

Econ 8181, Topics in Microeconomics: Welfare Economics

Homework Problem Set #1

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1 Excise taxes and welfare

Suppose a simple general-equilibrium economy with two commodities produced according to the Ricardian production-possibility frontier

$$(1.1) \quad b_1 y_1 + b_2 y_2 \leq l,$$

where y_i is the output of commodity i and l is the endowment of labor. b_1 and b_2 are fixed factor-output coefficients. Suppose all consumers have identical utility functions

$$(1.2) \quad U(x_1, x_2) = x_1^{\theta_1} x_2^{\theta_2}, \quad \theta_i > 0, \quad \theta_1 + \theta_2 = 1,$$

where x_i is the consumption of commodity i . The government now decides to impose an excise tax on commodity 1 equal to τ_1 times the unit cost of production $w b_1$ of commodity 1, where w is the wage rate of labor (taken as given).

Assume that consumers maximize (1.2) subject to the budget constraint

$$(1.3) \quad p_1 x_1 + p_2 x_2 \leq Y,$$

where p_i is the market price of commodity i , equal to its unit production cost including the tax, and where Y is disposable income, equal to the wage income plus the tax revenues, i.e.,

$$(1.4) \quad Y = wl + R(\tau_1, 0),$$

where the tax revenues are determined from

$$(1.5) \quad R(\tau_1, 0) = \tau_1 \cdot w b_1 \cdot x_1.$$

- (a) Compute the indirect utility function and express the community's welfare as this utility evaluated at the tax rate τ_1 .
- (b) Show that imposition of this tax reduces the community's welfare.
- (c) Calculate the welfare loss according to the method of consumer's surplus, where:
 - (i) the consumer's surplus is calculated on the assumption that disposable income remains at its initial level $Y = wl$;

- (ii) the tax revenues are also calculated on the assumption that disposable income remains at its initial level $Y = wl$.
- (d) Suggest a way to compare the two methods of evaluating the welfare effect of an excise tax.

2 Public goods and misrepresentation of preferences

Consider an economy in which two individuals start out with half a unit each of a private good, which can be invested to produce a public good according to the production transformation relation

$$(2.1) \quad F(x, y) = x + y - 1 = 0,$$

where $x = x_1 + x_2$ is the amount produced and consumed of the private good, x_i being the amount of the private good consumed by individual i , and y being the amount produced, as well as consumed by both individuals, of the public good. These two individuals are assumed to have identical preferences for the private and public good represented by the utility functions

$$(2.2) \quad U_i(x_i, y; a) = x_i + \sqrt{ay}, \quad i = 1, 2,$$

where $0 < a < 1$.

The government is trying to determine how much of the public good to produce, the cost of which (in terms of units of the private good) has to be jointly defrayed by the two individuals. It does so by providing a questionnaire to each individual asking how much of the public good that individual would like to have available, given each hypothetical share t_i in the cost of the public good. The government then chooses t_1 and t_2 in such a way that both individuals agree on the amount desired of the public good.

- (a) Assume that each individual i furnishes a demand function to the government specifying the amount of the desired public good for each level of t_i , and that this demand function is obtained by maximizing (2.2) subject to the budget constraint

$$(2.3) \quad x_i + t_i y = 1/2,$$

$1/2$ being the amount of the private good that each individual starts out with. Find y as a function of a .

- (b) Now assume that in order to evade full payment of their shares of the cost of the public good, both individuals furnish falsified demand functions on the questionnaire, derived from maximization of the respective utility functions

$$(2.4) \quad U_i(x_i, y; b) = x_i + \sqrt{by}, \quad i = 1, 2,$$

where $0 < b < a$. Show that, in terms of their true utility functions (2.2), both of the individuals will end up worse off in the equilibrium obtained by misrepresenting their preferences by (2.4) than if their true utility functions (2.2) had been used to generate the demand functions in the questionnaire.