Intertemporal Comparative Advantage, I*

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Abstract

A two-country, two-factor, two-commodity, two-period model is developed in which there is one tradable consumption good and one nontradable investment good which augments the initial capital stock. Labor supply in each country is constant. It is shown that, if the consumption-good industry uses a higher capital-labor ratio than the investment-good industry at all wage-rental ratios, the country with the higher initial endowment ratio of capital to labor has a comparative advantage in producing the present consumption good, and is therefore a natural lender. Assuming preferences and technologies to be identical and homogeneous as between two countries, and that the borrowing country does not default (i.e., trade is balanced in the long run), this results in an intertemporal Heckscher-Ohlin theorem.

0 Introduction

The model presented here has been developed in order to provide a framework for analyzing international payments and debt problems from a classical point of view. The model starts from the very simplest case: two commodities (a consumption good and an investment good), two factors of production (labor and capital), two countries, and two periods. The framework can in principle be extended to any number of commodities, factors, countries, and periods; but in order to obtain qualitative results and to develop an intuitive grasp of the model, one must start from the very simplest case.

In conformity with the hypotheses of the Heckscher-Ohlin-Lerner-Samuelson theory, the following additional simplifying assumptions will be made:

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1. The consumption good is freely tradable with no transport costs.

2. Factors of production (in this case, labor and capital) are freely mobile between industries within countries, but completely immobile between countries. In particular, this implies that the investment good is nontradable. (In Part II we take up the case of international mobility of the investment good.)

3. Production functions are neoclassical (concave, homogeneous of degree 1, and strictly quasi-concave) and constant over time.

4. In each country the endowment of labor is constant over the two periods, and the endowment of capital in the final period is equal to that of the initial period augmented by the output of the investment good in the initial period.

The following additional assumptions will be used in Theorem 1 of Section 3. The first is a *capital-intensity hypothesis* first introduced by Uzawa [52].[1]

5. In each country and each period, and for all wage rates and rentals of capital, the consumption-good industry employs a higher ratio of capital to labor than the investment-good industry. (In particular, this of course implies nonreversal of factor intensities.)

6. Production functions for the respective consumption and investment goods are identical as between the two countries.

7. Preferences as between the present and future consumption good are identical and homothetic within and between the two countries.

8. Trade of present and future consumption goods between the two countries is balanced over the two periods.

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[1]See also Inada [24], Oniki & Uzawa [37]. The main difference between the first part of the present model and the model of Oniki and Uzawa is that we assume the investment good to be nontradable, whereas they assume it to be tradable until installed as a factor, as we shall do in the sequel.

There does not appear to have been much empirical research concerning the validity of the capital-intensity hypothesis. However, studies by Grosse [22] and Sutton [46] lend at least indirect support to it. In Grosse's words (p. 225):

> It is of some interest to note the type of industry which is a heavy capital user. With the exception of steel works and rolling mills, no manufacturing or mining industry is included among the first 10. Most of the capital stock of the economy is in the hands of service industries such as home renting and trade, agriculture, transportation, electric public utilities, and the crude-oil industry. In 1939 practically all manufacturing and mining activity was carried on with some 20 percent of the fixed capital stock.

The main result of the paper (Theorem 1) is an intertemporal version of the principal lemma underlying the Heckscher-Ohlin theorem. Given assumptions 1-4 above, we may define a Fisherian intertemporal production-possibility set (cf. Fisher [19, pp. 264-91]) whose frontier, for each country, is the locus of efficient output combinations of the present and future consumption good. The price of the present relative to the future consumption good is the real interest factor (1 plus the real interest rate). Theorem 1—which constitutes the main technical result of the paper—states that if the capital-intensity hypothesis (condition 5 above) holds, then at any world price ratio (real interest factor), the ratio of each country's efficient outputs of the present and future consumption good is a monotone increasing function of that county's initial capital-labor endowment ratio. That is, the country which is initially relatively well endowed with capital has a comparative advantage in producing the present consumption good relative to the future consumption good. If conditions 6–8 also hold, then the country which is initially relatively capital abundant will lend present consumption goods to the other country in the initial period, and will be repaid in future consumption goods in the final period.

As is the case with any well-formulated theorem, the hypotheses of the Heckscher–Ohlin theorem are indispensable in the sense that if any one of them is omitted, a counterexample to the conclusion may be found. In the case of the Heckscher–Ohlin theorem such a situation has come to be known as a "Leontief paradox" (cf. Leontief [27]). In the present case it is obvious that if the inhabitants of the initially capital-abundant country have a strong relative preference for present over future goods compared to the inhabitants of the initially labor-abundant country, and if the capital-intensity hypothesis holds, the capital-abundant country may be a borrower in the initial period. An example of such a case would be a capital-abundant country whose government engages in deficit spending in the initial period to finance consumption of present goods, or which enacts tax legislation that discourages saving and encourages borrowing.

1 The Two-Period Model

The basic structure of the model will now be developed. The following notation will be used:

l_t = endowment of labor at time t $(t = 0, 1)$.
k_t = endowment of capital at time t $(t = 0, 1)$.
v_{ijt} = allocation of factor $i = L, K$ to industry $j = C, I$ at time $t = 0, 1$, where
factor $i = L$ denotes labor;
factor $i = K$ denotes capital, and
industry $j = C$ denotes the consumption-good industry;
industry $j = I$ denotes the investment-good industry.
y_{jt} = output of commodity $j = C, I$ at time $t = 0, 1$.

p_{jt} = price of commodity $j = C, I$ at time $t = 0, 1$.
w_t = wage rate at time $t = 0, 1$.
r_t = rental of capital at time $t = 0, 1$.

Production is carried out by means of homogeneous-of-degree-1 production functions

$$y_{jt} = f_j(v_{Ljt}, v_{Kjt}) \quad (j = C, I;\ t = 0, 1), \tag{1.1}$$

and subject to resource-allocation constraints

$$\begin{aligned} v_{LCt} + v_{LIt} &\leq l_t \qquad (t = 0, 1); \\ v_{KCt} + v_{KIt} &\leq k_t \qquad (t = 0, 1). \end{aligned} \tag{1.2}$$

Endowments obey the following rules:

$$l_1 = l_0; \tag{1.3}$$

$$k_1 = k_0 + y_{I0}. \tag{1.4}$$

That is, labor (population) is constant, and capital is augmented in period 1 by the output of the investment good in period 0.

The *present value of the domestic product* is defined as

$$p_{C0}y_{C0} + p_{C1}y_{C1}. \tag{1.5}$$

The prices must here be interpreted as in Lerner [28, Ch. 20]. The real interest rate may be defined as

$$\rho = \frac{p_{C0}}{p_{C1}} - 1. \tag{1.6}$$

Since by our assumption that the consumption good is freely traded in both periods with no transport costs, it follows that the real interest rate will be equal between countries in equilibrium. The *real interest factor* may be defined as

$$R = \frac{p_{C0}}{p_{C1}} = 1 + \rho. \tag{1.7}$$

In our model, it plays the role of an *intertemporal terms of trade.*

Since the capital produced in period 0 must be all used up in period 1, its price in period 0 must be equal to its rental in period 1:[2]

$$r_1 = p_{I0}. \tag{1.8}$$

[2]This may be shown analytically by carrying out the constrained maximization (1.17) below and employing the Kuhn–Tucker [26] conditions.

The national accounting then proceeds as follows. In period 0, consumption and investment are given by[3]

$$(1.9)\qquad \begin{aligned} C_0 &= p_{C0}y_{C0} = w_0v_{LC0} + r_0v_{KC0} \\ I_0 &= p_{I0}y_{I0} = w_0v_{LI0} + r_0v_{KI0}. \end{aligned}$$

Assuming full employment (i.e., equalities in (1.2)), these sum to the domestic product

$$(1.10)\qquad Y_0 = C_0 + I_0 = w_0l_0 + r_0k_0.$$

Consumption in period 0 is then equal to

$$(1.11)\qquad C_0 = w_0l_0 + r_0k_0 - p_{I0}y_{I0}.$$

In period 1 there is no production of investment goods, hence

$$(1.12)\qquad C_1 = p_{C1}y_{C1} = w_1l_1 + r_1k_1 = w_1l_0 + r_1k_0 + p_{I0}y_{I0}.$$

In order to avoid double-counting we may define the domestic product in period 1 as

$$(1.13)\qquad Y_1 = w_1l_0 + r_1k_0,$$

i.e., as the sum of then-current factor rentals times *initial* endowments. In the case of capital, this defines the return to capital in period 1 as the period-1 rental times the initial capital. Then $I_1 = -I_0$, i.e., the capital invested in period 0 is disinvested in period 1. From (1.10) – (1.13) the present value of the domestic product (as well as of domestic consumption), to be denoted by W, may be expressed as

$$(1.14)\qquad \begin{aligned} Y_0 + Y_1 &= (w_0 + w_1)l_0 + (r_0 + r_1)k_0. \\ &= C_0 + C_1 \equiv W. \end{aligned}$$

We may thus define for each factor the present value of its rentals:

$$(1.15)\qquad w = w_0 + w_1 \qquad r = r_0 + r_1.$$

Now let us define country k's *intertemporal production-possibility set* as

$$(1.16)\qquad \mathcal{Y}_C(l_0, k_0) = \{(y_{C0}, y_{C1}| \text{ conditions (1.1) to (1.4) are satisfied}\}.$$

Let us further define the *domestic-wealth function* by

$$(1.17)\qquad \Omega(p_{C0}, p_{C1}, l_0, k_0) = \max\{p_{C0}y_{C0} + p_{C1}y_{C1} \mid y_C \in \mathcal{Y}_C(l_0, k_0)\}$$

[3]By abuse of notation, we here employ the symbols C and I, previously defined as the *names* of the consumption-good and investment-good industries, to denote aggregate consumption and aggregate investment as well.

where y_C denotes the vector (y_{C0}, y_{C1}).

The following may be shown:[4]

$$\frac{\partial\Omega}{\partial l_0} = w; \quad \frac{\partial\Omega}{\partial k_0} = r; \quad \frac{\partial\Omega^k}{\partial p_{jt}} = y_{jt} \quad (j = C, I; t = 0, 1). \tag{1.18}$$

These generalize Samuelson's reciprocity relations (cf. [41]). In particular, the third equation of (1.18) defines the intertemporal Rybczynski functions (cf. Chipman [8])

$$y_{jt} = \overline{y_{jt}}(p_{C0}, p_{C1}, l_0, k_0) \quad (j = C, I; t = 0, 1). \tag{1.19}$$

2 Construction of the Intertemporal Production-Possibility Set

The shape of a country's intertemporal production-possibility set $\mathcal{Y}_C(l_0, k_0)$ depends entirely on the initial endowments l_0, k_0. In fact, owing to the assumption of constant returns to scale, as is the case with the static period-0 production-possibility frontier, the slope of a country's intertemporal production-possibility frontier on any ray from the origin depends entirely on its *relative* endowments k_0/l_0. We would like to determine how the shape of the production-possibility set changes as the relative endowment varies.

The four-quadrant diagram of Figure 1[5] shows how the shape of a country intertemporal production-possibility frontier (depicted in the northeast quadrant) can be traced out given knowledge of (1) its endowment of capital in period 0, k_0 (marked off leftwards on the left horizontal axis, which measures the country's endowment of capital in period 1, k_1); (2) the cross-section of its production function for the consumption good, $y_{C1} = f_C(l_0, k_1)$, corresponding to its fixed initial endowment of labor l_0 and variable endowment of capital k_1, shown in the northwest quadrant and drawn on the assumption that all of both resources are allocated to production of the consumption good in period 1; and (3) its static production-possibility set in period 0, $\mathcal{Y}(l_0, k_0)$, shown in the southeast quadrant. The northeast quadrant depicts a 45°-line and the southwest quadrant a displaced 45°-line emanating from the point $(k_0, 0)$.

Starting at the point $(k_0, 0)$ and moving vertically upwards to the production function one obtains the amount $y_{C1} = \widetilde{y_{C0}}$ of the consumption good the country would produce in period 1 if it allocated all its resources to that good in period 0; this is the maximum amount of the consumption good that can be produced in period 0, and is measured on the upward vertical axis as shown by the dashed horizontal line; and by extending this line to the right until it meets the 45°-line in the northeast quadrant (marking off one extreme efficient point of the country's intertemporal production-possibility frontier), we see that corresponding to this maximum output $y_{C0} = \widetilde{y_{C0}}$

[4] As in Chipman [8] this may be shown using the Kuhn–Tucker [26] conditions.

[5] This diagram was suggested to the first author by Fumio Dei.

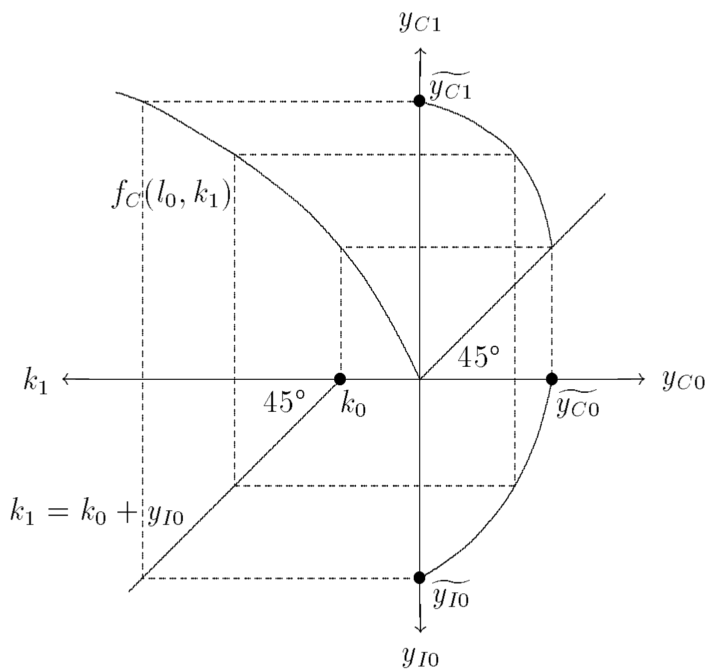

Figure 1

in period 0 is the equal amount $y_{C1} = \widetilde{y_{C0}}$ available in period 1. Proceeding from this point downward to the rightward horizontal axis measuring the output y_{C0} of the consumption good in period 0, this marks off the point $\widetilde{y_{C0}}$ on the country's period-0 static production-possibility frontier corresponding to zero output of the investment good, $y_{I0} = 0$, and maximum output of the consumption good $y_{C0} = \widetilde{y_{C0}}$. The period-0 production possibility frontier is shown in the southeast quadrant of Figure 1.

Starting in the southeast quadrant from the point of maximum output of the inestment good in period 0, $\widetilde{y_{I0}}$, corresponding to zero output of the consumption good y_{C0} in period 0, and moving horizontally leftward to the displaced 45°-line in the southwest quadrant, we mark off on the left horizontal axis the amount of capital $k_1 = k_0 + y_{I0}$ available in period 1, all of which (since period 1 is the last period) will be allocated to production of the consumption good in period 1; thus one traces a point upward from the displaced 45°-line in the southwest quadrant to the production function in the northwest quadrant and then horizontally rightward until it meets the upward vertical axis measuring the maximum output of the consumption good in period 1, denoted $\widetilde{y_{C1}}$. This determines the other extreme point of the country's intertemporal production-possibility frontier.

Intermediate points are obtained in the same way. Starting from any feasible point on the rightward horizontal axis (to the left of the maximum period-0 output of the consumption good)—which is to form the first component of a point on the intertemporal production-possibility frontier—the second component is determined (as shown by the dashed lines in the diagram) by moving downward in the southeast quadrant to the period-0 production-possibility frontier, then leftward to the displaced 45°-line in the southwest quadrant, then upward to the production function in the northwest quadrant; the corresponding intercept on the upward verical axis determines the remaining component of the point on the intertemporal production-possibility frontier.

It will be noted that the intertemporal production-possibility frontier contains an inefficient vertical segment from $(\widetilde{y_{C0}}, \widetilde{y_{C0}})$ to $(\widetilde{y_{C0}}, 0)$. The slope of the intertemporal production-possibility frontier at the point $(\widetilde{y_{C0}}, \widetilde{y_{C0}})$ is the maximum price ratio (interest factor) p_{C0}/p_{C1} for which the country will produce the future consumption good.

3 Intertemporal comparative advantage

The concept of "comparative advantage" is well defined in the case of a two-commodity one-factor Ricadian model in which each country has constant costs, but it needs to be redefined in the case of the more general Heckscher-Ohlin-Lerner-Samuelson model. In the static case, on which we build, the key property is that if technologies are identical between countries, the ratio of a country's outputs, *at any given world prices*, depends uniquely on the ratio of its factor endowments, and indeed is a monotone function of this factor-endowment ratio. Under the (admittedly stringent) assumptions of the Heckscher-Ohlin theorem (cf., e.g., Chipman [10], p. 938, or Riezman [38]), since a country will export the good which uses its relatively well-endowed factor relatively intensively, we may say that it has a "comparative advantage" in this good. In this section we show that when the following Factor-Intensity Hypothesis holds, this property extends to our model of intertemporal trade.

Factor-Intensity Hypothesis. In period 0, the consumption-good industry is more capital-intensive than the investment-good industry (i.e., the cost-minimizing capital-labor ratio is higher in the consumption-good industry than in the investment-good industry), at all factor rentals in period 0, i.e.,

$$\frac{b_{KC}(w_0, r_0)}{b_{LC}(w_0, r_0)} > \frac{b_{KI}(w_0, r_0)}{b_{LI}(w_0, r_0)} \quad \text{for all } (w_0, r_0), \tag{3.1}$$

where (by Shephard's theorem [42])

$$b_{Lj}(w_t, r_t) = \frac{\partial g_j(w_t, r_t)}{\partial w_t}, \quad b_{Kj}(w_t, r_t) = \frac{\partial g_j(w_t, r_t)}{\partial r_t}, \tag{3.2}$$

and

$$g_j(w_t, r_t) \qquad (j = C, I;\ t = 0, 1) \tag{3.3}$$

is the minimum-unit-cost function dual to the production function (1.1).

Our object now is to establish the following:

THEOREM 1. The intertemporal Rybczynski functions (1.19) have the property that the ratio

$$\frac{\overline{\overline{y_{C0}}}(p_{C0}, p_{C1}, l_0, k_0)}{\overline{\overline{y_{C1}}}(p_{C0}, p_{C1}, l_0, k_0)} = \frac{\overline{\overline{y_{C0}}}(p_{C0}, p_{C1}, 1, k_0/l_0)}{\overline{\overline{y_{C1}}}(p_{C0}, p_{C1}, 1, k_0/l_0)} \tag{3.4}$$

is a monotone increasing function of k_0/l_0.

PROOF: First we observe from definition (1.16) and the homogeneity of the production functions (1.1) that the intertemporal Rybczynski functions $\overline{\overline{y_{Ct}}}$ of (1.19) are homogeneous of degree 1 in (l_0, k_0), so the second equality in (3.4) follows. We then proceed to differentiate (3.4) with respect to k_0. We need then to show that the numerator of the resulting expression is positive, i.e.,

$$N(p_{C0}, p_{C1}, l_0, k_0) = y_{C1}\,\frac{\partial\,\overline{\overline{y_{C0}}}}{\partial k_0} - y_{C0}\,\frac{\partial\,\overline{\overline{y_{C1}}}}{\partial k_0} > 0. \tag{3.5}$$

Now since both goods are produced in period 0, the period-0 Rybczynski functions for given prices p_{C0}, p_{I0} are the solution of

$$\begin{bmatrix} y_{C0} \\ y_{I0} \end{bmatrix} = \begin{bmatrix} b_{KC} & b_{KI} \\ b_{LC} & b_{LI} \end{bmatrix}^{-1} \begin{bmatrix} k_0 \\ l_0 \end{bmatrix} = \frac{1}{|B|} \begin{bmatrix} b_{LI} & -b_{KI} \\ -b_{LC} & b_{KC} \end{bmatrix} \begin{bmatrix} k_0 \\ l_0 \end{bmatrix}, \tag{3.6}$$

where $|B| = b_{KC}b_{LI} - b_{LC}b_{KI} > 0$ on account of (3.1), hence

$$\frac{\partial\,\overline{y_{I0}}}{\partial k_0} < 0 \quad \text{and} \quad \frac{\partial\,\overline{y_{I0}}}{\partial l_0} > 0. \tag{3.7}$$

In order to obtain expressions for the intertemporal Rybczynski functions for the consumption good, we first obtain one for the investment good. Prior to that, however, we need to make use of (1.8) and the fact that all resources in period 1 are devoted to the consumption good, so that the marginal value productivity of capital in period 1 is

$$p_{I0} = r_1 = p_{C1}\frac{\partial f_C(l_1, k_1)}{\partial k_1} \equiv \overline{p_{I0}}\,(p_{C1}, l_1, k_1), \tag{3.8}$$

which defines the function $\overline{p_{I0}}$. Then making use of (1.3) and (1.4) we define the intertemporal Rybczynski function for y_{I0} implicitly by

$$\overline{\overline{y_{I0}}}(p_{C0}, p_{C1}, l_0, k_0) = \overline{y_{I0}}\Big(p_{C0}, \overline{p_{I0}}\,\big(p_{C1}, l_0, k_0 + \overline{\overline{y_{I0}}}(p_{C0}, p_{C1}, l_0, k_0)\big), l_0, k_0\Big). \tag{3.9}$$

(We verify that the Jacobian, J, of this transformation satisfies

$$J = 1 - \frac{\partial\,\overline{y_{I0}}}{\partial p_{I0}} \cdot \frac{\partial\,\overline{p_{I0}}}{\partial k_1} > 1, \tag{3.10}$$

since $\partial\overline{y_{I0}}/\partial p_{I0} > 0$ and $\partial\overline{p_{I0}}/\partial k_1 < 0$.) We then define the intertemporal Rybczynski functions for the consumption good in the two periods by

$$
(3.11)\quad \overline{\overline{y_{C0}}}(p_{C0}, p_{C1}, l_0, k_0) = \overline{y_{C0}}\Big(p_{C0}, \overline{p_{I0}}\left(p_{C1}, l_0, k_0 + \overline{\overline{y_{I0}}}(p_{C0}, p_{C1}, l_0, k_0)\right), l_0, k_0\Big),
$$

and

$$
(3.12)\qquad \overline{\overline{y_{C1}}}(p_{C0}, p_{C1}, l_0, k_0) = f_C\Big(l_0, k_0 + \overline{\overline{y_{I0}}}(p_{C0}, p_{C1}, l_0, k_0)\Big).
$$

respectively.

From (3.11), (3.12), and (3.9) we compute

$$
\begin{aligned}
(3.13)\qquad \frac{\partial\,\overline{\overline{y_{C0}}}}{\partial k_0} &= \frac{\partial\,\overline{y_{C0}}}{\partial k_0} + \frac{\partial\,\overline{y_{C0}}}{\partial p_{I0}}\frac{\partial\,\overline{p_{I0}}}{\partial k_1}\left(1 + \frac{\partial\,\overline{\overline{y_{I0}}}}{\partial k_0}\right), \\
(3.14)\qquad \frac{\partial\,\overline{\overline{y_{C1}}}}{\partial k_0} &= \frac{\partial f_C}{\partial k_1}\left(1 + \frac{\partial\,\overline{\overline{y_{I0}}}}{\partial k_0}\right), \quad \text{and} \\
(3.15)\qquad \frac{\partial\,\overline{\overline{y_{I0}}}}{\partial k_0} &= \frac{\dfrac{\partial\,\overline{y_{I0}}}{\partial k_0} + \dfrac{\partial\,\overline{y_{I0}}}{\partial p_{I0}}\cdot\dfrac{\partial\,\overline{p_{I0}}}{\partial k_1}}{1 - \dfrac{\partial\,\overline{y_{I0}}}{\partial p_{I0}}\cdot\dfrac{\partial\,\overline{p_{I0}}}{\partial k_1}}.
\end{aligned}
$$

We notice from (3.15) that the expression in parentheses in (3.13) and (3.14) reduces to

$$
(3.16)\qquad 1 + \frac{\partial\,\overline{\overline{y_{I0}}}}{\partial k_0} = \frac{1 + \dfrac{\partial\,\overline{y_{I0}}}{\partial k_0}}{1 - \dfrac{\partial\,\overline{y_{I0}}}{\partial p_{I0}}\cdot\dfrac{\partial\,\overline{p_{I0}}}{\partial k_1}},
$$

whose denominator is the (positive) Jacobian (3.10). Multiplying (3.5) through by J, our problem reduces to that of showing that the expression

$$
\begin{aligned}
(3.17)\quad JN \;=\; & y_{C1}\left[\frac{\partial\,\overline{y_{C0}}}{\partial k_0}\left(1 - \frac{\partial\,\overline{y_{I0}}}{\partial p_{I0}}\cdot\frac{\partial\,\overline{p_{I0}}}{\partial k_1}\right) + \frac{\partial\,\overline{y_{C0}}}{\partial p_{I0}}\cdot\frac{\partial\,\overline{p_{I0}}}{\partial k_1}\left(1 + \frac{\partial\,\overline{y_{I0}}}{\partial k_0}\right)\right] \\
& - y_{C0}\frac{\partial f_C}{\partial k_0}\left(1 + \frac{\partial\,\overline{y_{I0}}}{\partial k_0}\right)
\end{aligned}
$$

is positive.

Since $\partial\,\overline{y_{I0}}/\partial k_0 < 0$ from (3.7), we consider two cases:

Case 1: $1 + \dfrac{\partial\,\overline{y_{I0}}}{\partial k_0} > 0$. In this case we note that the second expression in brackets in (3.17) is positive, since: (1) $\partial\,\overline{p_{I0}}/\partial k_1 < 0$, owing to the fact that (from (1.8)) an increase in capital in period 1 lowers the rental of capital in the consumption-good industry in period 1, the consumption good being the only produced good in period 1; and (2) $\partial\,\overline{y_{C0}}/\partial p_{I0} < 0$ from the Factor-Intensity Hypothesis, since in period 0 (both goods being produced), a rise in the price of the investment good will, by the Rybczynski theorem, lower the output of the consumption good. Since moreover $J > 1$ and $0 < 1 + \partial\,\overline{y_{I0}}/\partial k_0 < 1$, we have

$$JN > y_{C1}\frac{\partial\,\overline{y_{C0}}}{\partial k_0} - y_{C0}\frac{\partial f_C}{\partial k_0}. \tag{3.18}$$

Now since all resources are devoted to the consumption good in period 1, we have by Euler's theorem, (1.8), and (3.2),

$$p_{C1} = g_C(w_1, r_1) = \frac{\partial g_C}{\partial w_1}w_1 + \frac{\partial g_C}{\partial r_1}r_1 \geq b_{KC}p_{I0} \tag{3.19}$$

where by (3.2)

$$\frac{\partial g_C}{\partial r_1} = b_{KC} = \frac{k_1}{y_{C1}} = \frac{k_0 + y_{I0}}{y_{C1}}. \tag{3.20}$$

Further, since the marginal physical productivity of capital in the consumption-good industry in period 1 equal to the price of the investment good in period 0 relative to the price of the consumption good in period 1, we have

$$\frac{\partial f_C}{\partial k_1} = \frac{p_{I0}}{p_{C1}}. \tag{3.21}$$

Substituting (3.19), (3.20), and (3.21) in (3.18) we obtain

$$JN\frac{p_{C1}}{p_{I0}} > (k_0 + y_{I0})\frac{\partial\,\overline{y_{C0}}}{\partial k_0} - y_{C0}. \tag{3.22}$$

Finally we evaluate y_{C0} and its partial derivative $\partial\,\overline{y_{C0}}/\partial k_0$ from the period-0 Rybczyski functions (3.6):

$$y_{C0} = \frac{b_{LI}k_0 - b_{KI}l_0}{|B|} \quad \text{and} \quad \frac{\partial\,\overline{y_{c0}}}{\partial k_0} = \frac{b_{LI}}{|B|}, \tag{3.23}$$

to obtain

$$JN\frac{p_{C1}}{p_{I0}} > (k_0 + y_{I0})\frac{b_{LI}}{|B|} - \frac{b_{LI}k_0 - b_{KI}l_0}{|B|} = \frac{1}{|B|}(b_{LI}y_{I0} + b_{KI}l_0) > 0. \tag{3.24}$$

Case 2: $1 + \dfrac{\partial\,\overline{y_{I0}}}{\partial k_0} \leq 0$. In this case it is clear from (3.17) that

$$\begin{aligned}\frac{JN}{y_C} &\geq \frac{\partial\,\overline{y_{C0}}}{\partial k_0}\left(1 - \frac{\partial\,\overline{y_{I0}}}{\partial p_{I0}}\cdot\frac{\partial\,\overline{p_{I0}}}{\partial k_1}\right) + \frac{\partial\,\overline{y_{C0}}}{\partial p_{I0}}\cdot\frac{\partial\,\overline{p_{I0}}}{\partial k_1}\left(1 + \frac{\partial\,\overline{y_{I0}}}{\partial k_0}\right)\\ &= \frac{\partial\,\overline{y_{C0}}}{\partial k_0}\left(1 - \frac{\partial\,\overline{y_{I0}}}{\partial p_{I0}}\cdot\frac{\partial\,\overline{p_{I0}}}{\partial k_1}\right) - \frac{p_{I0}}{p_{C0}}\frac{\partial\,\overline{y_{I0}}}{\partial p_{I0}}\frac{\partial\,\overline{p_{I0}}}{\partial k_1}\left(1 + \frac{\partial\,\overline{y_{I0}}}{\partial k_0}\right)\end{aligned} \tag{3.25}$$

since $p_{C0}\dfrac{\partial\,\overline{y_{I0}}}{\partial p_{C0}} + p_{I0}\dfrac{\partial\,\overline{y_{I0}}}{\partial p_{I0}} = 0$ from the homogeneity of degree 0 of $\overline{y_{I0}}(p_{C0}, p_{C1}, l_0, k_0)$ in (p_{C0}, p_{C1}). Hence from (3.25) we have

$$\frac{JN}{y_C} \geq \frac{\partial\,\overline{y_{C0}}}{\partial k_0} - \frac{\partial\,\overline{y_{I0}}}{\partial p_{I0}}\frac{\partial\,\overline{p_{I0}}}{\partial k_0}\left[\frac{\partial\,\overline{y_{C0}}}{\partial k_0} + \frac{p_{I0}}{p_{C0}}\left(1 + \frac{\partial\,\overline{y_{I0}}}{\partial k_0}\right)\right]. \tag{3.26}$$

Now $\partial\,\overline{y_{I0}}/\partial p_{I0} > 0$ from the convexity of the period-0 domestic-product function in the prices, and $\partial\,\overline{y_{C0}}/\partial k_0 > 0$ from the Factor-Intensity Hypothesis; moreover, $\partial\,\overline{p_{I0}}/\partial k_1 < 0$ from the diminishing marginal productivity of capital in the consumption-good industry in period 1; thus the expression (3.26) is positive provided the bracketed term is nonnegative.

Now, from the reciprocity of the partial derivatives of the period-0 Rybczynski and Stolper-Samuelson functions (cf. Samuelson [41], p. 10) we have

$$\frac{\partial\,\overline{y_{i0}}}{\partial k_0} = \frac{\partial\,\overline{r_0}}{\partial p_{i0}} \quad \text{for } i = C, I. \tag{3.27}$$

But the bracketed term in (3.26) is positive since

$$\begin{aligned} p_{C0}\frac{\partial\,\overline{r_0}}{\partial p_{C0}} + p_{I0}\left(1 + \frac{\partial\,\overline{r_0}}{\partial p_{I0}}\right) &= \frac{\partial\,\overline{r_0}}{\partial p_{C0}}p_{C0} + \frac{\partial\,\overline{r_0}}{\partial p_{I0}}p_{I0} + p_{I0} \\ &= r_0 + p_{I0} > 0, \end{aligned} \tag{3.28}$$

the last equality following from the homogeneity of degree 0 of the Stolper-Samuelson function $\overline{r_0}(p_{C0}, p_{I0}, l_0, k_0)$. □

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