, upon division by income (which in this case is the same as multiplication by the marginal utility of income as in (10) above), coincides with the *marginal* utility. This accounts for Knies's otherwise puzzling statement (1855, pp. 452–3):[67]

> One cannot emphasize enough that the use *value* of goods, in other words the *degree* of their usefulness, is determined by the *proportion* in which objects with particular characteristics are found in relation to certain human needs, and that the concrete use value depends not only on the particular quantities of available goods with certain characteristics, but rather on the proportion of these to the concrete needs of people.

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to be the single unit = 1, then the term could denote the fractions 1/1000 and 1/2000 in his illustration (i.e., $1/x_j$) by which the "need" $\theta_j Y$ must be multiplied to give the "concrete value", or the concrete value itself. I take the second interpretation as more likely, so that the "proportionate use value" is the same as the "concrete use value". However, for Knies's numerical illustration to be consistent with his definition, he would have to refer to the fraction as "the ratio *of* its quantity [= 1 unit] *to* the total available quantity [x_j]".

Streissler (1990, p. 49n) has remarked that the "proportionate use value" is Knies's term for marginal utility; this is certainly suggested by a passage quoted above, although I am inclined to the interpretation (10) rather than (6). Streissler further remarks on p. 49 that Hildebrand was "taken to task by Menger" for assuming that "total utility for every commodity always remained constant." This probably refers to the passage in Menger (1871, p. 109n; 1950, p. 298) in which he states, in criticism of Hildebrand, that: "A 'species value', in the proper sense of the term, therefore has no real nature and does not exist, unless 'utility' [*Nützlichkeit*], 'recognized utility', or the 'degree of utility' is confounded with 'value'. On the other hand, the species value, in the sense of the totality of the value of the concrete goods of a certain species to the individual members of human society, is not an unchanging magnitude, even if the needs of the various members of society remain unchanged, and hence the foundation on which Hildebrand builds his calculus is disputable." (I have made some changes to the 1950 translation). This shows that Menger did not follow Hildebrand's reasoning. Hildebrand did not use the terms "recognized utility" or "degree of utility" (these were Rau's terms); he spoke of "use value" or "utility value" (*Nutzwerth*), not of "usefulness" or "utility" (*Nützlichkeit*), and he ascribed constancy not to the total value of a species, but to its value relative to total income.

Menger made the same mistake in his criticism of Knies (1871, p. 111n; 1950, p. 300): "Although ... Knies also touches upon the concrete use value in the economy of an individual [*privatwirthschaftlich-concreten Gebrauchswerth*] ([Knies, 1855,] p. 461) this indeed is done only in order to elucidate the {frequent} contrast {in Rau} between the 'species value' (in truth, 'utility' [*Nützlichkeit*]) of goods and their concrete value, in other words, the proposition—quite correct—that the measure of the utility of things is something essentially different from the measure of their value" (again I have altered the 1950 translation, and I have restored two missing passages in braces). Here, Menger equates "species value" with "utility". However, for Hildebrand and Knies, it is the "total need" (or expenditure share) θ_j for every commodity that remains constant, but it just so happens that the *average need* θ_j / x_j is equal to the *marginal utility* of commodity j, so Hildebrand and Knies are both exonerated from Menger's criticism. See also footnotes 75, 79, and 115 below.

[67] See also footnote 54 above.

Knies's solution to the first "stumbling block" assumes that the "need" for a species of goods, implicitly defined as the share of income devoted to that species, remains constant, i.e., that the price-elasticity of demand is unitary. But this assumption is of course at variance with the data cited by Roscher (1854, §5, note 3, p. 5) from Cordier (1823) (see section 5 above); this gives rise to the second stumbling block. Lauderdale (1804, pp. 68–70; 1819, pp. 69–71) had shown that (with fixed money supply) a rise in output of a commodity could lead to a fall in its total value (the case of inelastic demand). Apparently neither Hildebrand nor Knies had consulted Lauderdale on this particular question. Knies bravely attempted an explanation.

The crux of Knies's argument seems to be contained in the following passage (1855, pp. 456–7):

> A greater quantity of wheat has without any doubt a greater *capacity* to perform, a greater *nutritive power*; but this—the intensity with which a quantity of goods *can* satisfy needs—is of course only *one* aspect of the concrete use value; the second is the intensity of the need which stands opposite it. The greater [*sic*] the former and the greater the latter—we see—the greater is the use value. In our case the relationship is *such that the decrease [sic] in the capacity to perform of the available quantity of goods is surpassed—is more than compensated—by a greater intensification of the need!* Then the effective use *value* of the smaller quantity is greater than that of the larger quantity.

The demand for a good may just as well be expressed in efficiency units; that is, it is the *effect* which the consumer is presumably interested in. The concrete use value (per efficiency unit) of good j to the community as a whole may then be expressed as $\theta_j Y/\nu_j x_j$. However, since there is no difference between a 10% increase (or decrease) in ν_j and a 10% increase (or decrease) in x_j, the discussion of "capacity to perform" seems irrelevant to this argument. Now, from the formula $\theta_j Y/\nu_j x_j$ for the concrete value of good j expressed in efficiency units, it is clear that an equal percentage increase in the numerator and denominator will have no effect on the concrete value. I conclude that Knies must have meant "the *smaller* the former" in the second sentence of the above passage—"the former" being $\nu_j x_j$ and "the latter" θ_j. This is confirmed by the third sentence, since only when the "need" θ_j increases will the concrete use value, $(\theta_j Y/\nu_j x_j) \cdot \nu_j x_j = \theta_j Y$, of the *entire harvest* of grain, $\nu_j x_j$ (measured in efficiency units), increase as $\nu_j x_j$ decreases. This is because the "concrete value" of the *entire* harvest is just its species value. And it is enough that θ_j (the "need", or equivalently, the species value) simply increase. This interpretation seems confirmed by the passage following the above-cited one (p. 457):

> Were the need for the consumption of bread-grains in a year of dearth manifested only just as strongly as in a bountiful year, then the smaller total quantity would have only *the same* use and exchange value as the larger one in the abundant year; ... Yet this is *not* the fact of the matter. For the common man a more intense need is expressed forthwith; how greatly it has grown we can see from the fact that, conscious of his needs for cereal-grains, he will make room for a far greater percentage of them in the totality of his needs.

Empirically this is no doubt correct; that is, it conforms to Cordier's data. But in the course of his argument Knies has been forced to modify the model he used to tackle the "first stumbling-block", and express θ_j as a function of the quantities $x_1, x_2, \ldots, x_n$, or indirectly as a function of the n prices and income. For example, as p_j/Y increases during a dearth, so must θ_j increase if the value of the total harvest is to increase.

Despite this criticism, it must be said that Knies had the great virtue of confronting his theory with historical data. Not till many decades later could one find a performance as impressive as his.

7 Roscher, 2

Roscher, in the second edition of his treatise (1857, §4, note 1, p. 6) cited Knies's 1855 article, particularly approving his sharp differentiation between use value [*Gebrauchswerth*] and usefulness [*Brauchbarkeit*], as well as between exchange value and capability of being exchanged. In §5 (p. 6) he withdrew the two sentences that Knies had criticized to the effect that use value and exchange value could move in opposite directions. And undoubtedly greatly stimulated by Knies's work, especially the latter's incomplete example of bread and lace, he introduced an important application of the Rau-Hildebrand-Knies analysis to the Smithian "paradox of value" (§6, pp. 7–10):

> Recently reference has been made, especially on the part of the socialists [e.g., Proudhon], to the dubious "contradiction" that is said to exist between use value and exchange value. A pound of gold, it is said, has a much higher exchange value than a pound of iron; and yet the use value of a pound of iron is said be altogether incomparably higher. I dispute the latter. Without question iron has a much higher species-use value than gold: in other words, the need for iron is much more urgent and universal than the need for gold. Despite this, a much greater proportional amount of the need for gold can be satisfied with a pound of gold than can the need for iron be satisfied with a pound of iron. Many a farmer uses annually 100 pounds of iron, while his requirement for gold for his entire life is covered by less than half an ounce in the form of two wedding rings. Now to be sure, these wedding rings are, for him, just as important as any iron tool of a thousand times the volume. Let us imagine that a people would annually require 10 centners of gold and $2\frac{1}{2}$ million centners of iron; then in the first case each individual centner would cover $\frac{1}{10}$, in the second case only $\frac{1}{2{,}500{,}000}$, of the total requirement. If it were then possible to compare the species value of the two metals precisely, and if, accordingly, that of iron were to turn out to be 10 times as great as that of gold, a single pound of gold would still have a concrete use value 25,000 times as high as that of a single pound of iron. This is currently the approximate relation of their exchange values.

This may be given a precise interpretation as follows. The unchanged "need" on the part of a farmer for good j may be identified with the proportion, θ_j, of his income spent on good j (consistently with Hildebrand's and Knies's usage); the "requirement" of good j may be identified with the *demand* for it, x_j (as given by (4)). Roscher assumes that

the need for iron is very much greater than that for gold, specifically, $\theta_{\text{iron}} = 10\theta_{\text{gold}}$ (e.g., $\theta_{\text{iron}} = 0.01$ and $\theta_{\text{gold}} = 0.001$); on the other hand, the "requirement" of gold is $x_{\text{gold}} = 10$ centners, while that of iron is $x_{\text{iron}} = 2{,}500{,}000$ centners. In accordance with formula (4), the "concrete values" of gold and iron are related by

$$\frac{p_{\text{gold}}}{p_{\text{iron}}} = \frac{\theta_{\text{gold}}/x_{\text{gold}}}{\theta_{\text{iron}}/x_{\text{iron}}} = 25{,}000.$$

On this analysis, cited approvingly by Knies (1873, pp. 125–6; 1885, p. 162), Block (1890, I, p. 135; 1897, I, p. 154) remarked: "This is very ingenious, but is it the best of explanations?" It was certainly bettered by Jevons (1871, p. 62); and presumably Gossen (1854) might have been able to provide a better explanation had he considered addressing the "paradox of value". But I am not aware of a more precise and logically satisfactory explanation of this paradox than Roscher's prior to that of Jevons.

Thus Rau (1847) and the three founding members of the older historical school, Hildebrand (1848), Knies (1855), and Roscher (1857), developed—although the latter three under Hildebrand's special assumptions concerning consumer preferences—the essential ideas of the marginal revolution later associated with the names of Gossen (1854), Menger (1871), and Jevons (1871).

8 Schäffle

Albert Schäffle was one of the most prolific economic (and sociological) writers of the period, was a professor at Tübingen and Vienna (preceding Menger), and served as Minister of Trade in the Austrian government. A very useful summary of his accomplishments is contained in Fabian-Sagal (1909).

In the first edition of his textbook Schäffle (1861a, Ch. 11, §52, 2, p. 92) followed Rau and Hildebrand in employing the distinction between abstract and concrete value:

> Use value is so-called abstract or species value if the average importance of each item of a species is, accordingly, taken into consideration abstractly; it is so-called concrete use value when one has in view the importance of a single object of a species in its economic application in a particular actual case.

He went on, however, to make a statement of a kind that he was to repeat quite often, and which seems to display a peculiar misunderstanding of the Rau-Hildebrand theory (1861a, §52, 3, p. 92):

> *A good has that much more value the greater and more intense is the requirement for it, and the smaller and weaker ... is the available total supply of the type of good.*

What is peculiar about it is that unlike Rau's formulation, in which the "total supply" refers to the amounts already held by the individual (see footnote 24 above), in Schäffle it appears to refer to the amount available on the market. We shall have occasion presently to return to this important distinction.

Schäffle went on (1861a, Ch. 12, §53, pp. 96–97) to quote (approvingly) the above-cited passage from Hildebrand criticizing Proudhon's antinomy, and to develop an argument similar to Roscher's (1857) (but evidently quite independently of Roscher) to explain the paradox of value (p. 97):

> When one often hears it said that many things—such as water—have high use value and slight exchange value, this is based on the same confusion between usefulness and the amount of use value. All water is capable of use [*nutzfähig*] in the highest degree. This, however, is value for use [*Werth für den Gebrauch*]; use value [*Gebrauchswerth*] is possessed by a particular quantity (despite higher objective capability of use) only insofar as there is not in practice an unlimited quantity at one's disposal [*zur Verfügung*] .... Insofar, however, as the quantity at one's disposal [*die verfügbare Quantität*] is practically infinite (∞), it results from the theorem that use value stands in inverse proportion to the total quantity of the good, that the amount of use value of one quantity ($= 1$) is $\frac{1}{\infty}$, i.e. 0. Such is the case with light, air, etc. So long as it is generally not actually related to human economic purposes, one can at best speak of the potential average importance of the species (the so-called *abstract use value*).

This is an unexceptionable statement in all respects: reference is made to the *quantity at one's disposal*, as opposed to the "available total supply". Note that the "theorem" referred to by Schäffle corresponds to formulas (6) and (10) above.

The following year saw the publication of Schäffle's important Inaugural Address as professor of political economy at Tübingen University, "The Ethical Aspect of the Theory of Value in National Economics" (1862). By the "ethical aspect" of the theory of value Schäffle understands the role of individuals with their free will in acting "with a consciousness directed at the complete fulfillment of a morally rational purpose of life" (1862, p. 7), which we may perhaps interpret simply as their optimizing behavior,[68] which he describes (following Hermann, 1832, p. 3), as "economizing" [*Wirthschaft*]. Thus (1862, p. 10):

> *Usefulness is the objective aspect; the value of the good is the subjective aspect.* Usefulness (serviceability, utility) is a thing's fitness to serve a human purpose. ... Value, however, is the importance a good possesses through its usefulness for the economic consciouness of purpose of an economizing individual. Value is thus the ethical aspect of the good .... To economize is to create value.

This provides at best a qualitative definition of value; however, he does proceed to discuss the factors that can cause value to rise or fall (p. 13):

> The more urgent is the personal need for a good and the more difficult it is to obtain this needed good, then the more energetically the activity of economizing will be stimulated. The more these two factors—intensity of desire (demand,

[68] Schäffle developed this theme in a preceding work (1861b, p. 240): "By ethical (ἔζειν, to set), moral (to set) orientation in the broadest sense we understand the realization ('setting') of rational life purposes by acts of human will", which does not get us much further. He also states (p. 236): "Not the acquired, or to be acquired, *good*, the *chrema* (χρῆμα), but rather *man* (*anthropos*, ἄνθρωπος) must be placed at the center of national economics", etc.

need, etc.) and the intensity of the difficulty of procurement (scarcity, lack of supply, etc.)—affect one another, the more important and meaningful this good becomes in the consciousness of economic activity.

Note that in its reference to "lack of supply" this does not state that if an individual's *own supply* of a good diminishes, its use value will increase; the implication seems instead to be that if the good becomes more scarce, i.e., the amount available on the market declines, then the "difficulty of procurement" (the price?) will increase. Until it is explained how scarcity "stimulates" the consumer to attach a higher value to the good, the cause has still not been accounted for.

Referring to Roscher's gold-and-iron example (section 7 above), Schäffle provided the explanation (1862, p. 29) that "that good [whose procurement] would occasion a greater amount of effort (labor, cost, etc.) is more valuable in its particular amount for use." This is quite different from Roscher's explanation, which was that there was a greater quantity of iron required for consumption than of gold. In referring to Knies's explanation for the decline in the use value of grain following a larger harvest, Schäffle (1862, pp. 29–30) stated: "This is also easy to explain: The intensity of the need, the main subjective element of value, is much greater in times of dearth, although an equivalent amount of grain of the same quality would be sufficient for the consumer's satisfaction. Similarly, the effort required to procure the same quantity is greater in times of dearth as compared with times of plenty." The second of these is quite different from Knies's own explanation.

Schäffle's ambivalence regarding the explanation of value is illustrated in the following passage, in which the first sentence provides two alternative explanations, the second of which reverses the direction of causation of the first (1862, p. 35):

> A scarce good is of great value because it costs a great deal of labor in order to fill the range of its entire existing requirement, or because much labor is devoted to it owing to its scarcity which therefore makes it valuable. ... the central point ... is that owing to its scarcity and the required economic exertion, the importance of an object is especially strong in the economic consciousness of the economically acting subject.

How it is that "scarcity stimulates the consciousness of value whereas plenty leaves it in slumber" (p. 36) still remains unexplained.

Schäffle made much of the distinction (and properly so) between what he called "objective usefulness" [*objective Brauchbarkeit*] (1862, pp. 32–33) and use value (1862, p. 27):

> Usefulness in the sense of utility, and use value, are ... not identical concepts. Usefulness is the general serviceability of a thing for people, while use value is the utility [of a thing] consciously valued by people in their economic activity.

However, this definition of use value does not explain its quantitative aspect: why one use value should be greater than another. As an illustration Schäffle contrasted the unconscious breathing of air with the conscious breathing of a trumpeter (p. 9). But if I consciously eat a second chocolate, there is nothing in Schäffle's definition to distinguish this quantitatively from my conscious eating of a third one; or indeed, from my conscious decision to resist the temptation to eat a fourth one. Yet the latter assuredly has no "use value".

In commenting on Hildebrand's and Knies's principle of diminishing marginal utility Schäffle nevertheless provided the following very interesting observations (1862, pp. 30–31):

> The progressive decline—and not the increase—of exchange and use value of equal quantities of goods is the triumph of economizing civilization; because it is the expression of the idea that the more man is emancipated from matter, the less valuable this becomes for him. Therefore, the observation that with a reduction in a quantity of goods, the use value of the remaining quantity may increase, characterizes indeed in each occurring case a deplorable misfortune to be resisted on all accounts. However, it does not demonstrate disharmony in the economic world order; because the task of economic behavior and the token of wealth or success of the progressive, economic conquering of the external world is found not in the rise of use value (although in the rise of usefulness) and of exchange value of equal quantities of goods, but rather in its fall. The misfortune itself is found in the increasing value of that which remains of a quantity of goods after being reduced. To counteract this condition is the goal of economically social institutions.

This indeed is a more felicitous (even if less precise) statement than that provided by what I above called Hildebrand's "law of conservation of value shares".

In his review of Mannequin (1863), Schäffle (1864, p. 561) discovered Turgot's definition of "esteem value" (*valeur estimative*) which Mannequin (1863, I, p. 31) had quoted from Turgot (1769; 1844, I, p. 87; 1919, III, p. 92; 1977, p. 144):

> ... the esteem value of an object, for the man in isolation, is none other than the ratio between that part of his resources [*facultés*] which a man can devote to the quest for this object and the totality of his resources. ...[69]

This is a much clearer definition than the one quoted by Friedländer (1852, p. 21) (see footnote 57 above), since the same units (of "resources" or "faculties" [*facultés*]) are used in both the numerator and denominator, which makes clear the resemblance to Proudhon's concept of value and to Hildebrand's and Knies's interpretation of Rau's species value. However, Schäffle did not notice this connection.[70]

The second edition of Schäffle's textbook (1867) quotes extensively both from the first edition (1861a) and the Inaugural Address (1862). It starts a kind of analysis that he was to emphasize increasingly later on, namely the balancing of "use value" with "cost value", although the latter term does not seem to have been introduced until 1870 (see footnote 76 below). Thus, (1867, II, §17, pp. 51–52):

> ... economic value has its basis partly in needs, the non-satisfaction of which is privation, and partly in the sacrifice which the attainment of the external means of satisfaction costs in the system of social satisfaction of needs. ...

[69] I have substituted "ratio" for "relation" in Groenewegen's translation of Turgot's *rapport*.

[70] See also Schäffle (1873, I, §100, p. 173) where this definition of esteem value and the similar definition of value supplied by Proudhon are discussed and compared, but again without reference to Hildebrand's similar particularization of Rau's concept, even though Hildebrand's concept was derived from Proudhon. See footnote 51 above.

> Non-satisfaction of the person is negative sacrifice, acquisition of the thing, positive sacrifice.

However, while there is discussion of the fallacy of Proudhon's antinomy (Ch. IX, §54, 4, pp. 121–122), what is most noteworthy for us is its complete omission of any mention of Hildebrand's analysis and of the second portion of the passage from the first edition quoted above dealing with abstract and concrete value. He "quotes" from the first edition as follows (p. 122):[71]

> "When one often hears it said {(Rau)} that many things—such as water—have high use value and slight exchange value, this is based on the same confusion between usefulness and the amount of use value. [All] water is useful [*Brauchbar*] in the highest degree, {but is frequently without value}. This, however, is value for use; use value is possessed by a particular quantity [...] only insofar as there is not in practice an unlimited quantity at one's disposal," {in other words, economic sacrifice is required for its acquisition.}

Note that the added explanation is illogical, since unless a limited quantity at one's disposal itself implies a high use value, there is no incentive to make a sacrifice to acquire more of the good. And the passage as a whole appears to imply that Rau was "confused" as between usefulness and use value, whereas on the contrary it was Rau who made the careful distinction between "usefulness" (which he called abstract or species use value) and "use value" (which he called concrete or quantitative use value).

In 1870 there appeared two notable contributions, closely related in content: Lecture III of his *Capitalism and Socialism* (1870a), and his review article (1870b) of the second edition of Hermann's work (1870). The two contributions are quite closely related. The first, whose declared purpose (1870a, p. 29) is to overcome the "neglect of a clear theory of value" in "bourgeois national economics", by presenting "the correct concept of value" so that students will not be seduced by the "captivating theories of value" offered by the socialists Proudhon and Marx, goes on to provide a lucid exposition for students of the "calculus of pleasure and pain" (1870a, p. 32) involved in equating use value and cost value. Since this material is also covered in his article (1870b), I shall concentrate on that.

This review article (1870b)[72] is a lengthy critique of both editions of Hermann's work, liberally combined with the author's own thoughts. As a critique of Hermann it is difficult to assess, since Schäffle rarely provides page numbers (or even the edition) of the passages he quotes. It is respectful but mostly critical of Hermann. The following passage is interesting in that it reveals what was absent in Schäffle's 1862 work: an explanation of the process by which a shortage "stimulates the consciousness of value" (1870b, p. 149):

> ... not only the satisfaction capability corresponding to the "need", and the *degree* of this capability, but also the *quantity* of available commodities—the

[71] I indicate with braces words or phrases added in the second edition to the passage in the first edition, and by bracketed English words or bracketed ellipses [...] passages expunged from the first edition; changes in wording from the first edition are indicated by the German wording in bracketed italics.

[72] The occasion for this article, as explained by Schäffle in the opening footnote, was the appearance of the jubilee issue of the *Zeitschrift für die gesamte Staatswissenschaft* (of which Schäffle was editor), together with the failure of a number of promised articles to arrive in time for the printing deadline. The hurriedliness of the article is quite evident from its content. The article promised a sequel which apparently never appeared.

> supplies—influences the use value, although in inverse proportion to the amount of the requirement. The larger the supplies that are known to the consumer, the less urgent is his demand, with the same degree and extent of need [*Bedürfens*]. Hermann himself states: "An increase in scarcity will simply increase the competitiveness of the demander."[73] One can observe this with quantity variations of, for example, foodstuffs, due to poor harvests, and has always done so. The degree of the need, the requirement, and the suitability of the good may have remained entirely the same, nonetheless a much more intense scrambling will take place around the smaller quantity; equal amounts of goods have, as is shown here, become higher in use *value*.

Note that Schäffle does not state that "the larger the supplies *that are in the possession of* the consumer, the less urgent is his demand" but that "the larger the supplies *that are known to* the consumer ...." And the explanation provided for this is that there will be less scrambling for these supplies when they are large, and more when they are small. But no explanation is provided for the scrambling, or the lack of it, itself. This could only be because people *expect* the price to fall when supplies are large, and *expect* it to rise when supplies are small.[74] This, then, is not a theory of *value*, but a theory of *speculation*! And the theory assumes what is to be proved! It seems, therefore, from this passage, that while Schäffle accepted the results of Hildebrand and Knies, he failed to understand their analysis!

In this article (1870b, pp. 150–151) Schäffle briefly discusses Rau's dual concepts of value. With regard to Rau's concrete value he makes the strange comment: "Rau has probably suggested and named this concept, but does not pursue it. Otherwise he could not refer to his 'abstract use value' as use value" (p. 150n). Then he goes on with the following passage, the second sentence of which reiterates even more clearly his misperception that what is

[73]This comes from Hermann (1870, pp. 401–2). The complete passage (which is a rewording of the corresponding passage in the first edition (1832, pp. 70–71)), reads as follows:

> If the good is of very great and widespread use value, and is difficult to replace, such as foodstuffs are to the greater population, and if it is impossible to suppress the need—indeed, even difficult just to limit it—then an increase in scarcity will simply increase the competitiveness of the demander, which can cause the price to rise without limit. Conversely, amid a surplus of it, the demand for an essential commodity does not increase nearly as much as that of a dispensable one, because even during a period of scarcity one cannot significantly limit its consumption.

Schäffle apparently overlooked the important sentence which immediately followed this passage in the 1870 edition (but which was not present in the 1832 edition), p. 402:

> It is of great overarching influence whether the consumption of larger quantities of a product provides enjoyment. With clothing this is the case to a high degree, with beverages, up to a rather narrow limit; with foodstuffs and wood, the increase of consumption beyond the point of satiation is even more limited; in household consumption, table salt surely allows little increase.

This idea—forming the rudiments of a principle of diminishing marginal utility—seems to come much closer to a correct explanation than that provided by Schäffle.

[74]This would be the case in a market economy, in which the only way information is transmitted to the consumer concerning the increased scarcity of an article is through its higher price. However, Schäffle (and Hermann too) seems often to reason as if one were discussing a small primitive economy in which there is perfect information about available supplies.

of relevance to the consumer is not the supply of goods *in his possession* but the supply available "on particular markets" (p. 151):

> ... use value is not the "abstract" judgment of a general ability to satisfy, but rather the measure of importance for a concrete-practical desire. Only with regard to the supply of goods on *particular markets* and at *particular points in time* is use value present as a particular amount. It has often been emphasized by many authors, and early on, that for price formation only supply and demand on a particular market and at a particular time are decisive. But this has not happened with the concept of use value, which has remained much too abstract even with Rau, despite his distinction between concrete and species-based use values.

This is said of the author who introduced supply-and-demand curves! In fact, Schäffle's concept of "usefulness" could be interpreted as corresponding to Hildebrand's interpretation of Rau's species or abstract use value, and his concept of "use value" as corresponding to Knies's interpretation of Rau's concrete use value[75]—except that Schäffle forgot that Rau's "concrete value" was a *quantitative* value, a function of the quantity consumed. It is of interest that in the above passage, Schäffle mentions "the supply of goods on *particular markets* and at *particular points in time*" but does *not* say "*in particular amounts*". Evidently Schäffle objected to the use of the word "value" in describing usefulness, but this descends from the tradition of Quesnay (1767), Turgot (1769), and Adam Smith (1776), and is not the fault of Rau.

Schäffle continues in his 1870 article with a development clearly influenced by Hermann, as well as by Senior (1836, p. 139; 1850, p. 26), of the equilibrium between use value and "cost value". He expresses this in the formula $g - k = 0$ where g is the use value (*Gebrauchswerth*) and k the cost value (*Kostenwerth*) (pp. 153–155).[76] This is preceded (pp. 140–141) by a discussion (again influenced by Hermann) of the distinction between the "perception of the sacrifice of labor" needed to procure a good (denoted aO for *Arbeitsopfers*) and the "perception of the sacrifice of privation" in going without the good (denoted eO for *Entbehrungsopfers*), which quantities he states must be equal in equilibrium. Now presumably the latter concept (eO) is the marginal utility of a commodity in a negative sense, that is, the marginal disutility of being deprived of a unit of it; then eO is presumably the same as g. The former concept (aO) is presumably the marginal disutility of labor; but this cannot be equal to the "cost value" k or marginal cost of the product unless it is divided by the marginal productivity of labor in producing the product (see the first equation in formula (18) below). But Schäffle does not provide a clear idea of what he means by "cost value", either in this paper (1870b) or in his book (1870a). He had defined it (1870b, p. 134) as "the amount of personal sacrifice which the good represents"; while this takes account of the labor *effort*, it takes no account of the *productivity* of labor in producing the good.

[75] According to such an interpretation, and in terms of the Hildebrand-Knies version of Rau's theory, the "usefulness" of good j would be the constant expenditure share θ_j, and the "use value" the average of this share over the consumer's holdings, θ_j/x_j. We shall see later in section 13.3 that this in fact was Wieser's interpretation. Note that this would also identify Schäffle's "usefulness" with Hildebrand's and Knies's "need", and his "use value" with marginal utility (see footnote 66 above).

[76] He states (pp. 155–6): "In my *Social System* I have set forth this difference as the decisive one." However, I have been unable to find this formula, or even the expression "cost value" (*Kostenwerth*), in Schäffle (1867).

The third and last edition of Schäffle's textbook (1873) resumes the discussion of the balance between cost value and use value (I, §95, pp. 166–168), saying that "economic deliberation is directed towards highest utility and least cost". Here he states (I, §90, pp. 160–161): "The pain of having to work, or having to do without, and the pleasure of effortlessly having something at one's disposal, and enjoyment, are magnitudes, primarily of internal valuation and perception, which are comparable and weighable against one another." Then (§95, p. 167): "the practical task of economics [is] the most beneficial balancing of minimal sacrifice and pain, and maximal satisfaction of the basic needs of life", and, in a statement which finally takes account of the *quantities* of commodities (p. 168): "We define *private-economic value* ... as the quantitative relationship of utility value [*Nutzwerth*] to cost value of a particular amount of commodities for a particular person at a particular place and at a particular time."

Some glimpses of diminishing marginal utility are present. Despite the statement (§99, p. 172): "A barrel of oil with 10 hundredweights of oil has twice as much use value as one with 5 hundredweights of oil", we later have (§100, p. 174):

> I can absolutely require a hundredweight of flour, and require a second one much less, and may no longer be able to pay for it.
>
> Depending upon differences in income (the ability to pay), the same units of goods have very dissimilar effective use value for different persons.
>
> Truffles and champagne have no effective use value for proletarians, while possessing a very high one for a wealthy retiree.

Schäffle does not provide a very satisfactory definition of "effective use value",[77] but assuming the expression to mean the marginal utility of a good divided by the marginal utility of income (i.e., the demand price for the good—see formulas (2) and (10) above), and assuming the latter to be decreasing in income, then if proletarians and wealthy retirees have similar tastes, this provides a good explanation for the third of the above statements.

The chief interest for us is his discussion of abstract and concrete use value. He states (1873, I, §97, p. 171):[78]

> ... the "abstract use value" is only a theoretical motivation for the "concrete" use value. Considered economically, only *concrete* use value exists; all economic value is *concrete* use value ...

[77] His definition reads (1873, §100, p. 173): "the being of value [*Werthsein*] [of a thing] for the use of a person who is inclined to give practical effect to his perception of use value through the sacrifice of his labor or wealth up to a particular amount, beyond which the cost value would outweigh the use value in his comparative perception of value." This was reproduced verbatim from his previous definition of Rau's "concrete use value" in Schäffle (1870b, pp. 150–151). Thus we may identify Schäffle's "effective use value" with Knies's "proportionate use value" which Knies in turn identified with Rau's "concrete use value"—both special cases of Rau's "value to the buyer"—formula (2) above. The expression "effective use value" was also employed by Knies (1855, p. 457).

[78] He also misquotes Roscher by saying: "Roscher believes that a pound of iron would have less use value than a pound of gold because the latter has higher concrete usefulness than iron" (p. 170). Roscher never used the expression "concrete usefulness"; rather, he used Rau's terms "abstract" or "species use value" and "concrete" or "quantitative use value". Besides, if the correct expression "concrete use value" is substituted for "concrete usefulness" in Schäffle's statement, it becomes a mere tautology. Roscher's reasoning is not supplied, and one can only suppose that Schäffle was unable to follow it.

then refers to Schäffle (1862). He repeats (p. 171): "We know no *abstract*, but rather only *concrete* economic value."[79] This is fair enough; indeed, Rau himself stated (1841a, §147, note (*a*), p. 154), as we saw above, that in the theory of price only concrete value is relevant. But with their "theoretical motivation", Rau, Hildebrand, Knies, and Roscher were able to make more headway in explaining market prices than Schäffle succeeded in doing, because they understood that the defining property of "concrete value" was its relation to the quantity consumed.

9 Michaelis

In the third edition of his textbook Schäffle complained quite bitterly regarding his concept of value (1873, I, §91, p. 162): "This conception of value has already been emphatically asserted in my monograph ... (1862). In view of repeated flagrant and disguised plagiarism, I draw attention to the ideas of this monograph." It is not hard to infer that the object of his ire was the paper by Michaelis (1863), which appeared one year after the publication of that monograph, and was reprinted in the same year as the third edition of Schäffle's textbook, and quite likely had already appeared before the latter had gone to press.

Let us, however, analyze Michaelis's contribution on its merits.[80] It is essentially a model of a Crusoe economy, developed in order to derive a concept of value independently of and prior to exchange (cf. Michaelis 1863, p. 2; 1873, pp. 239–240). It is consistent with a model in which "economic man" maximizes a quadratic utility function

$$(16) \quad U(x_1, x_2, \ldots, x_n, l) = \sum_{j=1}^{n} c_j \left(x_j - \frac{x_j^2}{2b_j} \right) - l - \frac{l^2}{2a} \quad (0 \leqq x_j \leqq b_j;\ a, b_j, c_j > 0),$$

where x_j is the quantity of the jth commodity consumed and l the amount of labor supplied by economic man for production, subject to

$$(17) \qquad x_j = f_j(v_j), \quad \sum_{j=1}^{n} v_j = l,$$

where v_j is the amount of economic man's labor allocated to the production of commodity j, and f_j is the (increasing and strictly concave) production function for commodity j. He adopts an unabashed numerical measure for reckoning the excess of satisfaction over the

[79] Here we may also record the views of Stein who stated (1858, p. 39): "Rau's distinction between *concrete* and *species value* §. 57.*d* is simply the distinction between the utility [*Nützlichkeit*] of the individual object and the average usability [*durchschnittlichen Nutzbarkeit*] of each object of the same species." This is quite different from Rau's own definitions! Stein defined the "usability" of a good as its "capability to serve the purposes of another good" (p. 38)—a rather circular definition. This discussion of the theory of value was entirely omitted from the second edition of Stein (1878).

[80] Michaelis was described by Lambi (1963, p. 24) as "one of the leading free trade theorists of Germany". His two-volume collection of papers (1873), which includes "The Topic of Value" discussed here, contains theoretical papers on paper currency and credit, applied papers on speculative transactions and business cycles, and many papers on railroad rates, but none on trade. According to Hentschel (1994), Michaelis's career was largely political; he was "one of the noteworthy spokesmen of the informal free-trade party", although less dogmatic and more pragmatic than his mentor Prince-Smith. Thus he could be regarded as a free-trader but not a free-trade *theorist*.

effort expended in attaining it (1863, pp. 2–4; 1873, pp. 240–243), and employs a peculiar terminology (1863, p. 8; 1873, p. 247), calling the commodities "usefulnesses" (*Brauchbarkeiten*), or sometimes "usabilities" (*Nutzbarkeiten*) or even "utilities" (*Nützlichkeiten*).[81] The reason for this seems to be partly that he sometimes wishes to measure the quantities of these commodities in Rau's efficiency units (calories, thermal units, etc.); I shall consider an example of this below.

We find a passage containing an explicit enunciation of the principle, or "law" as he calls it, of diminishing marginal utility, although stated negatively rather than positively (1863, pp. 12–13; 1873, pp. 252–253):

> It is a law founded on the nature of man that a diminution, by equal quantities, in consumption corresponding to a given need, imposes ever greater pain of privation. Thus if I require 40 bushels of grain and possess only 20, then by the acquisition of the 21st I will spare myself greater pain than if I require only 30 bushels and possess 20. Therefore in the first case I will be inclined to expend greater effort than in the second one.

To follow the explanation given in the second sentence of this passage we may consult the quadratic utility function (16), where the parameter b_j may be interpreted as the "requirement" (*Bedarf*) of commodity j, also interpreted by Michaelis as the "need" (*Bedürfnis*) for it. Let us suppose that $b_{\text{rye}} = 40$ and $b_{\text{wheat}} = 30$, and that $x_{\text{rye}} = x_{\text{wheat}} = 20$; then, evaluated at these latter consumption levels, we have $\partial U/\partial x_{\text{rye}} = \frac{1}{2} > \partial U/\partial x_{\text{wheat}} = \frac{1}{3}$, confirming Michaelis's statement.

Michaelis goes on to consider the relation of "value" (marginal utility) to "need" and to the productivity of labor. He expresses this in a formula (1863, p. 14; 1873, p. 254):

> Thus the value of a usefulness rises with the need, rises with the labor necessary for its production, and declines with an increase in the productivity of human labor. If we wish to express the mutual interaction of these factors in a mathematical formula, then the value-increasing aspects, by their intensification of the elements which increase value, are to be interpreted as multipliers, and by their intensification of the elements which reduce value, as divisors. If we therefore stipulate the aggregate of all usefulnesses of the type which a need requires as $= B$, and the aggregate of all usefulnesses of the same type which a day of labor produces as $= S$, then the value formula is: B/S. If B, the need, rises, then the value rises; if S—the productivity of labor with regard to the usefulness to be valuated—rises, then the value declines. If S declines, so that in other words the labor necessary for the production of B increases, then the value increases as well.

A precise interpretation of this result may be obtained by carrying out the maximization of (16) subject to (17):

$$(18) \qquad \frac{\partial U}{\partial x_j} = \frac{-\partial U/\partial l}{\partial f_j/\partial v_j} \quad \text{where} \quad \frac{\partial U}{\partial x_j} = c_j\left(1 - \frac{x_j}{b_j}\right) \gtreqless 0, \quad \frac{\partial U}{\partial l} = -1 - \frac{l}{a} < 0.$$

[81] See footnote 79 above. Michaelis did not, however, avail himself of the word *Brauchlichkeiten* coined by Zachariä as the German equivalent of the English "commodities"; see footnote 60 above.

Thus, if the "need" b_j (corresponding to Michaelis's B) rises (this is of course an exogenous change in tastes), the marginal utility of commodity j rises; and if the productivity of labor rises, i.e., if there is an exogenous increase in the marginal productivity of labor $\partial f_j/\partial v_j$, then the marginal utility of commodity j falls; and if the amount of labor required to produce commodity j rises, then the marginal productivity of labor in the production of commodity j falls, hence the marginal utility of commodity j must rise. Thus we may interpret Michaelis's parameter S, which he calls the "productivity of labor", as in fact the *marginal* productivity of labor. That he could have arrived at this formula without the above formal optimization procedure must be considered a very impressive achievement.

Of special interest to us are Michaelis's comments on Rau's "species value" (1863, pp. 16–18; 1873, pp. 257–259):

> Value is a property which is assigned not to things but to the usefulness represented in them in comparison with other usefulnesses; it is not objectively attached to the usefulnesses; it is, rather, of a subjective nature, that is to say, dependent upon the opinion (valuation) of economic agents. There is, therefore, no absolute but only relative and individual value. An individual thing, as an aggregate of usefulnesses which serve particular needs, has an individual value for a particular economic man. If one wanted to speak of "species value" in the objective sense, then, since the number of members of a species is not limited, the element of need—always limited—goes unconsidered. In contrast, of course, economic man assigns the same value to equal quantities of the same usefulness at each moment, and only in this relative sense may one speak of *species value*. But this is not species *value*, but rather species *usefulness*. Of two goods which are suited to serve the same need, the one which, relative to this need, unifies the greater sum of usefulness in itself therefore has the greater value, because the magnitude of the attribute "value" is dependent upon the extent of the object's usefulness. A bushel of wheat has, for the same reason, more value than a bushel of rye; ....[82] The relative value of equal quantities of rye and wheat corresponds to the ratios of their usabilities for [meeting] nutritional needs.[83] If one wished, however, to name this value relationship species *value*, then one would confuse value with usability;[84] for in this comparison one measures only the usability which is simultaneously valuable. Since, furthermore, nutritional needs are not alone decisive, but rather, simultaneously the need for good flavor, ease of digestion, and so forth also enter in, that abstraction upon which Rau bases his concept of species value leads to numerous errors in its application to concrete cases.[85] If we speak figuratively of the values of things themselves,

[82]Here, there appears to be a slip in Michaelis's text, uncorrected in the 1873 reprint. He goes on to say: "consequently two bushels of wheat have more value than one bushel of wheat." He must have meant to say: "consequently two bushels of wheat have more value than two bushels of rye." This is confirmed by the sentence in the above text that immediately follows.

[83]Of course, this is just the Rau-Knies formula (11) above.

[84]Conceivably Schäffle might have regarded this statement as influenced by his monograph (1862); Michaelis is presumably objecting to describing the *right* side of equation (11) as a relative value (Knies's relative subspecies value) when this clearly is the correct term for the *left* side.

[85]This criticism had also been raised by Roscher (1854, §4, p. 5; 1856, §4, p. 5, etc.): "Thus, for example,

> what we mean is the usefulnesses for our needs adhering to the things, multiplied by their values. When different economic agents who are not in an exchange relationship with one another or the outer world assign an approximately equal value to things of the same species, then the reason can only be found in the fact that under the same production and need conditions, they reckon the elements B and S more or less equally. A *unit value* based upon this can, for a certain time [and] within a certain area, lay claim to a certain objectivity, similarly to individually paid prices as compared with the market price. A historical manifestation of such an assumed unit value is found in the manbotes [*Wergelde*][86] of the ancient Germanic tribes, in the fines which were originally paid to libel victims, and similar estimates of the values of objects which are not subject to exchange even though they represent the result of expended effort, costs, and privations.[87]

I wish to draw attention to the sentence beginning "If we speak figuratively ..."; this appears to allow a single thing to have several different "usefulnesses" adhering to it. But this is just the "new approach" to the theory of consumer demand introduced by Lancaster (1971)! Michaelis's "usefulnesses", which are the same as Friedländer's "need-units" and Knies's "subspecies values", are in turn the same as Lancaster's "characteristics"![88]

Michaelis's analysis in this paper is not limited to stationary equilibrium, but is extended to capital and growth. Since there cannot be a stock of satisfaction upon which one can draw in the future, one has to produce and store durable means of satisfaction. Thus production requires division of labor over time; it also requires, along with human effort and the contribution of nature, intermediate goods which Michaelis also describes (1863, p. 8; 1873, p. 247) as "usefulnesses"and "usabilities". The only thing missing is a concept of time preference.

While undoubtedly there are evidences of stimulus provided by Schäffle (1862), it seems equally evident that a considerable influence went subsequently also in the opposite direction, since Schäffle's formulation in terms of the interaction between "use value" and "cost value" only began to be systematically developed in his two works of 1870.[89] Formula (18) expresses Schäffle's "cost value" of commodity j as the ratio of the marginal disutility of labor to the marginal productivity of labor in the production of commodity j. But I have

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the nutritive power of various dishes may readily be calculated, but not the quality of the taste, the pleasure to the eye associated with them, etc."

[86] According to Lyon (1960, 1980, p. 83), "Primitively all the German tribes based their organization on the kindred (*maegth*). ... Before artificial protections were devised, the individual secured his protection and rights through the kindred. Should he be slain it was the responsibility of the kindred to avenge his death through feud against the slayer and his kindred. In lieu of such vengeance it became acceptable for the slayer and his kindred to pay compensation to the dead man's kindred. This was the wergeld (man-price)." (I am much indebted to my colleague Bernard Bachrach for this reference.)

[87] Five years later there appeared a fascinating study by Inama-Sternegg (1878) of the judicial and quasi-judicial determinations of value in Roman and Mediaeval law, in their relationship to Rau's species value and Hermann's "general" (later "universal") use value (1832, p. 69; 1870, p. 108). In particular he discussed (pp. 203, 210–217, 223) the manbotes, hides of land, and other units of measurement of fines employed by the Germanic tribes.

[88] Lancaster referred to Menger (1871) but he evidently was not aware of the earlier German literature.

[89] See footnote 76 above.

been unable to find a precise definition of this kind in Schäffle's writings.

10 Roesler

In Roesler (1864) we find one of the best statements of the principle of diminishing marginal utility in the German literature (1864, p. 229):[90]

> There is an average use value which develops according to time and place from the average circumstances of need, uses, and knowledge; in practical life, minor deviations scarcely enter into consideration. Two aspects are important above all: 1. The amount of available supply of a good; because with the decline in supply, for each person the danger increases of not being able to satisfy his needs, and this thus becomes more urgent. Thus, use value stands in inverse proportion to supply. 2. Whether the need or the usefulness predominates. Needs are intrinsically capable of much greater and frequent fluctuation than is usefulness, because the judgments of people change more easily than do the characteristics of goods.

True, there is an ambiguity as to whether "with the decline in supply [*mit dem Sinken des Vorraths*]" refers to the consumer's own supply or the market supply; the ensuing explanation, however, makes clear that the prospect of a reduced supply to the consumer raises the good's use value. Other passages confirm his understanding of the principle of diminishing marginal utility, e.g. (p. 228): "A loaf of bread for one who would surely die of hunger without consuming it has, of course, incalculable use value; in contrast, it has very little for one who is satiated ...".

Roesler also commented on the Rau-Riedel concept of species or abstract value (p. 230):

> The distinction between abstract (species) and concrete (quantitative) value is important. The former is the value of goods with regard to their general characteristics as compared with a given need. Thus, in general water has a high use value because it is exceedingly indispensable—for the preservation of life, for health, for cleanliness, etc.; in contrast, a particular glass of water has almost no use value in an area with abundant water, because here the water is available in almost inexhaustible amounts; however, a glass of water in the desert can save one from dying of thirst, and is thus also capable of having a high concrete use value. ... Informative books have high abstract value, although, for the owner of duplicates, one of them has almost no value at all, and so forth.

[90] Roesler (1834–1872) received his training in law and economics in Erlangen and Munich and became a professor of political science at the University of Rostock. His works in economics include an early critique of wage theory (1861). His reputation is based largely on his 2-volume work on jurisprudence, *Soziales Verwaltungsrecht* (Erlangen, 1872–73), which is characterized by his sociological approach to law, influenced (according to Wani (1995)) by the catholic social theory of W. E. Kettler. Siemes (1968, p. 8) relates: "Finally Roesler entered the Catholic Church and, by this, lost his teaching chair at the University of Rostock. At about this time, however, he was invited by the Japanese Minister in Berlin, Aoki Shūzō, to become legal adviser to the Foreign Ministry in Tokyo." There he played a significant role in the writing of the Meiji Constitution, amply described by Siemes (1968), and published a 6-volume work in Japanese on constitutional law (1879–83) and a 3-volume work in German on Japanese commercial law (1884).

> The species value is the limit of the concrete [value]; it is, however rarely reached by the latter, because the supply rarely declines to a minimum. If the concrete value rises too high, then this could lower the abstract value, because one might then well change the need; this is particularly the case for all dispensable goods.

Here we have a clear statement of the identification of "species value" with Smith's "value in use", and the "concrete value" with Smith's "value in exchange". Now Roesler states that "the species value is the limit of the concrete [value]; it is, however rarely reached by the latter, because the supply rarely declines to a minimum." This may be interpreted to mean that the species value is the limit of the concrete value as $x_j \to 1$ from above, i.e., in terms of Knies's interpretation of Rau,

$$\lim_{x_j \to 1+} \frac{\theta_j Y}{x_j} = \theta_j Y, \tag{19}$$

in accordance with the definitions (3). This may be compared with the statement in the sixth edition of Rau introducing concrete value (1855, §61, p. 76): "The use value of a single individual (*concrete*) quantity of a material good, or of a single unit to a certain person (*concrete value*) ... very frequently does not coincide with the species value of this object, but rather remains far below it or disappears entirely." In terms of the Hildebrand-Knies formulation this states that

$$0 \leqq \frac{\theta_j Y}{x_j} \leqq \theta_j Y \quad \text{for} \quad x_j \geqq 1, \tag{20}$$

hence the species value is the upper limit as $x_j \to 1$ from above, and 0 the lower limit as $x_j \to \infty$.[91]

Finally, Roesler makes the interesting qualifying statement: "If the concrete value rises too high, then this could lower the abstract value, because one might then well change the

[91] Rau never subscribed to the Hildebrand-Knies extension of his theory; he lists Hildebrand (1848) among his references in §45, note (*c*), of the 6th–8th editions (1855, 1863, p. 51; 1868, p. 60), and Roscher (1866) in the latter, but does not appear to discuss their work. Nor does he appear to mention Knies (1855). According to his own 1868 formulation (see footnote 12 above), his statement from the 6th (1855) edition quoted above would correspond to

$$0 \leqq \frac{c_j b_j}{x_j} \leqq c_j \quad \text{for} \quad x_j \geqq b_j,$$

hence the species value c_j is the upper limit as $x_j \to b_j$ from above, and again, 0 the lower limit as $x_j \to \infty$. For low income levels, at which the demands, x_j, fall short of the "requirements", b_j, Rau's 1868 demand functions are multivalued and discontinuous, forcing the equilibrium prices to be proportional to the corresponding species values (compare Locke's statement quoted in footnote 10 above that "before the desire of having more than men needed had altered the intrinsic value of things, which depend only on their usefulness to the life of man ..."); however, at high incomes, at which demands exceed requirements ($x_j > b_j$ for all j), Rau's 1868 utility function also leads to constant expenditure shares, $p_j x_j / Y = c_j b_j / \sum_{k=1}^{n} c_k b_k$ and thus unitary price (and of course also income) elasticities of demand. In an intermediate case where, say, good 1 is unsatiated and the remaining goods are satiated ($x_1 < b_1$ and $x_j > b_j$ for $j = 2, \ldots, n$) we find that the demand for the unsatiated good is $x_1 = Y/p_1 - \sum_{j=2}^{n} c_j b_j / c_1$ whence the own price-elasticity of demand for this good is

$$-\frac{p_1}{x_1}\frac{\partial x_1}{\partial p_1} = \frac{1}{1 - \sum_{j=2}^{n} \frac{c_j b_j}{c_1}\frac{p_1}{Y}} > 1,$$

i.e., the demand is elastic. This could have accommodated Roesler but not Knies.

need; this is particularly the case for all dispensable goods." Thus, Roesler is worried that the Rau-Hildebrand theory cannot accommodate elastic demand—the opposite of the case that concerned Knies. That is, if the total supply of grain (commodity j) is reduced to one bushel, it is unreasonable to assume that people will pay the exhorbitant price $p_j = \theta_j Y$ when a large stock $x_k >> 1$ of potatoes (commodity k) remains available at a much lower price $p_k = \theta_k Y/x_k$. Thus if p_j/Y rises too high, one would have to allow the "abstract value" or "need" θ_j to fall in order to explain the decline in expenditure on commodity j, just as Knies, in tackling his "second stumbling-block", required it to rise to explain the increase in expenditure on grain during a dearth. Knies's formulation (cf. formula (15) above), apparently not known to Roesler, could accommodate infinite as well as unitary elasticity of substitution, but no other cases. The Rau-Hildebrand-Knies model is thus too rigid to command acceptance.

These doubts raised by Roesler in 1864 evidently grew stronger and led him in 1868 to a negative assessment of the Rauian system.

He started out (1868b) by outlining what he conceived to be the received theory of value, based on the definitions of "value" given by Rau and Roscher. Rau's definition (1855, §57, p. 70) was "the recognized degree of utility of a material good in human judgment"; Roscher's (1861, §4, p. 6) was "the significance which [a good] has for the consciousness of purpose of economic man." Neither of these definitions refers to the quantity of the good—an oversight, presumably, on the part of both authors. Roesler went on to present his interpretation (1868b, p. 280; 1871, p. 145):

> Let us remain with the last-mentioned definitions for a moment; for they give rise to questions which cannot be passed over lightly. What does "the degree of utility or serviceability of an object" mean? One may understand by this those physical characteristics and aspects of an object which constitute a particular amount of fitness for one or another kind of human life purpose, and which, be they naturally or artificially made, we shall once and for all to refer to as *technical*.

He proceeded to spell this interpretation out so as to make clear that by "technical utility" he meant a property inherent in an object; that (1868b, p. 282; 1871, p. 148) "the technical usefulness of an object is a given once and for all, and does not permit a rise and fall of its degree. Thus from the starting point of need and usefulness one cannot proceed to an actual concept of value." For, "since use value is supposed to be the basis of exchange value and the price of things," if, as he had argued, "use value is not capable of a variety of degrees relative to need and usefulness, how can the exchange value and the price of things change?" Apparently he had overlooked Hildebrand's and Knies's contributions, and forgotten his own earlier presentation of Rau's theory, ending up in effect with Rau's species value as the only type of use value and forgetting entirely about Rau's "concrete value".

Roesler did, however, discuss Roscher's ingenious attempt to resolve the "paradox of value" by use of Rau's concepts as interpreted by Hildebrand. His critique proceeded as follows (1868b, pp. 286–7; 1871, pp. 152-3):

> It may well be hardly provable that a farmer would compare his gold wedding ring with an iron implement 25,000 times as large. The two are entirely disparate

> objects, which, even taking into consideration their relative quantities, permit no abstract comparison between them. It has already been demonstrated above that the value relationships which originate from differences in quantity presume the existence of a simple or absolute value, but cannot give rise to it. A wedding ring on one's finger and a plow in the field have, strictly speaking, nothing in common with one another that could yield a point of comparison between them. This ingenious and seemingly felicitous thought thus cannot exceed the value of an illusory expedient; it also appears that within the sphere of the historical method, the value of gold relative to the value of iron must be explained not arithmetically, but rather historically.

It is clear that Roesler did not follow the details of Roscher's argument.[92] The above also brings out the limitations of Roesler's approach: a farmer is presumably perfectly capable of *choosing* whether to spend a given amount of money on a wedding ring or a tractor, given their prices, even though he might have difficulty estimating his *demand price* for a given quantity of each object. But the last sentence in the above quotation, addressed to the leader of the older Historical School, was a low blow, and must have stung Roscher particularly strongly; presumably as a consequence, he withdrew this brilliant analysis from the ninth (1871) and subsequent editions of his *Principles*.

Having thought that he had successfully rebutted Rau and Roscher, Roesler concluded as follows (1868b, pp. 287–8; 1871, pp. 153–5):

> The difference between abstract or species value and concrete value touched upon here, and introduced into the theory by Rau, is not capable of helping us past the contradictions in the Smithian theory of value.
>
> ...In order to find the abstract value of goods, we must transport ourselves into an order of things which in truth does not exist, although it lies implicitly at the basis of the Smithian economic doctrine.

Roesler's article was not merely destructive, however; he presented an alternative approach, based entirely on legal concepts (1868b, pp. 297–8; 1871, pp. 160–161):

> If there were no ownership of diamonds, then—despite their gleam and their scarcity—diamonds would be worthless; for what value could a thing have for me which at any moment could be taken away by the next person to come along and which finds no place in the order of life? The order of value is therefore an outgrowth of the legal system, especially the system of property rights. No value exists outside of the sphere of property, because outside of this sphere there is no social power which could establish and maintain a relationship of things between one another. If property is eliminated, then all goods—even those to which we assign the highest value—become worthless.

From this Roesler comes to the startling and radical conclusion (1868b, p. 302; 1871, p. 165): "economics cannot claim any independent existence outside of the legal sphere."

[92] In particular, Roscher compared a wedding ring with a tractor that was 250,000 (not 25,000) times as heavy; since he assumed the species value of iron to be 10 times that of gold, he concluded that the concrete value of gold would be 25,000 times that of iron (as shown in detail in section 7 above).

However, in the above reasoning Roesler appears to have confused a *necessary* condition for value with a *sufficient* one; for while it is true that some legal or extra-legal enforcement of property rights is a prerequisite to possession and voluntary exchange of goods, there would be no incentive for societies to establish such property rights, or even for individuals to hire mafias to secure their own property, in the case of economically useless objects. Property rights, where they exist, have their origin in the economic value of property.

Roesler's last writings continue in this vein. He argues the case (1872, p. 510): "*National economics is a component of the legal order, and the investigation of its laws is a task of jurisprudence.*" Finally (1878, pp. 17–18): "what is to be understood by value is exclusively exchange or wealth value. ... The explanation of value as the economic significance of goods is ... meaningless .... With far greater justification one could instead say that the economic significance of goods is found in their value."

11 Gossen

As promised in the introduction, I include (if only as a "placebo") a discussion of Gossen's work.

In his introduction to the English transation of Gossen (1854), Georgescu-Roegen (1983) referred extensively to a mimeographed study by Karl Robert Blum of Gossen's life and work, *Hermann Heinrich Gossen: Eine Untersuchung über die Entstehung seiner Lehre*, which was deposited in the Hayek Library of the School of Law at the University of Salzburg (1983, pp. xxvi; cxxxiv, note 17), and a copy of which was made available to him by Friedrich Hayek and deposited in the Vanderbilt University library (but which has since been lost).[93] In the words of Georgescu-Roegen (1983, p. lxxii), this study shows Rau's treatise, *Lehrbuch der politischen Oekonomie*, to be the "decisive influence" on Gossen.[94]

[93] I am most grateful to Prof. Werner Tschiderer of the University of Salzburg for his help in arranging for me to receive a photocopy of this manuscript from the Hayek Library there. Included was a page (evidently not included in the copy sent to Georgescu-Roegen) containing the handwritten indication "Giessener Dissertation von 1933". Since a different work was published as Blum's dissertation in 1934 (see Blum 1934, title page; *Jahresverzeichnis* 50 (1934), p. 661), this description must be incorrect, unless perhaps the monograph on Gossen is considered to be a draft of an alternative dissertation. To complicate matters, the unnumbered page 2 of Blum's *Wertlehren* states that Blum's dissertation was accepted by the Faculty of Philosophy, Department II, of the University of Gießen, on 8 July 1931, and this is confirmed in Blum's curriculum vitæ (*Lebenslauf*) (Blum 1934, p. 61). Whether any revisions were made during those three years we do not know. But it is significant that on the first page of the text of the *Wertlehren* (Blum 1934, unnumbered p. 3), Blum states that he has come across some "entirely new sources on Gossen's youth" but that "publication of this material must be reserved for a separate study." If the published 1934 work is unaltered from the approved 1931 dissertation, this is consistent with the "separate study" having been prepared in 1933. The most recent date mentioned in Blum's monograph on Gossen is 12 March 1931 (Gossen 1933, footnotes 11 and 13 on pp. 5 and 6), the date of a communication Blum received from Gossen's grand niece Frau Julie Webbe, four months before the date of approval of Blum's dissertation.

[94] A passage by Rau on the wine tax that Gossen is said to have developed for his civil-service examinations is quoted in Blum (1933, p. 65, note 82), and its source is said to be Part II of Volume III, p. 441, of the third edition of Rau's Treatise. However, the third edition of Vol. III.2 (published in 1851) contains just 413 pages, and §433a on the wine tax (*Weinaufschlag*), which occurs on p. 190, is quite different from the passage quoted by Blum. The passage does agree perfectly, however, with the opening paragraph of §433 of the *first* edition of Volume III.2, which was published in 1837, the same year of publication as that of the *third* edition of Volume I. To make matters still more complex, this volume has two pages numbered 441, the

There are two aspects of Gossen's work that are of particular interest here: (1) his criticism of the so-called concept of "absolute value", which I argue is his name for Rau's "species value" or what Riedel called "abstract value"; and (2) his simplifying assumption which I show reduces to Hildebrand's basic assumption of constant expenditure shares, which Hildebrand, Knies, and Roscher identified with Rau's "species value".

(1) Gossen included a strong critique of what he called "absolute value" (1854, pp. 45–8; 1983, Ch. 3, pp. 54–6). However, he did not say what he meant by this term, hence his meaning can only be inferred from his criticism of the concept (1854, pp. 46–7; 1983, pp. 54–5):

> *According to my views of the external world, there exists nothing to which a so-called absolute value may be attributed.*
>
> This is contrary to what is now assumed more or less explicitly by economists, for whom every object is conceived as having some definite inherent value. ... The cause for all this undoubtedly has been the fact that without the assumption of [an absolute] value, value seems to become subject to such tremendous fluctuations that it would appear difficult to make practical use of the term. ... It was believed that one could overcome this elusiveness of value if one could postulate an absolute value. If such an absolute value existed, it would indeed simplify all calculations. ... It was believed that absolute value was determined by making inherent physical characteristics of objects an integral part of the concept of value—namely, those physical characteristics that render objects capable of satisfying pleasure to some significant degree, either directly or indirectly.

As we noted in section 1.2, the term "absolute value" was used (along with "inner value") by Schlözer (1805, §. 50, p. 40) as a synonym for "use value", in contrast to "relative value" which was a synonym for "exchange value"; and Schlözer regarded it as independent of the quantity consumed. Boileau (1811)—see section 1.5—used "absolute value" in the same sense. This usage was also employed by Lloyd (1834, pp. 28–31), apparently with the same meaning. (We recall that Soden (1805) and Lotz (1811) had used "positive" and "comparative value" for these same concepts, and that Soden used "absolute value" for the use value of a necessity, "relative value" for that of a luxury.) Kraus (1808, §. 16, p. 102), on the other hand, used the term to indicate labor as the standard of value, and Malthus (1823) likewise used it to refer to a commodity that could serve as an invariable standard of value. Ricardo's (1823) posthumous tract on the subject (disagreeing with Malthus on the possibility of such an absolute standard) was not published until 1951, and his letters to Malthus on the subject were still unpublished when Gossen wrote, having been published for the first time by Bonar (1887). It thus seems very doubtful that Gossen had Kraus's and Malthus's concept in mind.

Rau pointed out in an endnote[95] that Beccaria had used the term "absolute value" to mean "use value", suggesting that the concept had preceded "exchange value" historically:

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first (where the passage quoted in Blum appears) being really page 241 (pages 239–242 were misnumbered). Thus Gossen must have been working with a set which contained the third edition of Vol. I and the first editions of the remaining volumes.

[95] Rau (1833, §62, note (a), p. 62; 1837, §62, note (a), p. 65; 1841a, §62, note (a), p. 68; 1847, §63, note (a), p. 84); 1855 and 1863, §63, note (*a*), p. 79; 1868, §63, note (*a*), p. 98).

> BECCARIA referred to these original valuations of things, not entirely fittingly, as *absolute value* as opposed to *relative value* or exchange value, which came later.

Beccaria had used the term in passing without giving it particular significance.[96] Presumably following Rau's discussion of Beccaria's concept, Riedel (1838, §31, p. 24) adopted Beccaria's terminology:

> The valuation of useful objects may ... be carried out from two points of view, according to their distinct types of use, namely either: 1. from the viewpoint of the personal or material benefits which a thing affords by retaining it in one's possession, utilizing it in its natural state, or using it materially, or 2. from the viewpoint of the extent of such benefits—and services affording such benefits which one can obtain from others—[obtained] by devoting its possession to relinquishment of the object to others through exchange and purchase. The determination of the degree of utility according to the first point of view, or the original type of use—which forms the basis of all the others—gives the *value as such*, as it is also called, or, for a closer differentiation, the *use value, need value [Bedürfnisswerth], utility value [Nützlichkeitswerth]*, or the *absolute value*. If, on the other hand, the value is determined according to the second above-mentioned point of view—the derived type of use which it is capable of, insofar as it is a means for the acquisition of material objects, or yields personal services from other persons for the possessor, and is surrendered as the price for this—then this leads to the determination of the *exchange value* or the *relative* and *derived value* which, in keeping with the confusion of the terms "cause" and "effect" common in everyday life, is also simply referred to as the *price*.

However, as Hildebrand (1848) showed, there is no reason to expect the (concrete) use value of any object to be free from fluctuation; and Gossen himself made use of the concept of marginal utility which he knew very well would fluctuate in response to changes in supply. Thus, by process of elimination, I am led to conclude that by "absolute value" Gossen must have meant Rau's "species value", or what Riedel called "abstract value".

This conclusion is reinforced by the following additional comment Gossen made regarding the "absolute value" (1854, p. 87; 1983, p. 102):

> The reason for not seeing the importance of exchange is obviously the fiction of an absolute value whose scale is conceived in terms of physical properties. With such a concept of value, exchange obviously can have no effect on value since physical properties do not undergo any change through barter.

[96] In his words: (Beccaria, 1769–1770: 1804, p. 339; 1822, p. 237) "And since at first nothing was evaluated except to the extent that it was suitable to satisfy the necessities and conveniences of life, there came the idea of, and the word, value, denoting the power, custom, and ability to fulfill an end; then in this last stage things began to be appraised according to their capability of procuring others. Whence the absolute value thereupon became relative and exchangeable, signifying the power that each thing has of being exchanged for others; and the quantity of each thing that one must give for another was determined, and called the price of the other." See further Beccaria (1769–1770: 1804, pp. 344–5; 1822, p. 241).

As we have seen in section 1, all of the German economists before Rau conceived of use value as being independent of quantity. But Rau was the first, in identifying this concept with his "species" or "abstract" value, to make specific mention of physical attributes of goods, such as the nutritive power of foods (see footnote 14 above), as being characteristic of this type of value. Thus the evidence is overwhelming that by "absolute value" Gossen meant precisely Rau's "species value", renamed "abstract value" by Riedel.

We learn from Blum (1933, pp. 59–66) that Gossen's concern with "absolute value" began already in the course of preparing for the written civil-service examinations for his first government post (see also Georgescu-Roegen, 1983). Rather surprisingly, it arose in the course of his study of the theory of rent in connection with his assignment to assess the shifting and incidence of the Prussian tax system. He proceeded to outline essentially the Ricardian theory of rent (Blum 1933, p. 60)—but without reference to Ricardo—to the effect that rent "is a consequence of the limited amount in which agricultural products can be produced", in contrast to what he conceived to be the "prevailing opinion" which perceives it to be "a consequence of the absolute value of the products obtained with the aid of land." It is natural to inquire whether this was Rau's opinion. One might suspect so, given the surprising statement by McCulloch (1845, p. 26), based on the 1839/1840 French translation of Rau's work: "He rejects the theory of rent, as explained by Ricardo and others .... He has, in consequence, no clear or accurate ideas in regard to many of the most important departments of the science ...". Evidently (cf. Schefold et. al., 1997, p. I, n. 2) the translation reviewed by McCulloch was of the third edition of Rau, whose section on the theory of rent reads as follows (1837, I, §212, p. 225):

> The *quality* of a piece of land itself has a powerful influence on its yield, so that with equal cost outlay, here more, there fewer raw products can be yielded, and thus on the better lands a given quantity, e.g. 1 bushel, would be obtained with less cost than on the worst. ... The former therefore yields a rent if the price of the product only just covers the cost of the use of the latter.

This is certainly the Ricardian theory! In an endnote Rau then refers to the classical theories of Malthus, West, and Ricardo, and even to that of McCulloch! This is substantially the same as the explanation given by Gossen (Blum 1933, pp. 60–62), and at no point did Rau invoke his "species value". Rau also referred to the treatment in Thünen (1826, §5, pp. 13–37).[97] For his analysis of tax incidence Gossen relied, as was indicated in footnote 94 above, on the first edition (1837) of Vol. III.2 of Rau's *Lehrbuch* (esp. §433, pp. 241–2—erroneously numbered 421–2), but this also made no use of Rau's concept of species value.

Thus, where did Gossen get the idea that "absolute value" was involved in the "prevailing theory" of rent? Blum (1933, pp. 145–154) made a strong case that Gossen must have been influenced by Soden's utilitarianism, which suggests that he may have been influenced by Soden in other regards as well. But Blum referred only to the first two volumes (1805, 1806) of Soden's *National Economics*. In Vol. 3, where there are also many utilitarian passages, Soden deals with what he conceives to be the physiocratic theory of the incidence of a land

[97] Thünen maintained that a rise in grain prices would bring about a rise in rent; but there is nothing in this that is inconsistent with the Ricardian theory, and no concept such as "absolute value" was invoked. In the second edition of his work, Thünen (1842, §5a, pp. 13–20) criticized Smith's theory of rent, which did not require further criticism by Gossen.

tax (1808, §581, pp. 168–9); this is repeated in Volume 4 (1810, §528, pp. 439–40)—the same section criticized by Roscher (1874, p. 674) for its statement that the physiocrats assumed a hermetically sealed economy.[98] Soden offers the following interesting summary of what he conceived to be the physiocrats' beliefs (Vol. 3, 1808, §581, p. 169; Vol. 4, 1810, §528, p. 440):

> The primary products—so they conclude—are the most indispensable ones, hence their value is *absolute*. The farmer [*Landbauer*] will of course add the sum total of the taxes onto the price of his primary product; the industrial and commercial producers will add to this price the value of their economic productive power; the industrial product thus *participates in* the payment of the land tax, and the state will thus receive its due share of each product, i.e., of the *entire* national wealth.

We may recall that Soden *defined* as "absolute" the value of any indispensable good; possibly Gossen relied on Soden but failed to consult Soden's definition, and may have confused Soden's "absolute value" with Rau's "species value" and Riedel's "abstract value". While Soden cannot be said to have had a theory of rent, the above passage suggests that he thought that the physiocrats' single tax on land's net product (rent) would (in a closed economy) be shifted to the purchaser in the form of a higher price of the product, as if it were an indirect tax on the farmer's product. Of course this is erroneous. But possibly this is what Gossen considered to be the "prevailing theory" of rent.[99]

[98] "The principle of national economics is cosmopolitan. The physiocrats imagine a hermetically sealed state. In this hermetically sealed state many ideas are right which evidently lead to wrong results as soon as the seal is broken." In Vol. 3 (1808, §581, p. 168), the second sentence of the above is the more elaborate "The physiocrats imagine a state surrounded by a Chinese wall, as well as hermetically sealed." Soden may have relied on Quesnay (1767, p. 79; Meek 1962, p. 162) who, in his Fifth Observation on his *Tableau économique* explicitly admitted that his analysis had assumed a closed economy. However, Soden's apparently sole explicit reference to Quesnay (Soden 1805, §166, pp. 236–7), in a section in which which he praised the "basis of the physiocratic system" but criticized its restriction of productive power to agriculture, was to his Maxims (Quesnay 1767, pp. 99–172; Meek 1962, pp. 231–262), which certainly deal with an open economy. An alternative (and it seems more likely) explanation for Soden's views on the physiocratic theory of taxation is that he was influenced by later antiphysiocratic writers such as Feder and Dohm (see footnote 99 below).

[99] It is interesting to compare Soden's reasoning with the reasoning attributed by Liebel (1965, p. 51) to Reinhard (1772), whom she styles as one of the "enlightened bureaucrats" of Baden-Durlach; according to Liebel, referring to the physiocrats' single tax on the *produit net* of agricultural land, which the Margrave attempted to implement in his principality (see footnote 4): "The new tax would not raise land values as the physiocrats supposed, Reinhard argued, because land values depended on prices and prices were not determined by taxes but by the relationship of supply and demand on the world market." This appears to be a summary of Reinhard's argument as quoted in Drais (1816, pp. 325n–326n), which reads as follows: "If, in a country where agriculture is to be fostered, one places all the burdens of the entire state upon it, it will be ruined. It is of course answered [?] that one must see to it that the prices of the agricultural products rise and hence all who use them share in the payment of the burdens. But this rise or fall does not depend upon whether the production and taxation of the farmer [*Ackermann*] costs much but rather upon the surplus or shortage of the product and upon the circumstance of whether it is much or little sought. Since, accordingly, the system itself esteems freedom of trade as its very soul, it follows that one will not want to impede the import of foreign produce. This, however, is brought in from countries in which not all burdens lie on agriculture, and because it is for this reason cheaper, the domestic farmer is compelled to provide his produce cheaply as well, or to keep it and eat it himself. His heavy assessment, however, remains

(2) Gossen himself in the first part of his work introduced a great simplification, namely the hypothesis that marginal utility was linear in each commodity (1854, pp. 9–10; 1983, p. 11). He later admitted that this hypothesis was "incorrect" (1854, p. 123; 1983, p. 147). He therefore replaced it by the assumption that at each income level individuals spend a fixed proportion of their income on any commodity (1854, pp. 126, 136; 1983, pp. 149, 160).[100] He went on to postulate that for each commodity the ratio of expenditure on that commodity to income at first rises as income rises until a maximum ratio is reached, after which it starts to fall (1854, pp. 141–2; 1983, p. 165). However, this is clearly impossible; in the 2-commodity case, for example, if as income rises the proportion of expenditure on commodity 1 rises to a peak and then falls, then the proportion of expenditure on commodity 2 must fall to a trough and then rise. In fact, the assumption that at any positive level of income, the share of that income devoted to any commodity is independent of prices, necessarily implies (if rational behavior is assumed) that this income share is also independent of the level of income, i.e., constant.[101] Thus, Gossen's system reduces to the Hildebrand-Knies development of Rau's!

Georgescu-Roegen (1983, p. lviii) cited the view of Spiegel (1968, p. 210) that Gossen's work was "fundamentally un-German"—a judgment with which he agreed, on the ground

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a burden upon him. This holds true particularly in a small country which is surrounded by larger countries rich in crops; and how would it go for those farmers who, producing only what their households consume, have no produce to sell?" Thus Reinhard apparently thought that the single tax on the net product (rent) of land would be shifted to the consumer rather than absorbed by the landlord. This of course is the exact opposite of physiocratic theory, according to which the virtue of the single tax is precisely that it is *not* shifted, as opposed to *indirect* taxes—in which they included income and commodity taxes and taxation of the *gross* product—which would lower the prices received by producers (*le prix des ventes de la premiere main*), leading to layoffs, and ultimately be shifted to the landlords (but with economic loss). Cf., e.g., Mirabeau (1760, 3^{e} Entretien: 12°, pp. 68–69; 8°, p. 55), Du Pont (1768, §§XV–XVI: 1910, pp. 21–24).

Reinhard's posthumaous tract (1771) was most likely provoked by the appearance of the essay by Schlettwein (1772) defending physiocracy and explaining how he had implemented the single tax while in the service of the Margrave of Baden (an appointment he held during 1763–1773). In his defense of physiocracy Schlettwein concentrated (pp. 45–47) on explaining the effect of an ad valorem tax on commodities used as inputs by artisans, holding that it would lead to higher market prices of their products (Soden's Chinese wall?) and thus to higher costs to agriculturalists who needed these products. This made him vulnerable to Reinhard's criticisms as well as to the remarkably similar ones later made by Feder (1778, p. 638) and Dohm (1778, p. 315; 1782, p. 59) (cf. Tribe 1995, pp. 338–340).

Reinhard went on to say (Drais, p. 327n): "Of course the intention is to frequently change the classification of properties as they increase in fertility; but is this a means to promote the improvement of properties, if one imposes doubled or such large assessments on hardworking farmers [*Bauern*]?" This assumes that the differential fertility is due to labor rather than land. The similarity of Soden's analyses to those of Reinhard and the later German anti-physiocrats such as Feder and Dohm leads one to suspect that he may have been influenced by these later anti-physiocratic writers.

[100] Cf. Jolink and van Daal (1998, p. 45), who have called attention to this fact.

[101] It is enough to present a formal proof for the 2-commodity case. Let $\theta(Y)$ be the proportion of income, Y, devoted to commodity 1; then $1-\theta(Y)$ is the proportion devoted to commodity 2. The demand functions for the two commodities are therefore

$$x_1 = \frac{Y\theta(Y)}{p_1} \quad \text{and} \quad x_2 = \frac{Y - Y\theta(Y)}{p_2}.$$

The Slutsky cross-substitution terms are then

that "no German economist felt any attraction for utility economics."[102] From these statements I can only infer that, with the exception of Hermann (1832),[103] neither Georgescu-Roegen nor Spiegel had bothered to read the works of the German economists.[104] Gossen's cumbersome and unsystematic mathematical exposition has been a barrier to communication for all economists regardless of nationality.

$$s_{12} \equiv \frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial Y} x_2 = \frac{[\theta(Y) + Y\theta'(Y)][Y - Y\theta(Y)]}{p_1 p_2}$$

and

$$s_{21} \equiv \frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial Y} x_1 = \frac{[1 - \theta(Y) - Y\theta'(Y)]Y\theta(Y)}{p_1 p_2}.$$

The difference between them is

$$s_{12} - s_{21} = \frac{Y^2\theta'(Y)}{p_1 p_2},$$

which vanishes for $Y > 0$ if and only if $\theta'(Y) = 0$, i.e., if and only if $\theta(Y)$ (and therefore $1 - \theta(Y)$) is independent of income, Y.

[102] In case there should linger any doubts concerning Rau's "attraction for utility economics", a passage contained in his discussion of "unproductive consumption"—by which he meant the using up of existing stocks—might help dispel it (Rau, ¶2 of §322): 1826, p. 253; 1833, p. 322; 1837, pp. 353–354; 1841a, p. 374): "that unproductive consumption is ... best which, with the same outlay of goods, furnishes the intended benefit to the greatest number of people [in the fullest measure] and for the longest time ..." ("in fullest measure" was added in the 4th edition (1841a)). This was repeated in §322 of the later editions (1847, p. 423; 1855, p. 414; 1863, p. 419; 1869, p. 129), with "unproductive consumption" (*unproductive Consumtion*) replaced by "destructive consumption" or "using up" (*Verzehrung*); the version from the 5th edition (1847) was reproduced in Friedländer's monograph (1852, p. 36).

[103] Georgescu-Roegen (1983, pp. lxxv–lxxvi) examined a paragraph containing a numerical example from Hermann (1832, p. 73), also found in Hermann (1870, p. 404), and concluded that it "could hardly suggest the law [of diminishing marginal utility] itself." The paragraph was preceded by an interesting definition of individual i's "ability to pay" for, or purchase, good j, as the ratio of the individual's income, Y_i, to the amount spent on the good, $p_j x_{ij}$, which under the Hildebrand-Knies assumption of constant expenditure shares would be $Y_i/p_j x_{ij} = 1/\theta_{ij}$. The reciprocal of this he called the "relative value" of good j to individual i. This of course corresponds to Hildebrand and Knies's "species value" of (or "need" for) good j, and Schäffle's "usefulness" of good j, θ_{ij}. He applied this definition to subsistence goods such as bread, but did not appear to notice that the same definition would apply to non-subsistence goods, since from individual i's budget constraint the θ_{ij}s must sum (over the goods, j) to 1. For the latter he instead defined the ability to pay (in the 2-good case) by $(Y_i - p_1 x_{i1})/(p_2 x_{i2})$ (where commodity 1 is the subsistence good); but this is necessarily equal to unity. His subsequent numerical example lacks coherence because of this failure to take account of the individual's budget constraint. His concluding sentence, to which he added the footnote stating that Laplace (1814, p. 21) had pointed out a similar valuation from Bernoulli (1738), seems to state (for the 2-good case) that if individual i has a residual income of $R_i = Y_i - p_1 x_{i1}$ (which from the budget constraint is necessarily equal to $p_2 x_{i2}$), then the "value" to him of a bottle of wine (good 2) would be $1/R_i$, i.e., $1/(p_2 x_{i2})$. With constant expenditure shares, at least this is proportional to the marginal utility θ_{i2}/x_{i2} (cf. (6)). The example may be compared to the analysis in Laplace's basic work (1812, pp. 187, 432) (to which Hermann did not refer) showing that if utility ("*fortune morale*") is a logarithmic function of income ("*fortune physique*" or "*bien total*"), then the marginal utility of income ("*valeur relative*") varies as the reciprocal of income—Rau's formula (9). See also Poisson (1837, p. 72).

The passage quoted in footnote 73 above (Hermann 1870, pp. 401–2) certainly comes much closer to a recognition of the principle of diminishing marginal utility, but is still very crude by comparison with Rau's statements, even those of the 1837 edition which must have been familiar to Gossen.

[104] An example of this is found in the following statement by Georgescu-Roegen (1983, p. lxvi): "Among the German economists, Gossen was the first to distinguish between economic and free commodities." This distinction had already been made by Hermann (1832, p. 3).

12 Non-German critiques of the German economists

Achille Loria (1882, pp. 33–4n) drew attention to some interesting critiques of the writings of German economists, one a Dutch thesis, and one that of a Russian (Ukrainian) economist.[105]

The thesis of van Houten (1859) refers to many German authors, including Rau, Riedel, Roscher, Knies, Lotz, Hermann, Friedländer, and Wagner. Houten (1859, p. 7, note 2) quotes the passage in Rau (1841a, §56, note (*b*), p. 59) stating: "If value is not to mean the *degree* of utility, but rather the utility itself, then one of the two expressions would be superfluous" then refers to the discussion in the first four pages of Knies (1855) on value as the "degree of utility". He then discusses Rau's distinction between species and concrete value (pp. 9–10), and comes to the conclusion that the former of these is "redundant" (p. 12). After covering the distinctions between various types of value in German economics he complains (p. 9): "Many German writers ... make so many distinctions in their theories of value that we certainly have to admire their acumen, but at the same time have to regret that the economic concept of *value* is almost lost because of those distinctions, or at least is blurred in such a way that reading their writings is a very difficult and troublesome task." Here he quotes De Quincey's (1844, Section II, p. 12) dismissive comment that "German books go for nothing here", saying that "it is a little too harsh"![106] Van Houten concludes (p. 16) that the German *Gattungswerth*, the English *value in use*, and the French *valeur d'usage* are best dispensed with, but nevertheless he has to return to these concepts to discuss Friedländer's monograph. He criticizes Friedländer's morally ranked hierarchy of wants by saying that "economics only studies the laws of society and should therefore take mankind as it is, not as it should be" (p. 18). Thus (p. 19) "Friedländer's work seems to be somewhat socialistic."

Loria (1882) also referred to a critique of Hildebrand's analysis by the Russian economist Nikolai Ziber (1871),[107] and Cossa (1893, p. 455) drew attention to this work as well as to that of Antonovich (1877). Both writers relied on Russian translations of Hildebrand's work (1848); in the one relied on by Ziber, Hildebrand's *Nutzwerth* (use value) was translated as *poleznosti*, which is the Russian for "utility" or "usefulness"; this may well have misled him. Thus, Hildebrand is quoted as saying that "the more the number of useful products increases, the more the usefulness of each separate product decreases, if the existing need for

[105] Loria also attempted to make the case that Hermann (1832) had plagiarized the work of Gioja (1815), by displaying passages of the two authors in parallel columns (pp. 48–50). Hermann (pp. 20–21) had cited Gioja, whose work he regarded as somewhat "obscure"; besides, the parallel passages are sufficiently dissimilar that at most they show some influence.

[106] Indeed, De Quincey's is a strange comment coming from one who puts the German writers to shame with the multiplicity of his different types of value, such as "intrinsic worth" (p. 25), "utility value" (pp. 29, 32), "difficulty value" (p. 34), "affirmative value" (pp. 44, 65–66, 69, 71, 75, 87), "scarcity value" (p. 51), "cost value" (p. 51), "latent value" (p. 53), "resistance value" (pp. 67, 76), "negative value" (pp. 68, 86–87), "power value" (pp. 71, 76), "potential value" (p. 75), "teleologic value" (pp. 75, 103–4), and even "market value" (p. 118) and "actual value" (p. 122)!

[107] Ziber was born in Sudak on the Crimean coast, of a Swiss father and Ukrainian mother, and died in Yalta (cf. Y., 1894; Mikhailov, 1916). His 1871 work discussed here was his master's thesis from Kiev University; it was praised by Marx in the preface to the second edition (1873) of *Das Kapital*, and cited by Luxemburg (1909–1910: 1925, p. 46; 1975, pp. 564–5). Ziber contributed many articles to journals in the fields of economics and anthropology, as well as articles on Marxian economics, and published Russian translations of Ricardo's works.

this product does not change. ... Therefore, the usefulness and exchange value not only do not contradict one another, but on the contrary are in greatest harmony with one another. The utility (usefulness) and exchange value of all products always rise and fall together." (Ziber 1871, p. 38n; 1885: 1959, pp. 64n–65n). In opposition to this, Ziber counters (1871, p. 38n; 1885: 1959, p. 65n):

> Bruno Hildebrand is ... wrong in trying to convince us that the more useful products there are, the lower is the usefulness of each one; who doesn't know that two socks are more useful than one? Within the limits of need, a new specimen of a product is an extra unit of usefulness. Outside the limits of the need, an extra unit is not needed at all, and therefore the usefulness of the remaining units remains unchanged. If we go backwards, decreasing the number of units, then even though the need is left partly unsatisfied, this does not imply that the *usefulness* of the available products has risen; only the *fear* of leaving part of the need unsatisfied has gone up. This fear is completely foreign to both usefulness and value, even though *in certain circumstances* it can exert such a pressure on the value as to lead Bruno Hildebrand to find a complete harmony between the two.

The passage about fear may well have been influenced by Roesler (1864) (see the reference to "danger" in the passage quoted above), since Ziber referred to this work (1871, p. 14, note 2; 1885: 1959, p. 46, note 2). In a further critique, this time of Proudhon himself, he goes on to say (*ibid.*):

> Proudhon is wrong when he states that the more of a product is produced, the cheaper it is sold: if the need is already satisfied, then the surplus *will not be sold at all*, and consequently the price of the remaining products will not decline; within the limits of complete unfulfillment of the need and its complete satisfaction, each new unit of a product does indeed cause a drop in the value of the remaining units, but this depends on independent factors.

While the misunderstanding displayed here may be partly accounted for by the bad translation of "use value" by "usefulness" that Ziber relied on,[108] still, it is remarkable that Ziber followed Rau's early (1833) doctrine to the effect that ("concrete") use value was positive and constant below satiation levels and zero above, and in holding that any smooth empirical relationship between the quantity supplied and price must be due to unspecified "independent factors".

Ziber made some other criticisms of the German economists' work which are of interest. One was a criticism of the the various value concepts, in particular the concept of abstract or species value (1871, p. 10, note 10; 1885: 1959, p. 42, note 1):

> Another reason that the number of definitions has grown in the modern German literature is that this literature (Roscher, Bruno Hildebrand, Schäffle, Roesler and others) started distinguishing between an abstract value, that is, a

[108] But we have seen in footnote 66 above that even though not suffering from Ziber's linguistic handicap, Menger made the same misinterpretation of Hildebrand!

> property of things to satisy human needs in general (oak is useful), and a concrete value, that is, a property of things to satisfy needs under certain conditions (two copies of a book are useless). However, no attention was paid to the fact that the first property is not really a property but just an exercise of the mind; that there exist only specific relationships between things and needs, and there are no other relationships.

The inclusion of Schäffle is fair enough, since as we have seen he accepted the concept of species value in the first edition of his textbook (1861, p. 92); but thereafter (1870b), as we have seen, he made essentially the same criticism of the concept as did Ziber a year later.

Ziber's most favorable comments on the writings of the German economists were reserved for Friedländer (1871, pp. 31–32n; 1885: 1959, pp. 59–60n):

> If I am not mistaken, Friedländer's "Theorie des Werthes" is the only attempt in the economic literature to point out the *simultaneous* relationship between different uses and different needs. A person's existence is assured through the *simultaneous* satisfaction of his various needs. Therefore, the quantity of goods satisfying a certain need makes up a unit of need. The aggregate of all goods satisfying a person's need, for instance the need for food, over a certain time period has the same use value as the aggregate of all goods that satisfy the person's need for clothing over a certain time period—at least in a cold country—because if either need is unmet, the person's life would be equally impossible. Now if different types of goods are ascribed to different needs, then the use value of each particular product is determined by its potential for fulfilling the goal assigned to it. For instance, the use value of a type of food is determined by the relationship between the actual amount of nutrients in this type of food and the average amount a person needs in a given time. The use value of each particular product thus consists of a fraction of the entire unit of need, and the use values of different goods stand in the same relation to one another as do the parts they represent to the different units of need ....

After this remarkably perceptive summary of a most difficult-to-interpret text, comes Ziber's criticism:

> Having recognized that different things that simultaneously satisfy needs have the same value, Friedländer then turns his main attention to the relationship between different use values. In his opinion, this relationship is based on the greater or less correspondence between the quantity of one or another product and the size of the need. Thus, Friedländer avoids the question of how his changes in classification alter the study of value. In addition he makes the mistake (which, by the way, is also shared by many other economists) of assuming that the shortage or excess of a product with respect to the unit of need is the cause of the increase or decrease in a product's use value; whereas it is obvious that a pound of bread or meat satisfies an absolutely equal need, both when there is much of it and when there is little. [!] Therefore, the use value or simply the usefulness of a pound of bread or of meat is always equal. The greater

> value ascribed to these products when there is not enough to meet demand is simply a way of expressing the fear of remaining without a suitable amount of the needed product. ([!] added.)

The "mistake" made by Friedländer may be contained in the passage he had ostensibly paraphrased but in fact precisely quoted (1852, p. 10) from Malthus (1820, p. 340), and which was quoted in footnote 39 above; for I have not found any other passage in Friedländer's text (except possibly a quotation from Rau[109]) which could be interpreted as enunciating the principle of diminishing marginal utility. But Ziber's response to such passages shows how hard it was for an extremely intelligent person to accept the idea of diminishing marginal utility!

Antonovich (1877, pp. 64–5), relying on a different Russian translation of Hildebrand which correctly translated *Nutzwerth* as *poleznaya tsennosti* (use value), proceeded with his criticism, responding to Hildebrand's passage by saying that (1877, p. 66):

> Any utility, as an economic fact, necessarily requires the presence of objective and subjective elements. Hildebrand declares directly that in this case the subjective element alone is sufficient, that use value is not something objective, that it does not exist outside of the subject who consumes. Obviously, B. Hildebrand is repeating Fichte's famous theory that the whole world is a product of our I, and that outside of this I the external world surrounding us does not exist.

This is certainly a most absurd and misleading statement! After continuing in this vein concerning Fichte, Antonovich proceeds as follows (1877, p. 67):

> Meanwhile it is precisely on this conclusion that B. Hildebrand bases his thesis that when need remains unchanged, an increase in [the amount of] products is equivalent to a decrease in the utility [*sic: poleznost'*] of each unit of the product. This means that a doubled quantity of bread, given unchanged need, does not mean a doubling of its utility, but on the contrary a decrease in this case of the utility of each unit of the product by half. If each unit of the product satisfied 1 unit of need previously, now it satisfies only $\frac{1}{2}$ of it and therefore the object's utility is $\frac{1}{2}$ as much. But as long as the manufactured product finds consumers, the fact that the consumer can use two quarters of bread instead of one does not change its utility at all. ... One cannot say that a product's utility decreases just because the given need is more fully satisfied, or because it is satisfied by two products instead of one, although obviously, at the same time, any product that is not consumed is useless.

[109] The passage in question appears in Friedländer (1852, p. 34) and comes from Rau (1847, §59, endnote (*a*)): "... the successive increases in value resulting from continued perfection of a type of good, for example from a wooden bench to the finest sofa, become steadily smaller as each new improvement provides a smaller increase in benefit to human life." This was already in the 3rd edition (1837, §58, endnote (*a*), p. 62); see also the 4th (1841a, §58, (*a*), p. 64), 6th and 7th (1855, 1863, §59, (*a*), p. 73) and 8th (1868, §59, (*a*), p. 91), the discussion in the last three editions also containing Rau's comments on Friedländer's monograph. Another passage in Friedländer (1852, p. 33), in which he discusses Hermann (1832), makes the intriguing observation that "the size of one's holdings modifies one's judgment of their use value"—but without suggesting the direction of the relationship!

Here, Antonovich inadvertently substituted "utility" or "usefulness" (*Nutzen* or *Nützlichkeit* in German) for "use value" (Hildebrand's *Nutzwerth*). For whatever reasons, the statement is a specific denial of the principle of diminishing marginal utility. Although published in 1877, Antonovich's book makes no mention of the work of Jevons or Menger. That Antonovich's confusion follows (at least in part) from the confusion between these two expressions is clear from the following additional passage (p. 68):

> B. Hildebrand proposes a case opposite to that presented in Proudhon. If a nation's total need equals 100 and the utility [*sic*: it should be "use value"] of iron is equal to 5% of this number, and if the need for iron does not change while its production increases, then the 5% of iron's use value will be redistributed among its larger quantity, and therefore each unit of iron will have less of this use value.

That Hildebrand himself used "use value" in the same paragraph to denote both the species and concrete use value ("total" and "average" use value in his formulation) obviously contributed to the confusion. However, in the last part of this quotation Antonovich did succeed in capturing Hildebrand's basic idea, though he did not seem to see how it successfully confuted Proudhon's antinomy.

Both Ziber and Antonovich also criticized Roscher's gold-and-iron illustration in the explanation of Smith's "paradox of value". (Ziber relied on an apparently accurate French translation of Roscher.) Ziber objected that "Roscher is ... wrong in thinking that the species use value of iron is higher than that of gold while a certain amount of the former has lower concrete use value. We already know that we cannot compare the usefulness of two objects, as each of them is good in its own place." (Ziber 1871, p. 39n; 1885: 1959: p. 65n). What Ziber is objecting to is the mere comparison of the use value and exchange value of two disparate commodities, since he holds these to be incommensurable. This is a legitimate stance; but in this case there would be no paradox at all, whereas most commentators, from Montanari to Locke to Law to Smith, have found such examples surprising. And of course, while one might object that comparison of *prices* of unrelated objects is meaningless, it is always possible.

Like Ziber, Antonovich did not challenge Roscher's logic, but criticized Roscher on other grounds (1877, p. 68): "Arguing with Proudhon, Roscher says that the total consumption of iron is greater than the total consumption of gold, and that the species value of iron is greater than the species value of gold. Clearly, Roscher is merely repeating Proudhon's words, formulating the latter's idea more precisely using Rau's theory of abstract and concrete value." Thus, neither Ziber nor Antonovich understood the arguments that Hildebrand and Roscher had put forth.

As is suggested by some of the preceding comments, the German economists were largely ignored in Britain, or else dismissed with contempt, until they received the appreciative attention of Marshall (1890).[110] In France, Block (1890, 1893) gave them excellent treatment; in contrast to most commentators, he actually read and understood the works he discussed.

[110] The Germans "have made careful and profound analyses which add much to our knowledge, and they have greatly extended the boundaries of economic theory" (1890, Book I, Ch. IV, p. 70).

13 Later German-language critiques

A number of critiques of the German theories of value (especially Rau's) emerged starting in the 1860s. I will only be able to touch on the most important.

13.1 Lindwurm

Lindwurm's (1865) paper, which is oriented more towards philosophy than economics, caused quite a stir. Referring apparently to the 6th edition of Rau (1855, §57, p. 70), Lindwurm (p. 167, note 6) quoted Rau's definition: "The degree of utility of a material good recognized in human judgment is its value." He went on to cite the passage from Rau quoted in the previous section that was referred to by van Houten. Lindwurm objected:

> What, however, one may ask, is the degree? To be precise, not the degree of *something*, but rather the degree as such? As I see it, a "degree of utility" can be only a certain amount, more or less, of utility. Thus when one specifies the "degree of utility" of a material good, then what this means is that utility is the general, value the particular [concept]. Since, however, value is then itself something general and requires a predicate for its particular designation, then in fact the case anticipated in advance by Rau arises—namely that one or both expressions become superfluous, or alternatively, since we know that in actual life one designates completely different things by the same thing, value and utility are by no means synonyms; thus a strong suspicion is justified as to the erroneousness of Rau's definition.

As we have seen in section 1.5 above, Rau was simply following the definition given by Jakob (1825) of "need value". In a rare (perhaps unique) instance of his answering a critic, Rau replied as follows (1868, §57, note (*c*), p. 87):

> Lindwurm ...asks ...: "what is the degree, to be precise, not of something, but rather the degree as such?" Such a degree surely does not exist, but the degree of utility is indeed no less comprehensible than the degree of warmth, heat retention, heat conduction, fusibility, hardness, transparency, etc., of bodies, and of memory, imagination, of people, etc. The only thing that is characteristic is that for utility in general we possess a word for its measure, for its gradation, and its degree.

That word, from the above definition, is of course "value". As we have seen, Rau used it to denote the degree of nutritive power, etc., in his discussion of species value, as did Knies in his concept of subspecies value. There seems to be no reason to exclude its application to concrete value, where it would mean the degree of utility as a function of the quantity consumed, i.e., marginal utility. This is precisely how Rau used the expression "concrete value".[111]

[111] And as we noted in section 6 above, this is exactly how Knies characterized Rau's concrete value.

13.2 Neumann

Friedrich Julius Neumann (1872, 1885–1896) was the writer who kept the subject alive for the longest time. In a consideration of the Rauian distinction between species and concrete value, he made a mistake shared by Lindwurm (1865), Roesler (1868b, 1871), and others of judging a concept by its formal definition rather than by the way it is used. He objected (1872, pp. 293–301) to Rau's and Roscher's notion of a one-to-one relationship between species of goods and species of needs (omitting to mention Friedländer's and Knies's development of this theme) on the ground that many goods served not one but a variety of different needs (e.g., wood as fuel, as material for furniture and construction, for paper). It is easier to make such an objection than to provide an alternative framework for analyzing complementarity and substitutability among various goods.

13.3 Wieser

Wieser's first work (1884), in which he introduced Jevons's concept of "final degree of utility" into German-language economics under the name of "marginal utility" (*Grenznutzen*) (p. 128), is particularly interesting in that it displays his background as a student of Knies[112] and retains many of the traditional German concepts, such as that of "requirement", which of course was also fundamental in Menger (1871)—a concept that requires the assumption of satiable preferences. Thus he speaks (p. 127) of "the basis of value for an individual good from a supply which does not exceed the requirement", and states (p. 130) that "value declines when, under otherwise equal circumstances, the quantity of goods becomes greater, or the desire for them—the requirement—becomes less." This is the same type of comparison as that made by Michaelis on the basis of the implicit utility function (16). Further, Wieser (and perhaps he had already learnt this from his reading of Menger (1871)) shows strong traces of the ranking of wants that ones finds in Lotz, Friedländer, and Knies, when in explaining the law of diminishing marginal utility in the case of a man travelling through a desert (p. 126) he says that the first ration is needed "in order to just keep himself alive, an additional ration ... to keep himself passably strong, and a third ration in order to maintain his complete vigor and mood", and of course a fourth one for his horse.

Of greatest interest to us is his discussion of species value, especially as interpreted by his teachers, Knies, Hildebrand, and Roscher. Wieser makes a distinction between *Nutzen*, which I translate here as "utility", and *Nützlichkeit*, which I translate here as "usefulness". He refers to Hildebrand's and Knies's version of Rau's theory (but without mentioning their names) as the theory of usefulness-value (*Nützlichkeitswerth*). He states (p. 131):

> From the facts of experience ... it is commonly inferred that the assumption that value is derived from utility cannot be upheld. Owing to the fact that an incongruity is observed between value and utility, it is then supposed that the measure of value could in no way be drawn from that of utility. In fact, however, nothing more is to be concluded than that the value of goods is not congruent with their usefulness, and that those numerous theories that have

[112] Knies succeeded to Rau's chair in Heidelberg, and Böhm-Bawerk and Wieser (who were close friends) studied under him there in 1875–1877 as well as with Roscher in Leipzig and Hildebrand in Jena. See Streissler (1990, pp. 44–45; 1999, p. 26n.)

> declared usefulness to be the source of value are false, either outright, or in their approach of, for example, sticking with the total utility of the supply, or to its average utility.

Note that in the last sentence Wieser departs from his own terminology in identifying "usefulness" with "utility". As suggested by the interpretation given in footnote 75 above, the Hildebrand-Knies theory could be interpreted as identifying species value with Schäffle's "usefulness" (Hildebrand's and Knies's "need") and concrete value with the "average usefulness"; then the *average usefulness* is congruent with the *marginal utility*, assuming, of course, constant expenditure shares. The Hildebrand-Knies theory can be legitimately criticized for making this special assumption, and for extending the resulting explanation to cases where the assumption clearly does not hold. But here Wieser seems to be making the same mistake as Menger (see footnote 66 above) in supposing that in the Hildebrand-Knies theory, it is the total *utility* (as opposed to the total *usefulness*) that is constant.

Wieser continues (pp. 131–132):

> The theories of usefulness-value—or as one can also refer to them, of species value—arose from the obvious idea of adopting a connection between value and utility in a form that was offered by common usage of the language, without having examined the causes which rendered the connection necessary, and without having clarified the nature of the connection through a determination of the economic conditions. One commonly speaks of the values of iron, of gold, and of food, and so forth, which, construed grammatically, mean the value of the species iron, the species gold, and so on. What in fact is meant, however, is not the species, or the quantities occurring over the entire earth or in entire national territories—because there is not the slightest reason to think of these from an economic perspective—but rather, what is meant is the goods-units with which, or with whose totals, one must pursue economic activity almost exclusively. If, for example, one says that the value of gold is greater than that of iron, one can only mean that the unit weight of gold is worth more than the unit weight of iron. This carelessness of expression, which in the course of business misleads no one, has become fatal for those theoreticians who have been under the charm of the common usage of the language, without having any other aids than those of the art of grammatical and logical interpretation. Owing to the fact that they adhered to the common expression instead of investigating the economic facts, they understood value to be species value, and had to point to the usefulness which is peculiar to the species, instead of to the marginal utility, which pertains to the unit—a mistake that never could have arisen if the theory of value had been conceived of from the outset as an empirical one and pursued with all empirical means.

If the first sentence is a criticism of Hildebrand and Knies ("without having examined the causes which rendered the connection necessary"), it is plain wrong, inasmuch as Hildebrand made clear his assumption (which he even called a "law") of constant expenditure shares. With regard to the second and third sentences, Rau explictly stated that the "species value"

was to be reckoned *per unit*,[113] so it is absurd to imagine that he meant the values of all the stocks in the world.[114]

As for the fourth and fifth sentences, I do not know how a factual statement that the value of gold per ounce is greater than the value of iron per ounce can involve "carelessness of expression"; this criticism seems to be directed towards Roscher, who referred to the facts of his day in saying that the ratio of these two values was 25,000, and for which he supplied a reasonable explanation under the assumption of constant expenditure shares. But the fifth sentence contains the greatest distortion of all: "they understood value to be species value"! Who was it, if not Rau, who first introduced "concrete value", which is "marginal utility" under another name, and *distinguished* it from species value? Hildebrand's main result was that marginal utility ("use value") remains proportional to price ("exchange value")—a proposition which up to that time had been controverted by most of the profession (although strongly affirmed by Rau—see especially formula (2) above). Finally, Wieser is accusing Rau, Hildebrand, Knies, and Roscher of not conceiving their problem as an empirical one! How anybody can read their works and draw this conclusion, I do not know.

A correct criticism would have been that Rau in the early editions of his work assumed marginal utility to be a discontinuous function of quantity in the case of goods for which demand was satiable—although he made a number of qualifications to this, and ended up with a continuous function (see footnote 12 above). And Hildebrand and Knies could have been legitimately criticized for adopting a model which assumed unitary price-elasticity of demand when the data suggested that demand for foodstuffs was inelastic; further, Knies could have been appropriately faulted for thinking that he could resolve this problem by changing his assumptions midstream, in reasoning that a decrease in consumption of foodstuffs x_j would not only lower the denominator of the quotient θ_j/x_j, but also raise its numerator θ_j, i.e., increase the "need" for food, resulting in an increase in the total value, $\theta_j Y$, of the x_j units—but then Wieser would not be entitled to accuse him of assuming that the "total utility" (his terminology for Knies's "need") of the amount consumed remained constant. Knies could also have been fairly criticized for making general statements to the effect that the concrete value of a good was equal to its *average usefulness* (see what I called his "puzzling statement" in section 6 above) when Hildebrand's theorem that *average usefulness* = *marginal utility* was true only in the case of constant expenditure shares (unitary price elasticity), and thus in declaring that *average usefulness* rather than *marginal utility* was the general principle. But Wieser seems to have avoided this real issue.

In his second major work (1889), Wieser devoted a chapter to the "Paradox of Value" (1889, §. 10, pp. 27–32; 1893, Ch. 10, pp. 27–32); this paradox has come to be known as "Wieser's paradox" (cf. Stigler (1950, p. 316n; 1965, p. 86n)). It amounts to this: if a linear

[113] This specification was not adhered to by Hildebrand and Knies, since as we have seen, they identified species value with the share of income devoted to the species. Thus, these are two quite distinct notions of "species value". Nevertheless, Wieser's suggestion makes no more sense with respect to the Hildebrand-Knies concept than with respect to Rau's.

[114] Here, Wieser seems to be simply following Menger, who in his criticism of Hildebrand stated (1871, p. 109; 1950, p. 297): "In the above context, nothing else can be properly understood by the value of a 'species of goods' than: the value which the totality of available goods of a species has to human society" (again I have revised the 1950 translation). Rau did not speak of a "value of a species of goods"; rather, the "species value" (*per unit*) of a good. And both Menger and Wieser appear to have overlooked Hildebrand's assumption of constant expenditure shares.

function is postulated for marginal utility (an assumption in accordance with the traditional German conception of satiable wants, but which Gossen (1854)—as we have seen—had already rejected, and Rau (1868) had already made obsolete), then the product $x_j u_j'(x_j)$ (the "total value" expressed as the quantity times the marginal utility) will rise to a maximum and then fall. (This product is of course the counterpart of the constant Hildebrand-Knies function $x_j u_j'(x_j) = \theta_j$ resulting from the subutility function $u_j(x_j) = \theta_j \log x_j$ of formula (5) above.) For example, choosing $u_j'(x_j) = c_j(1 - x_j/b_j)$ as in (18), the "total value" reaches a maximum at $x_j = b_j/2$ and becomes zero at $x_j = b_j$.[115] What to Wieser was "paradoxical" was the declining portion where demand is inelastic (this of course was Knies's "second stumbling-block"; the rising elastic portion was what had concerned Roesler). This then is no different from Proudhon's paradox. While Proudhon's name was mentioned (1889, §. 11, p. 32; 1893, Ch. 11, p. 32), no reference was made to his work, nor was any mention made of Hildebrand, of Knies, or of Cordier's data cited by Roscher. Indeed, Wieser ignored these data in his statement: "As a matter of fact, human economies move almost entirely in the ascending branch. ... It is always some unusual accident when individual branches of economy are transferred to the descending branch in the movement of value" (1889, §. 10, p. 31; 1893, Ch. 10, pp. 31–32). So much for his censure that the German economics was not "conceived of from the outset as an empirical one and pursued with all empirical means"! As Stigler (1950, p. 316; 1965, p. 86) has remarked concerning this work and the one by Böhm-Bawerk next to be discussed: "These men did not improve on the substance of the theory—in fact it deteriorated in their hands—so we shall pass them by."

13.4 Böhm-Bawerk

Finally I consider the opinion of Knies's other illustrious student, Böhm-Bawerk. In his 1886 retrospective he cites Neumann's (1885) survey of value theory quite frequently, and follows Neumann in evaluating various value concepts in terms of their formal definitions rather than according to how they are used. Citing the eighth edition of Rau (1868, §62, pp. 94–5) (the same pages on which Rau introduced the continuous marginal-utility function shown in footnote 12), here is what Böhm-Bawerk has to say (1886, p. 17):

> I regard the "abstract species value" as a completely misguided conception. It does not exist—to the extent that one understands by value in general an actual significance of goods for people; rather, all value that exists at all is concrete value.

Then he cites approvingly the passage from Schäffle (1873) quoted above following footnote 78. He does not say what he means by value "existing at all". As we have seen, "species

[115] Wieser did not display his marginal-utility function in algebraic form, but only in tabular form in terms of an illustration in which he assumed $c_j = 11$ and $b_j = 1$. Nor did he attempt to integrate his marginal-utility function to obtain the (quadratic) total (sub-)utility function $u_j(x_j) = c_j(x_j - x_j^2/(2b_j))$ of (16), which reaches a maximum at $x_j = b_j$ and becomes zero at $x = 2b_j$. In his text he sometimes appears to treat his expenditure formula $x_j u_j'(x_j)$ as if it were the same thing as the subutility function $u_j(x_j)$ (e.g., he refers to the case $x_j > b_j/2$ —as opposed to the case $x_j > b_j$—as one of "superfluity" (*Ueberfluss*) on p. 31 of both editions); in fact he even describes it as "an abridged utility formula" (1889, p. 34n; 1893, p. 34n). It is perhaps for this reason that he (and Menger) thought that the corresponding value formula $x_j u_j'(x_j) = \theta_j$ under Hildebrand's and Knies's assumptions corresponded to "utility". See footnotes 66 and 75 above.

value" was the name given by Rau to the old Smithian concept, while "concrete value" was his own innovation; and Rau himself stipulated that the concept required for explaining market prices is exclusively concrete value. And Böhm-Bawerk does not address the question of whether current market prices are the correct prices to use in evaluating a country's wealth or welfare, which was one of Rau's chief concerns.

Böhm-Bawerk goes on to say (pp. 17–18):

> ... mere membership in a species does not accord to the goods anything other than participation in the objective attributes of the species, and therewith in the capability of use characteristic of the species. This is, however, too little to justify some kind of importance for human welfare .... An actual importance always presumes a dependency of human well-being on goods, and these, in turn, ... presuppose a certain scarcity of supply [*Vorrat*]. This latter element, however, is never specific to a species as such, but rather always emerges only out of a concrete situation in which a species is "scarce".

Of course, this is what Rau had said himself, and is the reason for his introduction of the "concrete value". And had Böhm-Bawerk turned the page in his copy of Rau and read the next section, he would have found the following statement (1868, §62a, p. 96):

> The influence of the requirement and [one's] holdings on the estimation of use value, particularly in the case of luxuries, is most decisive. If, further, one cannot precisely indicate how much one needs of a number of goods which serve for pleasure (luxury items), there is indeed an amount of them the excess over which is perceived as surplus, and even within that amount the concrete value of an individual unit or quantity typically becomes smaller the higher an owner's total supply rises.

This of course is Rau's final statement of the principle of diminishing marginal utility, starting with the statements from the fifth and subsequent editions quoted in section 2 above. It is disappointing indeed that a scientist of Böhm-Bawerk's stature would so distort the contributions of the one predecessor who, more than any other, made the contributions of the Austrian school possible.

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