A Reconsideration of the "Elasticity Approach" to Balance-of-Payments Adjustment Problems

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1 Introduction

According to Hegelian dialectics, thought is supposed to progress by means of a process of thesis, antithesis, and synthesis. One would like to think, therefore, that those branches of economic theory that are most characterized by continued controversy are the ones making the most rapid progress, and that those fields in which theses are put forward with the greatest degree of dogmatism, and antitheses with the greatest amount of vehemence, are the ones in which our knowledge of the actual workings of the economy is the greatest. Unfortunately, however, the opposite seems to be true. Dogmatic self-assurance, coupled with ringing denunciation of the work of others, seems not to be an indication of knowledge, but rather a symptom of our deep ignorance of the actual facts.

The following thesis was put forward by John Stuart Mill in 1848 (Vol. 2, Book 3, Chapter 22, §3, p. 175): "It thus appears, that a depreciation of the currency does not affect the foreign trade of the country: this is carried on precisely as if the currency maintained its value." A century and a quarter later, essentially the same thesis—but with a subtle and significant difference—was put forward in the following words (Dornbusch, 1975): "It is well known by now, and indeed may have been known to the attentive reader of Meade's work for twenty years, that an exchange rate change in and of itself will exert no real effects. It is rather because some other nominal variable is fixed that an exchange rate change represents a real change and can therefore be expected to affect other real variables, among them the trade balance."

Mill's thesis was stated in a form that was empirically refutable, at least in principle; and one of his most faithful followers, upon the conclusion of a number of painstaking empirical studies of the the international adjustment mechanism, had this to say about it (Taussig, 1927, pp. 338–9):

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> The older writers assumed that the main currents of international trade were not affected by the substitution of paper money for metallic ... Differences between the monetary systems of the trading countries simply caused the counters of exchange in each of them to be different, nothing more...
>
> It seems never to have occurred to these writers to inquire what would happen, if, under paper conditions, a *status quo* were disturbed.

Taussig went on to state (p.345):

> This then is the first proposition, and a fundamental one. In the absence of a common monetary standard, the rate of foreign exchange depends on the mere impact of the two quantities on hand at the moment.

There followed (pp. 346–358) an account that can be easily recognized today as containing the essence of what Machlup (1955, p. 172) has called the "relative-prices approach"—but which is still commonly referred to as the "elasticity approach"—to the the theory of balance-of-payments adjustment under variable exchange rates. Particularly noteworthy in Taussig's account—presented as a description of a hypothetical course of events following a disruption of the *status quo* by an autonomous capital movement—is his explicit assumption (p. 350) that "nothing happens to change the quantity of money in either country, or to change the velocity of circulation, the use of credit substitutes, and so on," from which follows "the fixity of the *total money income* of the population" and consequently "stability of domestic prices." In sum (p. 348): "Under specie the level of domestic prices in each country will be changed ... But with dislocated exchange, the level of domestic prices in each will remain as it was before ... Under paper, ... a new and different normal quotation for foreign exchange will be established."

Taussig's theory is the antithesis of Mill's. On the face of it, the doctrine put forward by Dornbusch in the above quotation would seem to be an admirable synthesis. It affirms Mill's thesis—with the qualification that a change in the exchange rate must be "an exchange rate change in and of itself." It proceeds to adopt the Taussig antithesis—but couched as a theory of exchange-rate change not "in and of itself." Unfortunately, however, no method is furnished to help us distinguish empirically between an exchange-rate change that is and one that is not "in and of itself." In effect, an exchange-rate change "in and of itself" is defined as one for which Mill's thesis is true. The synthesis has transformed the thesis into a tautology.

Taussig had already elaborated his theory in 1917, and one among his many illustrious students—Frank Graham—had subjected it to empirical test in 1922. In this important paper Graham made a clear distinction between internationally traded commodities and "domestic commodities"—i.e., those not entering international trade—and he found confirmation of Taussig's theory in the fact that in periods of heavy borrowing, prices of domestic commodities in the United States would rise relatively to international commodities, and the value of gold in terms of dollars would fall. Graham reaffirmed this thesis in 1948 (p. 199), and in his final 1949 account was as explicit as he could be; referring to "the case of a currently debtor country of independent currency and freely mobile exchange rates" he said (Graham, 1949, p. 2): "The fall in the exchange value of the currency of any such country would raise the *domestic-currency* price of the internationally traded commodities but would leave the prices of domestic commodities unchanged."

I have made a point of going into some of the background of the "relative-prices approach" to balance-of-payments theory in order to counter an impression that has recently been created that this approach, as exemplified, say, by the precise analytic statement to be found in Machlup's seminal paper on the foreign exchanges (1939, 1940), is no more than an historical aberration of the depression years.[1] In the words of Frenkel and Johnson (1976b, p.29):

> As documented in the next section, the monetary approach to balance-of-payments theory has a long, solid, and academically overwhelmingly reputable history. The continuity of its development, however, was reversed and the approach suppressed in international economic theory for upwards of a quarter of a century by the events of the 1930s.

It is not my aim to prove that the relative-prices approach has a longer, solider, or more overwhelmingly reputable history than the monetary approach. The proposition that relative prices have an important—in fact, essential—role in the adjustment process is one that can be defended quite well on its merits without reference to the length of its pedigree. However, recollection of the historical origins of the relative-prices approach is helpful as a reminder that it is not as narrowly based as its critics are wont to affirm, and that there is more to it than will be found in the caricature usually set up by its adversaries and indeed by some of its proponents.

That the "elasticity approach" should have come under fire is not surprising, for it does need defending. The problem with it is that it is, and has remained for far too long, just an "approach" and not a tightly-knit "theory". It has never been shown how it can be imbedded in a neoclassical general-equilibrium model. My aim in this paper is to show how this can be done.

The formal framework of the "elasticity approach" goes back—not as early as Hume's times, I regret to say, but a good deal earlier than the Great Depression—to Bickerdike's verbal and geometric analysis of 1906, supplemented by his important algebraic analysis of 1907, greatly enriched by Edgeworth's masterly treatment of 1908, and capped by Bickerdike's 1920 contribution to the theory of foreign exchange. While the latter contribution was apparently buried in obscurity until unearthed by Metzler in 1948, the same cannot be said for the three earlier ones; on the contrary, being the analyses that gave birth to the theory of optimal tariff, they were widely cited and the subject of lively discussion in the 1940s in Cambridge, by Kaldor (1940), Kahn (1947–8), Graaff (1949–50), and others, and in particular were well known to Mrs. Robinson (1946–47, p. 107n).[2]

[1] Taussig and his students were not the only ones to have rejected Mill's thesis on the basis of empirical evidence accumulated before the Great Depression. Bresciani-Turroni (1924) did likewise. And not surprisingly, we can trace the Taussigian point of view at least back to Malthus (1811, esp. pp. 342–3).

[2] Taussig's book was also cited by Mrs. Robinson in the same article (1946–47, p. 11), and his analysis was described as "the 'neoclassical' account of capital movements." In her 1947 treatment of the Foreign Exchanges (1947a, p. 134n), she stated: "It will be obvious that my main endeavor is to elaborate the hints thrown out by Mr. Keynes in his *Treatise on Money*, Chap. 21." If we consult this chapter (Keynes, 1930), which was written before the Great Depression, we find a neoclassical

But then a curious thing happened. In her 1947 treatment of the theory of foreign exchange, Mrs. Robinson set forth her famous expression for the effect of devaluation on the balance of trade in a brief footnote with the starkest of explanations (p. 142). Essentially the same formula was displayed by Metzler (1948, p. 226), with no explanation at all. These papers, together with Machlup's 1939–49 article, were required reading for a generation of graduate students, who demanded clearer explanations of the formulas and more precise clarification of the relations between the various demand and supply elasticities for imports and exports and the elasticities of supply and demand for foreign exchange. There ensued the well-known expositions of Haberler (1949), who acknowledged the assistance of Hyman Minsky; of Ellsworth (1950), with a mathematical appendix by Bronfenbrenner (1950); and of Allen (1954), with an appendix due largely to Lionel McKenzie. These were all within the kind of framework with which economists were most comfortable at the time—that of partial equilibrium—although these authors believed—correctly, as it turns out, but they were unable to show it—that their analyses could be given a valid general-equilibrium interpretation. The "elasticity approach" has come to be identified very largely with these types of expositions.

On the other hand, the original Bickerdike-Edgeworth analysis—despite a distressing gap which, by their own acknowledgment, they were unable to fill—is essentially a general-equilibrium one. My task is to show how the gap can be filled and the analysis made complete. The first important point to notice is that, following in the tradition of Jevons and Marshall, they took prices to be the dependent variables and quantities (of import and exports) as the independent variables; that is, they dealt with *inverse demand functions* (cf. Samuelson, 1950, p. 377) rather than with the more familiar (to us) *direct demand functions* expressing quantities as functions of prices and income. This has three important implications in the analysis to be presented below: (1) income in this formulation, far from being neglected, becomes a dependent variable, along with expenditure ("absorption") and thus the balance of payments; (2) the natural adjustment process is one of Marshallian non-tâtonnenment type (Marshall; 1879, Samuelson, 1947, p. 266) as opposed to the Walrasian tâtonnement type as formalized by Samuelson (1947, p. 270) and Arrow, Block, and Hurwicz (1959); (3) the use of inverse demand functions allows one to handle the case (arising from flat or ruled production-possibility surfaces) of

argument along Taussigian lines, in terms of relative price changes and resource reallocation, with not even the contemplation of unemployment. It is obviously and strongly influenced by Taussig, of whose work Keynes says (p. 334n): "his treatment of the influence of international investment on the price-levels in different countries is far in advance of any other discussion of the subject." There follows an analysis that is pure Taussig (Keynes, 1930, p. 358): "If the exchange-rate is altered so as to depreciate the local money to an appropriate extent, equilibrium is restored by *raising* the price of foreign-trade goods whilst leaving that of home-trade goods unchanged, thus attracting entrepreneurs towards an increased production of the former with the consequence of increasing the surplus production of foreign-trade goods." Essentially the same amount is found subsequently in Meade (1951, p. 234), but without reference to Keynes: "The consequence ... will be that the price in B of all B's foreign-trade products will tend to rise relatively to the price of B's home-trade products ... The process of readjustment of the balance of trade ... can now be regarded as essentially a matter of the consequential shift of demand and supply between foreign-trade and home-trade products in A and B." It should be added that all these accounts owe much for their development to Ohlin (1928).

multivalued excess-demand functions with simple calculus methods.

The second point concerns the constant unit of measure in which prices of the import or export goods are expressed in each country. With the benefit of hindsight it seems surprising that Bickerdike and Edgeworth found such difficulty justifying this assumption. In apologetic tones Bickerdike said (1906, p. 533): "it is implied that money can be regarded as a constant measure, which is not a legitimate supposition when we consider the producers and consumers to belong to different 'nations'." Edgeworth suggested (1908, p. 542): "We might imagine the national money in Mr. Bickerdike's system to be an inconvertible (or at least unexportable) currency, regulated, as some theorists have proposed, so that its value should remain constant. Constancy of value might be secured by one of the methods of measuring the value of money which I have elsewhere described ...", which *could* mean stabilizing the average price level of other, nontraded, commodities, though Edgeworth does not say so explicitly. The obvious (to us) solution was suggested by Graaff (1949–50, p. 53n), of introducing a domestic good which plays the role of numéraire.

So formulated, the "elasticity approach" is simply a model of a neoclassical two-country world with two traded goods and one nontraded good in each country, the latter acting as numéraire in each case, this being made possible by inconvertible currencies, appropriate monetary policies, and a flexible exchange rate. These assumptions are essential, but no others. To reproduce the standard Robinson-Metzler formulas (and to furnish the traditional Haberler-Ellsworth diagrammatics with a correct general-equilibrium interpretation) one must make the rather artificial assumption of zero cross-elasticites of inverse trade-demand functions; but this assumption is easily removed, resulting of course in still more fearful formulas (and in sacrificing the diagrammatics). The case of "infinite elasticity of supply of exports" becomes simply the case of a single factor of production and specialization in the export and domestic goods, yielding a flat (straight-line) Ricardian production-transformation locus between these two goods. And finally, the demand and supply curves for foreign exchange, introduced by Haberler (1933) and Bresciani-Turroni (1934) and most fully elaborated and developed in the classic article by Machlup (1939–40), turn out to be equilibrium loci of precisely the same kind as Marshall's reciprocal demand curves; and they can be defined quite generally without the need to assume zero cross-elasticities.

So reconstituted, how well does the "elasticity approach" stand up to the "absorption approach" and to the "monetary approach" to balance-of-payments adjustment theory? I take this question up in the final section.

2 The Reconstituted Model

It will be assumed that there are two countries, each of which is capable of producing three commodities, the first two of which are tradable (with zero transport costs) and the third nontradable.

Let p_i^k denote the price of commodity i in country k, denominated in country k's currency, and q_i^k the same, denominated in gold; let r^k be the value of country k's currency in terms of gold. Then

$$p_i^k r^k = q_i^k \quad (i = 1, 2, 3;\ k = 1, 2). \tag{2.1}$$

Let the exchange rate χ be defined as the price of a unit of country 1's currency in units of country 2's currency. Then, since $q_i^1 = q_i^2$ for $i = 1, 2$ (this equilibrium condition will be assumed to hold instantaneously), we have

$$\chi = \frac{r^1}{r^2} = \frac{p_1^2}{p_1^1} = \frac{p_2^2}{p_2^1} = \frac{p_3^2}{p_3^1} \cdot \frac{q_3^1}{q_3^2}. \tag{2.2}$$

It will be convenient to introduce the parameter

$$\mu = \frac{p_3^1}{p_3^2}, \tag{2.3}$$

so that (2.2) becomes

$$\mu\chi = \frac{q_3^1}{q_3^2}. \tag{2.4}$$

Under a gold standard, χ is fixed; under an inconvertible paper currency regime with flexible exchange rates of the type specified by Taussig and Graham, it is μ that is fixed. Each regime carries with it specific implications about monetary policy and monetary behavior; in a complete model these should be brought in explicitly, as proponents of the monetary approach quite properly insist. This will not be done here, since my purposes are more limited. Suffice it to say that in all the analysis that ensues, it is the ratio q_3^1/q_3^2 of gold prices of the domestic goods in the two countries that is the important variable; as far as the model is concerned, there is nothing to distinguish the gold-standard and the flexible-exchange-rate regimes other than the trivial substitution of $1/\mu$ for χ in the dynamic-adjustment process. Of course, this does not mean that there are not significant differences between such regimes in the real world, from which we are abstracting only a small part.

The rates of consumption and production of commodity i in country k will be denoted x_i^k and y_i^k respectively, and the excess of consumption over production by $z_i^k = x_i^k - y_i^k$, for $i = 1, 2, 3$ and $k = 1, 2$. Country k will be assumed to have a production-possibility set (or "production block" in Meade's terminology) denoted $\mathcal{Y}^k(\ell^k)$, where ℓ^k is a vector of factor endowments in country k; for example, if $y_i^k = f_i^k(v_i^k)$, where f_i^k is the production function in industry i in country k and v_i^k the vector of factor inputs in industry i, then under the usual Heckscher-Ohlin-Lerner-Samuelson assumptions of perfect factor mobility, cost minimization, and competitive factor markets, $\mathcal{Y}^k(\ell^k)$ is the set of output vectors $y^k = (y_1^k, y_2^k, y_3^k)$ which can be obtained from these production functions subject to the constraint

$$\sum_{i=1}^{3} v_i^k \leqq \ell^k$$

(see for example Chipman, 1974a, p. 27).

I shall continue in the time-honored tradition of international-trade theory and assume that individual preferences are identical and homothetic, so that an aggregate utility function $U^k(x^k) = U^k(x_1^k, x_2^k, x_3^k)$ exists which the community as a whole may be assumed to be maximizing (see Chipman, 1974b). It is more convenient to deal with consumption and production simultaneously and to express utility as a function of the excess of consumption over production, in the manner of Meade (1952). To obtain a formal counterpart of Meade's "trade-indifference curves" we define country k's *trade-utility function* (see Chipman, 1974a, p. 34n) by

$$\hat{U}^k(z^k;\ell^k) = \max\left\{U^k(x^k) : x^k \in \mathcal{Y}^k(\ell^k) + z^k\right\}. \tag{2.5}$$

We now introduce the concept of an *inverse trade-demand function*, which is the straightforward counterpart of the inverse demand function (also called "indirect demand function") introduced by Samuelson (1950) (see also Katzner, 1970, p. 44):

$$\begin{aligned} P_i^k(z^k;\ell^k) &\equiv \frac{\hat{U}_i^k(z^k;\ell^k)}{\hat{U}_3^k(z^k;\ell^k)} \qquad (i=1,2) \\ P_3^k(z^k;\ell^k) &\equiv z_1^k P_1^k(z^k;\ell^k) + z_2^k P_2^k(z^k;\ell^k) + z_3^k. \end{aligned} \tag{2.6}$$

The first two functions define the "marginal trade-rate of substitution" between commodities i and 3, or the "excess-demand price of commodity i" in terms of commodity 3 (the numéraire). The third function defines the excess of desired expenditure (or "absorption" in Alexander's 1952 terminology) over the national income that will be forthcoming given the amounts by which consumption exceeds production.

We shall stipulate as one of the defining properties of "equilibrium" that there is no inventory accumulation or decumulation. In that case, the amount z_i^k must correspond in equilibrium to the quantity of commodity i imported into country k (if positive) or exported from country k (if negative); accordingly, in equilibrium we have

$$z_i^1 + z_i^2 = 0 \quad (i=1,2); \quad z_3^k = 0 \quad (k=1,2). \tag{2.7}$$

(We shall assume that, initially and throughout the analysis, country 1 exports commodity 1 to country 2 and imports commodity 2 from country 2; hence, z_1^2 are the (positive) exports of commodity 1 from 1 to 2, and z_2^1 the (positive) imports of commodity 2 into 1 from 2.) In equilibrium, therefore, the third function of (2.6) also stipulates the deficit in country k's *balance of payments on current account* (expressed in terms of country k's currency) as a function of the quantities exported and imported.[3] Just as we assumed above that the equilibrium condition $q_i^1 = q_i^2$ $(i=1,2)$ holds instantaneously, henceforth the equilibrium conditions (2.7) will also be assumed to hold instantaneously.

Accordingly, defining country k's balance-of-payments deficit on current account (in terms of its own currency) as

$$d^k = p_1^k z_1^k + p_2^k z_2^k \quad (k=1,2) \tag{2.8}$$

[3]This formulation brings out a difference between the concepts employed here and the conventional accounting practices. In the usual formulation (see, for example, Alexander, 1952; Machlup, 1955; Johnson, 1958), involuntary inventory accumulation is (implicitly) included in investment and therefore in output and national income, so that the equality between the excess of income over expenditure and the balance of payments on current account is merely an accounting identity.

the following six equations must be satisfied in equilibrium:

$$(2.9)\qquad \begin{aligned} \frac{p_1^1}{p_3^1} &= P_1^1(-z_1^2, z_2^1, 0; \ell^1); & \frac{p_1^2}{p_3^2} &= P_1^2(z_1^2, -z_2^1, 0; \ell^2); \\ \frac{p_2^1}{p_3^1} &= P_2^1(-z_1^2, z_2^1, 0; \ell^1); & \frac{p_2^2}{p_3^2} &= P_2^2(z_1^2, -z_2^1, 0; \ell^2); \\ \frac{d^1}{p_3^1} &= P_3^1(-z_1^2, z_2^1, 0; \ell^1); & \frac{d^2}{p_3^2} &= P_1^1(z_1^2, -z_2^1, 0; \ell^2). \end{aligned}$$

Letting T denote an autonomous transfer from country 1 to country 2, if positive, or an autonomous transfer from country 2 to country 1, if negative, expressed (in either case) in terms of country 1's currency, we have

$$(2.10)\qquad d^1 = T, \quad d^2 = \chi T.$$

To complete the system, we have from (2.2)

$$(2.11)\qquad \chi = p_1^2/p_1^1, \quad \chi = p_2^2/p_2^1.$$

For fixed p_3^1 and p_3^2, this is a system of ten equations in the nine unknowns p_1^1, p_1^2, p_2^1, p_2^2, d^1, d^2, z_1^2, z_2^1, and χ. The equations are not independent, since from the definition of P_3^k in (2.6), either one of the equations on the third line of (2.9) follows from the remaining nine equations. That is, an excess of absorption over income in one country is necessarily equal (when expressed in the same currency) to an excess of income over absorption in the other country—a point that we shall have occasion to stress later on. In order to make our results comparable to Machlup's 1939–40 analysis of the supply and demand for foreign exchange, it will be convenient to retain the sixth equation of (2.9).

Accordingly, eliminating the variables p_1^1, p_1^2, p_2^1, p_2^2, d^1, d^2 from (2.9), (2.10), and (2.11), setting $p_3^2 = 1$ for convenience, and recalling the definition of P_3^k in (2.6) as well as condition (2.7), the system reduces to a system of three independent equations

$$(2.12)\qquad \begin{aligned} \mu\chi &= \frac{P_1^2(z_1^2, -z_2^1, 0; \ell^2)}{P_1^1(-z_1^2, z_2^1, 0; \ell^1)}; \quad \mu\chi = \frac{P_2^2(z_1^2, -z_2^1, 0; \ell^2)}{P_2^1(-z_1^2, z_2^1, 0; \ell^1)}; \\ \chi T &= z_1^2 P_1^2(z_1^2, -z_2^1, 0; \ell^1) - z_2^1 P_2^2(z_1^2, -z_2^1, 0; \ell^2); \end{aligned}$$

in the three unknowns z_1^2, z_2^1, and χ (the variables ℓ^1, ℓ^2, $\mu = p_3^1$, and T being parameters).

Under certain conditions[4] the first two equations of (2.12) may be solved for z_1^2, z_2^1 to obtain functions

$$(2.13)\qquad z_1^2 = F_1(\chi; \mu, \ell^1, \ell^2), \quad z_2^1 = F_2(\chi; \mu, \ell^1, \ell^2).$$

[4]The requirement is that

$$\begin{vmatrix} \pi_{11}^2 - \pi_{11}^1 & \pi_{12}^2 - \pi_{12}^1 \\ \pi_{21}^2 - \pi_{21}^1 & \pi_{22}^2 - \pi_{22}^1 \end{vmatrix} \neq 0$$

where the π_{ij}^k are defined by (3.2). As will be clear from formula (3.4), this is equivalent to the condition that a transfer have *some* effect on the exchange rate, or that the supply or demand for foreign exchange be not infinitely elastic.

For the case $T = 0$ we may now define the supply and demand functions for foreign exchange by substituting the functions (2.13) in the expressions in the third equation of (2.12):

$$
(2.14)\quad \begin{aligned} S_2(\chi;\mu,\ell^1,\ell^2) &= F_1(\chi;\mu,\ell^1,\ell^2)P_1^2[F_1(\chi,\mu,\ell^1,\ell^2),-F_2(\chi;\mu,\ell^1,\ell^2),0;\ell^2];\\ D_2(\chi;\mu,\ell^1,\ell^2) &= F_2(\chi;\mu,\ell^1,\ell^2)P_2^2[F_1(\chi;\mu,\ell^1,\ell^2),-F_2(\chi;\mu,\ell^1,\ell^2),0;\ell^2]. \end{aligned}
$$

Except for the unimportant difference that Machlup (1939–40) defines the exchange rate as $1/\chi$ rather than χ, these may be identified precisely with his supply and demand curves for foreign exchange, the parameters μ, ℓ^1, ℓ^2 allowing for shifts in these curves. For example, if $T = 0$ then it is clear from (2.12) that an "inflation" in country 1, which we can identify with a rise in μ, leads to a proportional devaluation of country 1's currency, that is, an equal percentage reduction in χ.[5] This could be described in terms of simultaneous shift in both curves of (2.14). Such an example was explicitly mentioned by Machlup (1939–40, p. 27).

3 Comparative Statics and Dynamics

As a simple dynamic-adjustment process corresponding to (2.12) we may postulate

$$
(3.1)\quad \begin{aligned} \dot{z}_1^2 &= \kappa_1\left\{\frac{P_1^2(z_1^2,-z_2^1,0;\ell^2)}{P_1^1(-z_1^2,z_2^1,0;\ell^1)}-\mu\chi\right\}\\ \dot{z}_2^1 &= \kappa_2\left\{\mu\chi-\frac{P_2^2(z_1^2,-z_2^1,0;\ell^2)}{P_2^1(-z_1^2,z_2^1,0;\ell^1)}\right\}\\ \dot{\chi} &= \kappa_3\left\{z_1^2P_1^2(z_1^2,-z_2^1,0;\ell^2)-z_2^1P_2^2(z_1^2,-z_2^1,0,\ell^2)-\chi T\right\} \end{aligned}
$$

where the κ_i are positive speeds of adjustment. For example, the first equation of (3.1) ensures that if the marginal rates of transformation between commodities 1 and 3 in the two countries are such as to result in the cost of producing the domestic good in country 1 relative to the cost of producing the domestic good in country 2 being less than the corresponding price ratio (computed in any common currency at the prevailing exchange rate), exports of commodity 1 from country 1 to country 2 will increase.

If $T \neq 0$, say $T < 0$, then the "equilibrium" state $\dot{\chi} = 0$ of the third equation of (3.1) will in general be one in which there is a continually recurring transfer of funds (whether in the form of reparations, immigrants' remittances, foreign aid, or private investment) from country 2 to country 1. It will not, except in an interesting special case discussed in the final section, entail any movement of international reserves from country 1 to country 2, and will therefore not be a situation of "balance-payments deficit" in Johnson's (1958) sense. This is worth emphasizing, since it brings out the fact that the "elasticity approach" has been designed primarily to deal with phenomena (international capital movements) specifically ruled out by assumption by proponents of the "monetary approach" (Johnson, 1958, p. 54).

[5] This example also brings out the importance of assuming $T = 0$ in order to obtain this purely "monetary" effect on the exchange rate. In fact, proponents of the monetary approach are well aware of this when they require stock-adjustment conditions that ensure $T = 0$ in equilibrium.

In the following development, it will be convenient to assume $T = 0$ in the initial state. This will enable us to reproduce the classical elasticity formulas, as well as to avoid inessential algebra.

Defining the elasticities of the inverse trade-demand functions

$$\pi_{ij}^k = \frac{z_j^k}{P_i^k}\frac{\partial P_i^k}{\partial z_j^k} \quad (i,j=1,2) \tag{3.2}$$

—termed "flexibilities" by Bronfenbrenner (1942), following Frisch—we may write the Jacobian matrix of (3.1), evaluated at $T = 0$, as $J = C_1 A C_2$ where

$$C_1 = \operatorname{diag}\left\{\kappa_1/(z_1^2 P_1^1), \kappa_2/(z_2^1 P_2^1), 1\right\}, \quad C_2 = \operatorname{diag}\left\{P_1^2, P_2^2, \mu z_1^2 P_1^1\right\}$$

and

$$A = \begin{bmatrix} \pi_{11}^2 - \pi_{11}^1 & \pi_{12}^2 - \pi_{12}^1 & -1 \\ \pi_{21}^1 - \pi_{21}^2 & \pi_{22}^1 - \pi_{22}^2 & 1 \\ 1 + \pi_{11}^2 - \pi_{21}^2 & -1 - \pi_{22}^2 + \pi_{12}^2 & 0 \end{bmatrix}. \tag{3.3}$$

From (2.12) we find that

$$\frac{1}{\chi}\frac{d\chi}{dT} = \frac{1}{\mu z_1^2 P_1^1}\frac{\begin{vmatrix} \pi_{11}^2 - \pi_{11}^1 & \pi_{12}^2 - \pi_{12}^1 \\ \pi_{21}^1 - \pi_{21}^2 & \pi_{22}^1 - \pi_{22}^2 \end{vmatrix}}{\begin{vmatrix} \pi_{11}^2 - \pi_{11}^1 & \pi_{12}^2 - \pi_{12}^1 & -1 \\ \pi_{21}^1 - \pi_{21}^2 & \pi_{22}^1 - \pi_{22}^2 & 1 \\ 1 + \pi_{11}^2 - \pi_{21}^2 & -1 - \pi_{22}^2 + \pi_{12}^2 & 0 \end{vmatrix}}. \tag{3.4}$$

From Metzler's (1945, pp. 282–3) results, as amended by Arrow (1974, pp. 184–5), we know that a *necessary* condition for the system (3.1) to be dynamically stable for all positive speeds of adjustment κ_i is that the diagonal elements of A be nonpositive, the second-order principal minors of A be nonnegative, and the determinant of A be negative. The latter condition is also necessary for stability given any positive speeds of adjustment (see Samuelson, 1947, p. 431). The condition that a system be stable independently of the adjustment speeds may for convenience be described as "Metzler stability," and the necessary conditions just described as the "modified Hicks conditions." Metzler stability then implies that $d\chi/dT \leqq 0$, that is, that a transfer from country 1 to country 2 will give rise to a devaluation of country 1's currency if it has any effect on the exchange rate at all.[6]

That there is a real possibility that a transfer will have no effect on the exchange rate at all follows at once from a previous result (Chipman, 1974a, Theorem 5) to the effect that if (i) production and utility functions are identical and homogeneous in both countries, (ii) all three commodities are produced in each country, and (iii) factor rentals are initially equalized between them (a virtual certainty under those conditions), a transfer will leave all prices unaltered. These conditions assure that the exchange rate will play a purely monetary role, that is, that it will be affected by μ but not by T.

[6]It is significant that this result depends on the particular dynamic-adjustment process (3.1) postulated and on its assumed Metzler stability at the initial point. Under a different type of dynamic-adjustment process, the corresponding result in terms of the relative gold prices of the domestic commodities in the two countries need not follow; see Chipman (1974a, p. 71).

Our analysis can also be used to examine the so-called "small-country case." This may be given a finite interpretation by assuming that country 2 is endowed with only a single factor of production so that, with constant returns to scale, its production-possibility surface will be a flat triangle. If country 2 is large enough relative to country 1, it will be a "focal country" in Graham's sense (1948, p. 58), its cost determining world prices. Then $\pi^2_{ij} = 0$ for $i, j = 1, 2$, and $d\chi/dT < 0$ if and only if $\pi^1_{11}/\pi^1_{12} > \pi^1_{21}/\pi^1_{22}$. If, now, we assume that country 1 also has a single factor of production, and specializes in commodities 1 and 3, then $\pi^1_{11} = \pi^1_{12} = 0$ and $d\chi/dT = 0$. Here, then, we have another interesting case in which the exchange rate plays only a purely monetary role.

4 The Separable Case

We now come to the special assumption that lies behind the "elasticity approach": additively separable trade-utility functions. This is the condition that $\partial P^k_i/\partial z^k_j = 0$ for $i, j = 1, 2$ and $i \neq j$, which is characteristic of preferences that can be represented by (trade-) utility functions of the form $\hat{U}(z) = \hat{u}_1(z_1) + \hat{u}_2(z_2) + \hat{u}_3(z_3)$. It is important to note that separability of the original utility function $U(x)$ does not imply that $\hat{U}(z)$.[7]

Let us set $\pi^k_{12} = \pi^k_{21} = 0$ for $k = 1, 2$, and define the "supply flexibilities" σ_k and "demand flexibilities" δ_k by

$$\sigma_1 = \pi^1_{11}, \quad \sigma_2 = \pi^2_{22}, \quad \delta_1 = -\pi^1_{22}, \quad \delta_2 = -\pi^2_{11}. \tag{4.1}$$

A necessary condition for stability of the system (3.1) is that

$$|A| = (1 - \delta_1)(1 - \delta_2) - (1 + \sigma_1)(1 + \sigma_2) < 0. \tag{4.2}$$

This condition was first obtained by Edgeworth (1908, p. 544n), via a different route (see Section 6 below).

From the modified Hicks conditions mentioned in the last section, a necessary condition for Metzler stability of (3.1) is that both $\sigma_1 + \delta_2 \geqq 0$ and $\sigma_2 + \delta_1 \geqq 0$. If either one of these terms is zero, then from (3.4) we have $d\chi/dT = 0$. If, on the other hand, both $\sigma_1 + \delta_2 > 0$ and $\sigma_2 + \delta_1 > 0$, then necessarily $d\chi/dT < 0$, and (3.4) yields (putting $c = 1/\mu z^2_1 P^1_1$)

$$\frac{1}{\chi}\frac{d\chi}{dT} = c \cdot \frac{-1}{\dfrac{1 - \delta_2}{\sigma_1 + \delta_2} + \dfrac{1 + \sigma_2}{\delta_2 + \sigma_2}}. \tag{4.3}$$

This is Bickerdike's original formula (1920, p. 12). A necessary condition for Metzler stability, when $\sigma_1 + \delta_2 > 0$ and $\sigma_2 + \delta_1 > 0$, is then

$$\frac{1 - \delta_2}{\sigma_1 + \delta_2} + \frac{1 + \sigma_2}{\delta_1 + \sigma_2} > 0. \tag{4.4}$$

[7]Separability of $U(x)$ is innocent enough, and is satisfied by many utility functions (e.g., the functions in the constant-elasticity-of-substitution family—see (8.1) below) employed in econometric work. It should be emphasized that additive separability of $U(x)$ does *not* imply zero cross-elasticities of the *direct* demand function. On the contrary, if $U(x) = \sum_i u_i(x_i)$, zero cross-elasticities would imply $u'_i(x_i) + x_i u''(x_i) = 0$, and this can be satisfied only by $u_i(x_i) = a_i + b_i \log x_i$, i.e., only if $U(x)$ is of the Mill-Cobb-Douglas type with unitary elasticity of substitution.

If, following Bickerdike (1907, p. 100n) and Edgeworth (1908, p. 541n), we define the "elasticities of demand for country 1's imports and exports" and the "elasticities of supply of country 1's exports and imports" (to avoid confusion I shall refer to them as *indirect elasticities*) as the *reciprocals* (when they exist) of the corresponding demand and supply flexibilities of (4.1), that is, as

$$\iota_k = \frac{1}{\delta_k}; \quad \varepsilon_k = \frac{1}{\sigma_k} \quad (k = 1, 2), \tag{4.5}$$

then Bickerdike's "stability condition" (4.4) becomes[8]

$$\frac{\iota_1\iota_2(\varepsilon_1 + \varepsilon_2 + 1) + \varepsilon_1\varepsilon_2(\iota_1 + \iota_2 - 1)}{(\varepsilon_1 + \iota_2)(\iota_1 + \varepsilon_2)} > 0. \tag{4.6}$$

This is Metzler's famous formula (1948, p. 226), to which Joan Robinson's (1947, p. 142n) reduces when $T = 0$. However, it is not a correct stability condition unless $(\varepsilon_1 + \iota_2)/\varepsilon_1\iota_2 > 0$ and $(\iota_1 + \varepsilon_2)/\iota_1\varepsilon_2 > 0$. On the other hand, Edgeworth's stability condition (4.2) becomes

$$\frac{\iota_1\iota_2(\varepsilon_1 + \varepsilon_2 + 1) + \varepsilon_1\varepsilon_2(\iota_1 + \iota_2 - 1)}{\iota_1\iota_2\varepsilon_1\varepsilon_2} > 0. \tag{4.7}$$

We now consider the case of "infinite elasticity of supply" of exports and imports, where $\varepsilon_1 = \varepsilon_2 = \infty$ (or $\sigma_1 = \sigma_2 = 0$). This means that the marginal trade rate of substitution between the export and domestic good is zero in both countries. The most natural condition leading to this result is the assumption that there is a single factor of production (say, labor) in each country, with each country specializing in its domestic and export goods. In this case, since the modified Hicks conditions require $\delta_k \geqq 0$, provided these are both positive the Edgeworth and Bickerdike conditions both reduce to the notorious and misnamed "Marshall-Lerner condition"

$$\iota_1 + \iota_2 - 1 > 0. \tag{4.8}$$

But before we too quickly accept this result we must consider the question: is the assumption of "infinite supply elasticities" consistent with the hypothesis that the trade-utility function $\hat{U}^k$ is separable? A moment's reflection will cause us to realize that the answer must be in the negative—except for an uninteresting freak case.

Let us assume that country 1 produces commodities 1 and 3 only, with a single factor of production (labor), by means of the production functions $y_i^1 = f_i^1(v_i^1) = b_i^1 v_i^1, i = 1, 3$, where $v_1^1 + v_3^1 = \ell^1, \ell^1$ being country 1's endowment of labor. Country 1's production-possibility set is then defined by

$$\mathcal{Y}^1(\ell^1) = \left\{(y_1^1, y_2^1, y_3^1) \geqq (0, 0, 0) : y_1^1/b_1^1 + y_3^1/b_3^1 \leqq \ell^1, y_2^1 = 0\right\}. \tag{4.9}$$

[8] It was noted by Jones (1961, p. 209) that for this formula to make sense it was "necessary to introduce a third, nontraded commodity serving as *numéraire*." However, he added that the analysis leading to (4.6) was necessarily of a "*partial*-equilibrium nature." The above development shows that this is not the case.

Writing the production constraints in the form

$$(4.10) \qquad \frac{x_1^1}{b_1^1} + \frac{x_3^1}{b_3^1} \leqq \frac{z_1^1}{b_1^1} + \frac{z_3^1}{b_3^1} + \ell^1; \qquad x_2^1 = z_2^1,$$

and noting that the first will hold with equality (i.e., there will be full employment) as long as U^1 is an increasing function, upon substituting (4.10) in the utility function we see from the definition (2.5) that the trade-utility function $\hat{U}^1$ has the form

$$(4.11) \quad \max_{x_1} U^1 \left[x_1^1, z_2^1, b_3^1 \left(\frac{z_1^1}{b_1^1} + \frac{z_3^1}{b_3^1} + \ell^1 \right) - \frac{b_3^1}{b_1^1} x_1^1 \right] = V^1 \left(\frac{z_1^1}{b_1^1} + \frac{z_3^1}{b_3^1} + \ell^1, z_2^1 \right)$$

for some function V^1. Denoting its partial derivatives by V_1^1 and V_2^1 respectively, we have from (2.6)

$$(4.12) \qquad \begin{aligned} P_1^1(z^1;\ell^1) &= \frac{b_3^1}{b_1^1}, \\ P_2^1(z^1;\ell^1) &= b_3^1 \frac{V_2^1(z_1^1/b_1^1 + z_3^1/b_3^1 + \ell^1, z_2^1)}{V_1^1(z_1^1/b_1^1 + z_3^1/b_3^1 + \ell^1, z_2^1)}. \end{aligned}$$

Thus $\pi_{11}^1 = \pi_{12}^1 = 0$ and we have an "infinite elasticity of supply of exports." However, in order to have $P_{21}^1 \equiv \partial P_2^1/\partial z_1^1 \equiv 0$ we must require that the function V_2^1/V_1^1 depend only on its second argument, z_2^1. It is not hard to see that this implies that V^1 must have the "parallel" form $V^1(\zeta_1^1, z_2^1) = \psi(\zeta_1^1 + W(z_2^1))$, where ψ is an increasing function and W is an increasing concave function (see Samuelson, 1942, p. 85; Chipman and Moore, 1976, p. 89). From (4.11) it follows immediately that

$$(4.13) \qquad \hat{U}^1(z^1;\ell^1) = \psi \left[\frac{z_1^1}{b_1^1} + \frac{z_3^1}{b_3^1} + \ell^1 + W(z_2^1) \right].$$

Now it can be shown that the original utility function U^1 is recoverable from the trade-utility function $\hat{U}^1$ by *minimizing* the latter subject to the production constraints (4.10); accordingly, the utility function must have the form

$$(4.14) \qquad U^1(x_1^1, x_2^1, x_3^1) = \psi \left(\frac{x_1^1}{b_1^1} + \frac{x_3^1}{b_3^1} + W(z_2^1) \right).$$

This is what is implied by allowing ε_1 to approach infinity and requiring $\hat{U}^1$ to remain separable.

In words, (4.14) states that consumers in country 1 regard one labor-hour's worth of commodity 1 as a perfect substitute for one labor-hour's worth of commodity 3, and that this amount of labor time can be considered as a standard of value for all three commodities (i.e., taking ψ to be the identity function, the marginal utility of either of the produced commodities is equal to the constant amount of labor time required to produce a unit of it). This requires tastes to be completely tied to the technology. It is surely a freak possibility.

Given this result, is there any way to salvage the "stability condition" (4.8)? One way to accomplish this *partially* is the following. Let us assume "infinite elasticities of supply of exports" in both countries. That is, let us assume that the relative (to the domestic commodity) supply price of the export good is a constant, equal to $P_1^1(z^1;\ell^1) = b_3^1/b_1^1$ for country 1, as in (4.12) above, and analogously, to $P_2^2(z^2;\ell^2)b_3^2/b_2^2$ for country 2. Then we have

$$\pi_{11}^1 = \pi_{12}^1 = 0; \qquad \pi_{22}^2 = \pi_{21}^2 = 0. \tag{4.15}$$

Under these conditions a necesary stability condition for (3.1) is, from (3.3),

$$|A| = \pi_{22}^1 + \pi_{22}^1\pi_{11}^2 + \pi_{11}^2 + \pi_{21}^1 - \pi_{21}^1\pi_{12}^2 + \pi_{12}^2 < 0. \tag{4.16}$$

The first three terms in this expression sum to $-(\iota_1+\iota_2-1)/\iota_1\iota_2$, so that a *sufficient* condition for (4.16) to follow from (4.8) is that the ι_k be positive (as assumed in deriving (4.8)) and that $\pi_{21}^1 \leqq 0$ and $\pi_{12}^2 \leqq 0$, that is, that in each country the export good should be weakly relatively trade-substitutable with the import good in relation to the domestic good (see Chipman 1974a, p. 71). From the above result this would actually require $\pi_{21}^1 < 0$ and $\pi_{12}^2 < 0$ almost everywhere, that is, that the export and import goods be strong relative trade-substitutes in this sense. This would allow us to say that fulfillment of (4.8) guarantees fulfillment of (4.16). But (4.16) is only a necessary, not a sufficient, condition for stability.

We can conclude that the separability assumption constitutes the one principal and serious defect of the "elasticity approach" as formulated heretofore. As applied to a pure-exchange model, for example to Haberler's (1950) short-run model, the assumption is innocuous enough, since as applied to the original utility function it is limiting but not far-fetched; but it leads only to trouble when extended to allow for production adjustments. But while it is a serious defect, it is by no means a fatal one; for it can easily be dispensed with, and there is no need to adopt it, as the above analysis has made clear.

5 The Supply and Demand for Foreign Exchange

In introducing the concepts of supply and demand for foreign exchange, Machlup (1939, p. 10) stated: "Every demand and supply curve must refer to a certain period of time which is allowed for the depicted changes and adjustments to take place," which he referred to as the "short period." He went on to state (p. 11):

> . . . it shows, for example, how the quantities of foreign exchange supplied by exporters will react upon a rise in the price of foreign currency after the export industries have adapted their selling prices in dollars to the increase in business.
>
> In other words, the short-period demand and supply curves of foreign exchange are not drawn on the basis of "given commodity prices" in the two countries, but on the basis of "given demand and supply conditions" in the commodity markets of the two countries.

This makes quite clear the *general-equilibrium* nature of Machlup's curves, showing that they are loci portraying equilibria in the commodity markets; they are thus of the very same nature as Marshall's reciprocal demand curves. This justifies the definition (2.14) given above.

From (2.12), (2.13), and (3.2) we may readily compute the elasticities of country 1's exports and imports with respect to the exchange rate:

$$
(5.1)\qquad
\begin{aligned}
\zeta_1^2 &\equiv \frac{\chi}{F_1} = \frac{\begin{vmatrix} -1 & \pi_{12}^1 - \pi_{12}^2 \\ -1 & \pi_{22}^1 - \pi_{22}^2 \end{vmatrix}}{\begin{vmatrix} \pi_{11}^1 - \pi_{11}^2 & \pi_{12}^1 - \pi_{12}^2 \\ \pi_{21}^1 - \pi_{21}^2 & \pi_{22}^1 - \pi_{22}^2 \end{vmatrix}};\\
\zeta_2^1 &\equiv \frac{\chi}{F_2} = \frac{\begin{vmatrix} \pi_{11}^1 - \pi_{11}^2 & -1 \\ \pi_{21}^1 - \pi_{21}^2 & -1 \end{vmatrix}}{\begin{vmatrix} \pi_{11}^1 - \pi_{11}^2 & \pi_{12}^1 - \pi_{12}^2 \\ \pi_{21}^1 - \pi_{21}^2 & \pi_{22}^1 - \pi_{22}^2 \end{vmatrix}}.
\end{aligned}
$$

The elasticities of supply and demand for foreign exchange are readily obtained from these by the formulas

$$
(5.2)\qquad
\begin{aligned}
\phi_S &\equiv -\frac{\chi}{S_2}\frac{\partial S_2}{\partial \chi} &=& -(1+\pi_{11}^2)\zeta_1^2 - \pi_{12}^2\zeta_2^1;\\
\phi_D &\equiv -\frac{\chi}{D_2}\frac{\partial D_2}{\partial \chi} &=& (1+\pi_{22}^2)\zeta_2^1 + \pi_{21}^2\zeta_1^2.
\end{aligned}
$$

(The signs appearing in these definitions are due to the fact that χ has been defined as the price of country 1's currency in terms of country 2's, rather than the other way around). In the separable case, in terms of the notations (4.1) and (4.5) these reduce to

$$
(5.3)\qquad \phi_S = \frac{\varepsilon_1(\iota_2 - 1)}{\varepsilon_1 + \iota_2}; \qquad \phi_D = \frac{\iota_1(\varepsilon_2 + 1)}{\iota_1 + \varepsilon_2}.
$$

These are precisely the same as the expressions given in Harberler (1949) and Bronfenbrenner (1950). If the third equation of (2.12) is replaced by the equation corresponding to country 1's balance-of-payments deficit, we obtain analogous expressions for the elasticities of supply and demand for country 1's currency as opposed to country 2's by permuting the subscripts 1 and 2 in (5.3); these are just as in Harberler (1949) and Allen (1954).

If we think of the dynamic-adjustment process as a two-stage one, with the commodity markets clearing first and the foreign-exchange market second, then we can imagine that the speeds of adjustment of the first two equations of (3.1) are infinitely rapid by comparison with that of the third. The latter differential equation then becomes

$$
(5.4)\qquad \dot{\chi} = \kappa_3\left\{S_2(\chi;\mu,\ell^1,\ell^2) - D_2(\chi;\mu,\ell^1,\ell^2)\right\}
$$

for the case $T = 0$. In this case we can express the stability condition in terms of elasticities rather than slopes (see Machlup, 1950, p. 53), namely in the form $\phi_S + \phi_D > 0$, and it is not hard to see that in the separable case the Metzler condition (4.6) results.[9]

[9] Since the condition of footnote 4 is assumed to hold, and since under the assumption of separability we have $\pi_{kk}^k \geqq 0$, $\pi_{ii}^k \leqq 0$ for $i \neq k$, and $\pi_{ij}^k = 0$ for $i \neq j$, the four elasticities of (4.5) are all positive. Consequently the conditions (4.6) and (4.7) become equivalent. Under the dynamic process (5.4), the Edgeworth condition (4.7) is therefore necessary and sufficient in

6 Marshallian Offer-Curve Analysis

We could just as well adopt the converse assumption that the foreign-exchange market adjusts infinitely rapidly by comparison with the commodity markets. An idealization of this kind leads to a variant of Marshallian offer-curve analysis.

If to (2.12) we add the redundant equation

$$(6.1) \qquad -T/\mu = -z_1^2 P_1^1(-z_1^2, z_2^1, 0; \ell^1) + z_2^1 P_2^1(-z_1^2, z_2^1, 0; \ell^1)$$

we can subtract either one of the first two, say the second. Country 1's offer curve can now be defined as the set of points (z_1^2, z_2^1) satisfying (6.1), that is, as the implicit function

$$(6.2) \quad \Omega^1(z_1^2, z_2^1; \ell^1; \mu, T) \equiv T/\mu - z_1^2 P_1^1(-z_1^2, z_2^1, 0; \ell^1) + z_2^1 P_2^1(-z_1^2{}_{,2}^{\,1}, 0; \ell^1) = 0.$$

Analogously, but with the substitution of the first equation of (2.12) in the third in order to eliminate the exchange rate, we may define the implicit function

$$(6.3) \quad \Omega^2(z_1^2, z_2^1; \ell^1, \ell^2; \mu, T) \equiv \frac{T}{\mu}\frac{P_1^2(z_1^2, -z_2^1, 0; \ell^2)}{P_1^1(-z_1^2, z_2^1, 0; \ell^1)} - z_1^2 P_1^2(z_1^2, -z_2^1, 0; \ell^2) + z_2^1 P_2^2(z_1^2, -z_2^1, 0; \ell^2) = 0.$$

Unfortunately, this is not a genuine Marshallian offer curve for country 2, since it is contaminated by the exchange-rate term, which involves one of country 1's inverse trade-demand functions. But this cannot be helped, and it simply shows that Marshallian offer-curve analysis is not a very suitable tool for handling these problems, except in the special case $T = 0$. For our limited purposes, however, it will be enough to examine the stability conditions in the neighborhood of a point where $T = 0$; for that case, the definitions (6.2) and (6.3) reduce to Edgeworth's (1908, p. 544n), and they describe genuine Marshallian offer curves.

If, at an initial equilibrium with $T = 0$, $\partial\Omega^1/\partial z_1^2 \neq 0$ and $\partial\Omega^2/\partial z_2^1 \neq 0$, the offer curves in the neighborhood of the equilibrium point $(\bar{z}_1^2, \bar{z}_2^1)$ can be expressed in the form

$$(6.4) \qquad z_1^2 G_1(z_2^1; \ell^1), \quad z_2^1 = G_2(z_1^2; \ell^2),$$

with imports as a function of exports in each country. This is always possible (globally) when the $\hat{U}^k$ are strictly quasi-concave and separable, since then $\partial\Omega^1/\partial z_1^2 = z_1^2 \partial P_1^1/\partial z_1^1 - P_1^1 < 0$ and $\partial\Omega^2/\partial z_2^1 = z_2^1 \partial P_2^2/\partial z_2^2 - P_2^2 < 0$. Following Alexander (1951, p. 386), we may define the "elasticities of trade" α_k of the two countries as

$$(6.5) \qquad \alpha_1 = \frac{z_2^1}{G_1}\frac{\partial G_1}{\partial z_2^1}; \qquad \alpha_2 = \frac{z_1^2}{G_2}\frac{\partial G_2}{\partial z_1^2}.$$

---

the separable case, whereas under the process (3.1) it is necessary but not sufficient. This may be what lies behind Machlup's doubts (1939, p. 12) as to whether it is legitimate "to transform operational time into clock time." Note that under (3.1), arguments to the effect that (4.6) or (4.7) is fulfilled (e.g., Machlup, 1950, p. 56) do not in themselves provide sufficient grounds for "elasticity optimism," since (4.7) is not in a general a sufficient condition for stability of exchange equilibrium.

As a dynamic-adjustment process we may postulate as in Samuelson (1947, p. 266)

$$\text{(6.6)} \qquad \begin{aligned} \dot z_1^2 &= \kappa_1 \{G_1(z_2^1;\ell^1) - z_1^2\}; \\ \dot z_2^1 &= \kappa_2 \{G_2(z_1^2;\ell^2) - z_2^1\}. \end{aligned}$$

Then it is readily verified that a necessary and sufficient condition for local stability is[10]

$$\text{(6.7)} \qquad \alpha_1\alpha_2 < 0.$$

From (6.2) and (6.3) we find that the elasticities of trade (6.5) are related to the flexibilities (3.2) by

$$\text{(6.8)} \qquad \alpha_1 = \frac{1-\pi_{12}^1+\pi_{22}^1}{1+\pi_{11}^1-\pi_{21}^1}; \qquad \alpha_2 = \frac{1+\pi_{11}^2-\pi_{11}^2}{1-\pi_{12}^2+\pi_{22}^2}.$$

In the separable case these reduce to $\alpha_k = (1-\delta_k)/(1+\sigma_k)$, whence Edgeworth's stability condition (4.7) follows immediately from (6.7). In fact, it is in this way that the condition was originally obtained by Edgeworth (1908, p. 544n).

The condition (6.7) may also be stated in terms of the Marshallian elasticities of excess demand ω_k (Marshall, 1923, p. 377). As is well known (see Johnson, 1950–51, p. 29) these are related to the elasticities of trade by $\alpha_k = (\omega_k - 1)/\omega_k$, or

$$\text{(6.9)} \qquad \omega_k = \frac{1}{1-\alpha_k} \quad (k = 1, 2),$$

so that provided ω_1 and ω_2 are both positive (6.7) becomes

$$\text{(6.10)} \qquad \omega_1 + \omega_2 - 1 > 0.$$

This is the well-known Marshallian stability condition (Marshall 1923, p. 354).

The formal similarity between (6.10) and (4.8) has unfortunately given rise to considerable confusion. Condition (4.8) was attributed by Mrs. Robinson (1947a, p. 143n) to Lerner (1944, p. 377), who provided a verbal analysis of exchange stability, and subsequent writers (e.g., Harberler 1949, p. 203) have agreed with this attribution. And yet, Hirschman (1949, p. 52) has called (4.8) the "Marshall-Lerner condition"—a term which seems to have stuck despite the fact that they are quite different conditions except in the freak case discussed above of infinite supply elasticities.[11] Machlup—always a stickler for correct terminology—is almost alone in eschewing this expression, employing instead the somewhat awkward but

[10]The Jacobian matrix of (6.6) is the matrix product

$$\begin{bmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{bmatrix} \begin{bmatrix} -1 & \partial G_1/\partial z_2^1 \\ \partial G_2/\partial z_1^2 & -1 \end{bmatrix}.$$

If its characteristic roots are complex or repeated, then $\alpha_1\alpha_2 \leqq 0$ and (6.7) is certainly satisfied; if they are real, they are both negative if and only if (6.7) holds.

[11]According to Hirschman (1949, p. 50): "Marshall was first to point out that devaluation might produce an unfavorable effect on the balance of trade ... on the condition that 'the total elasticity of demand of each country be less than unity, and on the average less than one half ...'." However, in the passage in question (Marshall, 1923, Appendix J, p. 354), Marshall makes no mention either of devaluation or of the balance of trade. Marshall was discussing the question

at least unobjectionable phrase "the theorem of the 'critical value' of the sum of the demand elasticities in international trade" (Machlup 1950, p. 55). We verify readily that in the special case of separable trade-utility functions (6.9) becomes $\omega_k = \iota_k(\varepsilon_k+1)/(\iota_k+\varepsilon_k)$—quite similar in form to the expressions (5.3) for the elasticities of supply and demand for foreign exchange—and that (6.10) then reduces to

$$\omega_1+\omega_2-1=\frac{\iota_1\iota_2(\varepsilon_1+\varepsilon_2+1)+\varepsilon_1\varepsilon_2(\iota_1+\iota_2-1)}{(\iota_1+\varepsilon_1)(\iota_2+\varepsilon_2)}. \tag{6.11}$$

This looks like (4.6), but the denominators are different. Both formulas reduce (formally) to (4.8) in the special case $\varepsilon_1=\varepsilon_2=\infty$, because then $\omega_k=\iota_k$ for $k=1,2$. However, condition (6.10) is much more general, not requiring the assumption of separable trade-utility functions, let alone the still more special assumption of infinite supply elasticities. It is not even clear that Lerner really had in mind (4.8) rather than (6.10)—in which case the linking of his name with Marshall's would at least be consistent, although still erroneous as a description of (4.8).

7 Devaluation and the Terms of Trade

It was pointed out by Machlup in 1955 (p. 195) that here had been a "remarkable change of opinion" among economists during the preceding decade concerning the effects of a devaluation on the terms of trade, the earlier opinion having tended to be that a devaluation necessarily was accompanied by a deterioration in the terms of trade.[12] For the kind of reasoning underlying these earlier opinions we can do no better that quote Machlup himself (1939, p. 10).[13]

> A rise in the price of foreign currency makes imported commodities more expensive in terms of dollars (assumed here to be the domestic currency) and exported commodities cheaper in terms of the foreign currency.

One could read similar explanations in the newspapers during the 1971–73 period of successive devaluations of the dollar; and when, instead, it turned out that the prices of exportables rose as well, it was of course concluded by much of the public that this was proof of a conspiracy on the part of wicked speculators and profiteers. But the newspaper's analyses had lagged considerably behind those of at least the more perceptive economists, for already in 1950 Machlup has this to say (p. 55):

of unstable and multiple equilibria, in terms of the dynamic movement of what he called the "exchange-index" which he defined (p. 340) as the point in his phase diagrams which we have here denoted (z_1^2, z_2^1); that is, he was discussing the properties of a system of differential equations which we have represented here (following Samuelson) by (6.6). The only price concepts mentioned by Marshall in his analysis are the "rate of interchange" (p. 353) and the "terms of trade" (p. 345), expressions which are defined as synonymous and presented with a warning (p. 161): "The phrase 'rate of exchange' is avoided; because it is already specialized, in connection with the Foreign Exchanges, to indicate the rate at which command over the currency of one country can be obtained in terms of the currency of another country." Taussig (1927, p. 9) also warned of this linguistic trap, which Hirschman apparently fell into.

[12] While this is true, if we go back far enough—to Pigou (1922)—we find the emphatic statement that "the two are not connected with one another at all" (p. 150).

[13] The citation follows the clarificatory 1964 rewording but is otherwise identical with the original 1939 passage.

> When an analyst, for example, assumes that a depreciation by ten per cent lowers the prices of export goods to foreigners by ten per cent, he implicitly assumes that the domestic prices remain unchanged. In postulating that these prices remain unchanged although the volume of exports is increased, he makes the implicit assumption that the elasticity of supply of these exports goods in infinite.

To the extent that a transfer from country 1 to country 2 causes a devaluation of country 1's currency, the question is formally identical with that of the effect of a transfer on the terms of trade, that is, with the transfer problem. In terms of the model (2.12), this is readily determined. Expressing the solution values of (2.12) as functions of T, substituting these in the functions P_1^1 and P_2^1, and differentiating P_1^1/P_2^1 totally with respect to T, we obtain

$$\text{(7.1)} \qquad \frac{1}{p_1^1/p_2^1}\frac{d(p_1^1/p_2^1)}{dT} = (\pi_{11}^1 - \pi_{21}^1)\frac{1}{z_1^2}\frac{dz_1^2}{dT} + (\pi_{12}^1 - \pi_{22}^1)\frac{1}{z_2^1}\frac{dz_2^1}{dT}.$$

We find readily from (2.12) that

$$\text{(7.2)} \qquad \frac{1}{z_1^2}\frac{dz_1^2}{dT} = \frac{\chi}{z_1^2 P_1^2}\frac{\Delta_{31}}{\Delta} \quad \frac{1}{z_2^1}\frac{dz_2^1}{dT} = \frac{\chi}{z_1^2 P_1^2}\frac{\Delta_{32}}{\Delta}$$

where Δ is the determinant of the matrix A of (3.3), and Δ_{ij} is the cofactor of the element of the ith row and jth column of A. Substituting (7.2) in (7.1) we obtain

$$\text{(7.3)} \qquad \frac{1}{p_1^1/p_2^1}\frac{d(p_1^1/p_2^1)}{dT} = \frac{\chi}{z_1^2 P_1^2}\frac{1}{\Delta}\begin{vmatrix} \pi_{11}^1 - \pi_{21}^1 & \pi_{12}^1 - \pi_{22}^1 \\ \pi_{11}^2 - \pi_{21}^2 & \pi_{12}^2 - \pi_{22}^2 \end{vmatrix}.$$

From the stability condition $\Delta < 0$, we conclude that a transfer strongly worsens country 1's terms of trade if and only of

$$\text{(7.4)} \qquad (\pi_{11}^1 - \pi_{21}^1)(\pi_{12}^2 - \pi_{22}^2) > (\pi_{11}^2 - \pi_{21}^2)(\pi_{12}^1 - \pi_{22}^1).$$

In the special case of additively separable trade-utility functions (7.3) reduces to

$$\text{(7.5)} \qquad \frac{1}{p_1^1/p_2^1}\frac{d(p_1^1/p_2^1)}{dT} = \frac{\chi}{z_1^2 P_1^2}\frac{\delta_1\delta_2 - \sigma_1\sigma_2}{(1-\delta_1)(1-\delta_2) - (1+\sigma_1)(1+\sigma_2)}.$$

Since the denominator is negative by Edgeworth's stability condition (4.2), we obtain the criterion: the paying country's terms of trade deteriorate (strongly) if and only if

$$\text{(7.6)} \qquad \delta_1\delta_2 > \sigma_1\sigma_2,$$

or, in terms of the "indirect elasticities" of (4.5), if and only if

$$\text{(7.7)} \qquad \varepsilon_1\varepsilon_2 > \iota_1\iota_2.$$

This is Pigou's formula (1932), derived independently by Yntema (1932, p. 91). It is also the formula subsequently and apparently independently derived by Mrs. Robinson (1947b, p. 163n), in the form $\varepsilon_1/\iota_1 > \varepsilon_2/\iota_2$, for the conditions under which a devaluation would lead to a deterioration in the terms of trade of the devaluing country.[14]

In the special case of constant supply prices of exports, in which (4.15) holds, we may note from (7.3) and (3.4) that

$$\frac{1}{p_1^1/p_2^1}\frac{d(p_1^1/p_2^1)}{dT} = \frac{1}{\chi}\frac{d\chi}{dT} = \frac{c}{\Delta}\begin{vmatrix} \pi_{11}^2 & \pi_{12}^1 \\ \pi_{21}^1 & \pi_{22}^1 \end{vmatrix}, \tag{7.8}$$

that is, the percentage deterioration in the terms of trade is the same as the percentage devaluation. This is of course follows from the fact that, in the notation of (4.9) above, under the assumed conditions we have from (2.2) and (2.3)

$$\frac{p_1^1}{p_2^1} = \chi\mu\frac{b_3^1/b_1^1}{b_3^2/b_2^2}. \tag{7.9}$$

This is the basic assumption adopted by Hirschman (1949) and underlying the models of Harberger (1950) and Laursen and Metzler (1950). It should be noted that the result holds independently of separability.

8 Elasticity Optimism, Pessimism, and Skepticism

Two different types of "elasticity" concepts emerged in the preceding analysis: an *indirect* elasticity, which is the *reciprocal* of an elasticity of an inverse or *indirect* demand function; and a Marshallian elasticity of excess demand with respect to the terms of trade. On the other hand, the elasticity concept that has been used for purposes of statistical estimation has almost invariably been an elasticity of a *direct* demand function (see for example Leamer and Stern 1970, pp. 9–12). Thus, we have three distinct concepts of "elasticity of demand for imports." How are they related? The case made by Orcutt (1950) against what Machlup (1950) has called "elasticity pessimism" was based entirely on the premise that the relevant elasticity concept was of the third, "direct" type, involving the quantity of imports as a dependent variable and the price of imports as an independent variable. This being assumed, he argued that the conventional direct least-squares estimator of the elasticity of demand for imports was biased downwards. But where does all this discussion lead us if it turns out that the wrong parameter has been estimated in the first place?

I shall not attempt to provide a general answer to this question, which seems to be difficult (and probably not worth pursuing), but will instead limit myself to providing a precise answer in a special case.

[14]It should be stressed that Pigou explicitly defined the elasticities in the *indirect* form (4.5), following Jevons (see also Pigou, 1947). The distinction between the direct and indirect elasticities was explicitly remarked upon by Kahn (1947–48, p. 17) and Graaff (1949–50, p. 54n). For the derivation of (7.7) see also Viner (1937, p. 341), Bronfenbrenner (1942), Pigou (1947, p. 180), and Johnson and Carter (1950). An analysis along Robinsonian lines was presented by Harberler (1952). Condition (7.7) is equivalent—in the case of the pure-exchange model with separable preferences—to Samuelson's (1952, p. 286) well-known inequality expressed in terms of income propensities.

It should be observed first of all that there are difficulties involved in defining an aggregate, direct demand function for imports in a competitive economy. For one thing, once prices are specified in such an economy, the value of the national product if determined, since efficiency requires this value to be maximized at those prices over the production-possibility set. This means that the only component of national income that is free to vary independently of prices is the deficit in the balance of payments on current account. Secondly, once the prices and the foreign deficit are given, there is no guarantee that the aggregate demand for the domestic, nontraded, good will be equal to the aggregate supply; in order to describe a general-equilibrium situation, therefore, it is necessary to assume that prices and the foreign deficit do not vary arbitrarily, but are so constrained in their variation that the market for domestic goods will be cleared.

Now suppose, at least provisionally, that we may regard the prices of a country's export and import goods (denominated in its own currency) as exogenous to that country. Then the above reasoning implies that the country's nominal national income and the price of its domestic commodity must be regarded as endogenous.[15] In general they cannot be assumed to be held constant even if the import price alone varies. For, unless the country produces no import-competing goods, a rise in the (domestic-currency) price of imports, by increasing the value of the domestic output of import-competing goods, will necessarily increase the nominal national income. Further, if the price of the export good is held constant, then unless the price of the domestic good is tied to that of the export good by constant costs, a rise in the price of the import good will necessitate some adjustment in the price of the domestic good so as to permit the market for the latter to be cleared. These difficulties can be assumed away by postulating that production possibilities have the special form (4.9) analyzed in an earlier section. In that case, the direct elasticity of demand for import coincides (as is easily verified) with the Marshallian elasticity.

How does the direct elasticity of demand for imports compare with the corresponding *indirect* elasticity? Since neither is constant in general, it does not seem possible to provide a straightforward answer that will cover all cases; however, we can provide one for the case in which the country's utility function is of the constant-elasticity-of-substitution (C.E.S.) type

$$U^k(x^k) = -\sum_{j=1}^{3} \theta_j^k (x_j^k)^{(\rho_k - 1)/\rho_k} \quad \left(\theta_j^k > 0, \sum_{j=1}^{3} \theta_j^k = 1\right). \tag{8.1}$$

Assuming production possibilities to be given by (4.9) for country 1, and analogously for country 2, it is possible by carrying out the constrained maximization (2.5) to

[15] What this means is that instead of having one equation for each country we should have at least three, the import-demand equation being supplemented by two additional equations indicating the dependence of income and the price of the domestic commodity on the export and import prices. (For a complete system one would also not want to ignore the market for country 1's export good.) This would be a system of "block-recursive" type, and the least-squares method would be justified if one could assume that the random influences on consumption of importables were independent of those on national income and the price of the domestic commodity.

compute $\hat{U}^k$ explicitly,[16] and we find that the flexibilities (3.2) become

$$\pi^k_{kk} = -\frac{1}{\rho_k}; \qquad \pi^k_{kj} = -\frac{1}{\rho_k}\frac{s_k}{1-s_k} \quad (j \neq k) \ \ j,k = 1,2, \tag{8.2}$$

where s_k is the share of imports in country k's national income, and ρ_k is the constant elasticity of substitution. Thus we have, from (4.1) and (4.5)[17]

$$\iota_k = \rho_k \quad (k = 1,2). \tag{8.3}$$

Defining country k's direct demand function for commodity i, namely $x^k_j = h^k_i(p^k_1, p^k_2, p^k_3, I^k)$, by the condition that $h^k(p^k, I^k)$ maximizes $U^k(x^k)$ subject to the budget constraint $\sum_{i=1}^k p^k_i x^k_i \leqq I^k$, where I^k is country k's nominal national income, if no import-competing goods are produced at home, so that consumption of importables coincides with imports, we may define the *direct elasticities of demand for imports* by

$$\eta_1 = \frac{p^1_2}{h^1_2}\frac{\partial h^1_2}{\partial p^1_2}; \qquad \eta_2 = \frac{p^2_1}{h^2_1}\frac{\partial h^2_1}{\partial p^2_1}. \tag{8.4}$$

In the case of C.E.S. utility functions (8.1) we find readily that

$$\eta_k = (1-s_k)\rho_k + s_k \quad (k = 1,2). \tag{8.5}$$

Combining this with (8.3) we have the result, under the special conditions (4.9): *the direct elasticity of demand for imports is in between the indirect elasticity of demand for imports and unity.* Under those same special conditions we also have $\eta_k = \omega_k$.

In making his case against "elasticity pessimism", Machlup stated (1950, p. 56): "a correct critical value of [the sum of the] demand elasticities for more realistic conditions, that is, for situations with lower supply elasticities, must lie *below* unity." He did not say that the correct elasticities would be higher. Thus, his concept appears to be closer to our ι_k than to the Marshallian ω_k. But (8.3) and (8.5) imply that if $\eta_k < 1$, then $\iota_k < \eta_k < 1$, so the "true" *indirect* elasticities are smaller than the *direct* ones being estimated; hence, even if the η_ks are underestimated, the statistical estimates of the η_ks need not underestimate the ι_ks. On the other hand, as we saw from (4.16) above, (4.8) is an incorrect stability condition; even with infinite supply elasticities the critical upper bound to $\iota_1 + \iota_2$ is below unity (under the above assumption $\pi^k_{kk} < 0$ for $i \neq k$). It is a wonder if anybody can draw firm conclusions out of this situation!

[16]For country 1 the formula is (dropping country suffixes for notational convenience)

$$\hat{U}^1(z^1_1, z^1_2, z^1_3; \ell^1) = -\theta_2(z^1_2)^{(\rho-1)/\rho} - (\theta_1^\rho b_1^{\rho-1} + \theta_3^\rho b_3^{\rho-1})^{1/\rho}\left(\frac{z^1_1}{b_1} + \frac{z^1_3}{b_3} + \ell^1\right)^{(\rho-1)/\rho}.$$

[17]This result depends critically upon the assumption that country k specializes in its export and domestic goods. If, say, we replaced (4.9) by a fixed-output production possibility set $\mathcal{Y}^1 = \{\bar{y}^1_1, \bar{y}^1_2, \bar{y}^1_3\}$ (which we could interpret as Harberler's 1950 short-run specific-factors technology, with the formal identity $\bar{y}^1 = \ell^1$), then it is not hard to see that in place of (8.3) we would obtain $\rho_1 x^1_2 / z^1_2$ (for country 1), that is, the elasticity of substitution divided by the share of imports in the consumption of importables. With positive domestic production of imports this would yield $\iota_1 > \rho_1$.

Let us consider now the allegation that the conventional least-squares estimator of η_1 will tend to underestimate its true value. Orcutt's argument was based on geometric and intuitive considerations, but the following interpretation may be offered. Suppose we assume (and what this means will be discussed in the next paragraph) that world equilibrium at time t can be represented by the intersection of a demand and supply curve for country 1's imports, defined by the equations $z_2^1(t) = \alpha_i + \beta_i p_2^1(t) + \varepsilon_i(t)$ for $i = 1, 2$ respectively, where the $\varepsilon_i(t)$ are random variables with zero means and covariances $E\varepsilon_i(t)\varepsilon_j(t) = \sigma_{ij}$. The least-squares estimator of either one of the β_is is defined by

$$(8.6) \qquad b = \frac{\sum_{i=1}^{n}[p_2^1(t) - \bar{p}_2^1][z_2^1(t) - \bar{z}_2^1]}{\sum_{t=1}^{n}[p_2^1(t) - \bar{p}_2^1]^2}$$

where

$$\bar{p}_2^1 = \frac{\sum_{t=1}^{n} p_2^1(t)}{n}, \qquad \bar{z}_2^1 = \frac{\sum_{t=1}^{n} z_2^1(t)}{n},$$

given a sample of observations at times $t = 1, 2, \ldots, n$. Solving the demand and supply equations it is not hard to see that

$$(8.7) \qquad (\operatorname{Plim}_{n\to\infty} b) - \beta_1 = \frac{(\beta_2 - \beta_1)(\sigma_{11} - \sigma_{12})}{\sigma_{11} - 2\sigma_{12} + \sigma_{22}}.$$

If the demand curve is downward sloping ($\beta_1 < 0$) and the supply curve upward sloping ($\beta_2 > 0$), as explicitly assumed by Orcutt (1950, p. 127), then $|\beta_1|$ will be underestmated by $|b|$, in the sense that $|\operatorname{Plim} b| < |\beta_1|$, if and only if $\sigma_{11} > \sigma_{12}$. In particular this will be the case if $\sigma_{12} < 0$, that is, if shifts in supply and demand are *negatively* correlated. This appears to be what Orcutt meant by saying (1950, p. 123) that the "demand and supply schedules for imports... shift up and down together."[18] The result also follows if the shifts are *uncorrelated* ($\sigma_{12} = 0$). If $\sigma_{11} > \sigma_{12}$, note that the result still follows if the supply curve is backward sloping ($\beta_2 < 0$) provided it is steeper than the demand curve ($\beta_1 < \beta_2$), that is, *provided the equilibrium is stable.* To the extent that Orcutt's reasoning is used to argue that devaluation will be effective, because equilibrium is stable, it is worth noting that this conclusion has been assumed in advance.

In fact, it is illegitimate to assume, as Orcutt does, that the foreign supply curve is upward sloping. Under the special assumptions we have considered, the appropriate supply curve for country 1's imports is country 2's *reciprocal demand curve* as defined by Viner (1937, p. 539), which could be backward bending as Viner noted.[19] There would then be nothing to prevent multiple intersections (alternately stable and unstable) of the two curves, and, if the premise is (at least provisionally)

[18]One must be careful about the use of words here. On the assumption that the demand and supply curves have negative and positive slopes respectively, and that price is measured vertically and quantity horizontally, the curves "shift up and down together" if and only if they shift to the left and right in *opposite* directions.

[19]This should not be interpreted as meaning that Viner's diagrams are valid only under these

accepted that only equilibria are observed (and, obviously, this means that only *stable* equilibria are observed, since we could no more observe unstable ones than we could obseve eggs standing on end) then we cannot in principle rule out the possibility that what we observe at different points in time are alternate stable intersection points of the same curves, rather than stable intersection points of shifted curves. In that case, the least-squares procedure would provide excellent estimates of the average slopes of *both* curves.

But it is of course very unreasonable to assume that we observe equilibria, as Machlup (1958) has argued with great perceptiveness. In fact, as he says (p. 122): "I cannot recognize an equilibrium in international trade no matter how hard I look." That being the case—and I agree—it is hard to be anything but skeptical concerning the correct values of the elasticities, however these might be defined.

9 Do Relative Prices Matter?

Judging by the long litany of accusations recited in Frenkel and Johnson's book (1976a), the relative-prices approach to the analysis of balance-of-payments adjustment problems is in terrible shape and in deep trouble. In fact, "it is hopelessly defective as an approach to devaluation" (Johnson, 1977, p. 254). What are the charges with which this approach is faced? Here are some of the most oft-repeated ones:

1. It assumes that "all goods are traded" (Frenkel and Johnson, 1976b, p. 27). One the contrary, the *essence* of the relative-prices or elasticity approach is the role of nontraded goods, as has been recognized from the beginning by Taussig (1927) and Keynes (1930), and quite explicitly by Machlup (1955, p. 183).

2. "Changes in the terms of trade ... are the center-piece of the elasticity approach" (Frenkel and Johnson, 1976b, p. 27). One the contrary, this was specifically rejected by Taussig (1927), Pigou (1922, 1932), Graham (1948, 1949), Robinson (1947b), Haberler (1952), and Machlup (1950, 1955, 1956), and is instead the position associated with the orginator of the "absorption approach" (Alexander, 1952).

3. It "implicitly assumes that changes in domestic income consequent on an increase in export earnings ... have no further effects on demand", (Johnson 1976, p. 266) or "income is implicitly held constant" (Johnson 1977, p. 254). One need only consult the third equation of (2.6) and of (2.12) above, which provide the source of Machlup's (1939–40) supply and demand for foreign exchange (2.14), to see that this is not so.

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assumptions, but rather that it is only under these assumptions that they are applicable to the analysis of exchange-rate determination—a purpose which there is no reason to believe Viner himself had in mind (*pace* Hirschman 1949). For *his* purposes, Viner's curves had more general validity.

4. It assumes “wage rigidity” and “mass unemployment” and is based on the “implicit assumption of the existence of unemployed resources” (Johnson 1972, pp. 149–50). One will find not even the glimmer of a hint of wage rigidity and unemployment in the accounts of Taussig (1927), Keynes (1930), Graham (1948), and Machlup (1939–40). And rightly or wrongly, full employment has been assumed all along in this paper.

5. It “invariably uses partial-equilibrium real analysis concepts” (Johnson 1976, p. 262). It has been one of the main objects of this paper to disprove this contention.

6. It “provides no analysis ... of the sources of increased production” (Johnson 1977, p. 254). An imperfect analysis, perhaps, by means of supply elasticities and the assumption of zero cross-flexibilities—but surely not “no analysis” !

A somewhat more substantial criticism arises in the course of Johnson’s 1958 analysis of balance-of-payments deficits, which starts out with the acceptance of Alexander’s (1952) view that it is more helpful to think of a balance-of-payments deficit as “an excess of aggregate payments by residents over aggregate receipts by residents” (Johnson, 1958, p. 49), than as an excess of international payments over international receipts. While the point may be accepted, the “absorption approach” has a pitfall of its own: it is all too easy to forget that, in equilibrium, an excess of payments over receipts in country 1 must be exactly offset by an equal excess of receipts over payments in country 2. Johnson asserted (1958, p. 51) that a country could not undergo a continuing balance-of-payments deficit (defined (Johnson, 1958, p. 49; 1976, p. 262) as a state in which official reserves were declining—presumably at a constant rate) unless it were sustained by continued credit creation. (This followed from an assumption, later made explicit (1958, p. 54), that he was “abstracting altogether from international capital movements (other than reserve transactions between foreign exchange authorities)”.) But if is true that country 1’s deficit cannot be sustained unless the depleted cash balances are being constantly replenished by continued credit creation, then it must be equally true that country 2’s equal and opposite surplus cannot be sustained unless country 2 insists on sterilizing the money inflow or offsetting it by continued credit contraction. The complete picture, then, is one of “involuntary foreign lending” from country 2 to country 1 (see Machlup 1965, pp. 62–4). We need not go into (and could not possibly settle) the philosophical question of whether the onus for the deficit should be placed on country 1 for creating the situation or on country 2 for not allowing the upward price movements (the “imported inflation”) to take place. But from the point of view of positive analysis, in terms of the present model we would have to characterize the situation described by Johnson as an autonomous (we need not say “voluntary” if this causes offense) capital movement from country 2 to country 1. In fact, in terms of our model no conceptual distinction is possible between “voluntary” and “involuntary” autonomous capital movements, yet only the second kind would give rise to a balance-of-payments deficit in Johnson’s sense. The analysis of the relative-prices approach therefore confirms rather than rebuts Johnson’s contention (1972, p. 150) that a “fully employed economy cannot use devaluation alone as a policy instrument for correcting a balance-of-payments deficit"; for it has surely never been alleged by proponents of the relative-prices approach that a devaluation brought about by an autonomous capital movement would succeed in choking off that capital movement itself!

Finally we may consider the criticism that "the familiar elasticity condition (sum-of-the-elasticities-of-demand-for-foreign-exchange-greater-than-unity[20]) for exchange market stability ... is completely irrelevant to a monetary international economy ... because it is the condition for stability of exchange in a barter economy" (Johnson, 1976, p. 281). The reasoning appears to be that if such conditions are not required in a monetary *model*, which "for simplicitly" rules out relative-price variations *by assumption* (Johnson, 1972, p. 154), then they are irrelevant to the *real world* in which money and relative prices both play a role. This is a good example of the fallacy of misplaced concreteness (Machlup, 1958, p. 122).

The basic idea of the monetary approach appears to be that the dichotomy between barter and monetary theory which unfortunately prevails in economic thinking is also a basic characteristic of the world that our imperfect theories try to describe; that there is one set of forces, or markets, that takes care of real or barter adjustments, and another that takes care of monetary ones, in complete isolation from one another. In a Walrasian (Cartesian?) world with n commodities, n prices surely suffice; an exchange rate is, according to this way of thinking, a superfluous $(n+1)$th price—a fifth wheel—whose role must therefore be "purely monetary". But if spare tires are needed in real cars, might not spare prices be needed in real economies? In this paper we have seen that exchange rates play an essential role if some prices (those of domestic goods) are *completely inflexible*. Might this not remain the case if they are only *somewhat inflexible*?

We have not answered this in the present paper, but an answer suggests itself. Imagine that, over time, the adjustment process represented by the third equation of (3.1) takes place for a time; and that at a specified moment, the exchange rate becomes fixed and variations in the nominal price of the domestic commodity take its place. In the final equilibrium, relative gold prices would remain as before, but the final exchange rate would be "indeterminate" (to use the prevailing unfortunate expression). It would be "indeterminate" only in the sense that *we* are unable to come up with a theory of its determination with our static methods. A more complete analysis than we have been able to present here would probably lead us to the following conclusion:

No satisfactory theory of exchange-rate determination is possible within the confines of the method of comparative statics.

[20]Johnson must have meant "greater than zero." See the discussion at the end of Section 5. If we formally allowed ε_1 and ε_2 to approach infinity in (5.3), we would obtain $\phi_S = \iota_2 - 1$ and $\phi_D = \iota_1$, yielding (4.8). But as indicated in Section 4, this procedure is not in general legitimate.

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