Protection and Exchange Rates in a Small Open Economy*

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Abstract

In a small-open-economy model with two tradables and one nontradable, if a price index of these three goods is stabilized and the exchange rate is flexible, conditions are obtained in the cases of two and of three or more factors for an export subsidy or an import tariff to result in currency appreciation. In the case of three or more factors, conditions are obtained under which either an export-subsidy or an import-tariff policy (or a combination) can take the place of a flexible exchange rate in accommodating the necessary resource allocation to an exogenous capital outflow, generalizing Keynes's 1931 proposition.

1 Introduction

In his Addendum to the Macmillan Report to the British Parliament, Keynes (1931, p. 199; 1981, p. 296) declared:

> Precisely the same effects as those produced by a devaluation of sterling by a given percentage could be brought about by a tariff of the same percentage on all imports together with an equal subsidy on all exports, except that this measure *would leave sterling international obligations unchanged in terms of gold.*

Keynes wrote during the Great Depression when the main object was to increase employment; his argument may be interpreted as saying that either a currency devaluation or a "tariffs plus bounties" scheme would, by raising domestic prices of tradable goods, have the effect of cutting real wages and increasing employment (1931, p. 195; 1981, p. 290).[1] Keynes's proposition is universally valid, however; in this paper a generalized version of it will be applied to a neoclassical model in which wages are

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flexible and full employment of resources prevails. In this model it turns out that the causal relation between the trade balance and the exchange rate is reversed: under a flexible exchange rate, an exogenous transfer out of a country will lead to a depreciation of its currency, causing the domestic prices of its tradables to rise and those of its nontradables to fall, thus encouraging the necessary resource reallocation from the nontradables sectors to the tradables sectors.

It has long been observed that the policies of "import substitution" and "export promotion" pursued in many Latin American countries have resulted in "overvalued" currencies. To the extent that the tariff and subsidy rates on all commodities are equal, this is simply a prediction of Keynes's proposition. In fact, a currency may be said to be "overvalued" if it is above what it would be in the absence of trade restrictions.

In this paper I analyze the relationships between trade restrictions and exchange rates in the case of a "small open economy", in terms of the traditional Lerner-Samuelson model with neoclassical production functions and full employment of resources.[2] It is assumed that the "home country" produces three commodities: an export good, an import good, and a nontradable good, with two or more factors of production. In analyzing the case of flexible exchange rates, it is assumed further that the country's monetary authority pursues a policy (by whatever means) to stabilize a domestic price index of the three commodities. Under the assumption (sections 2 and 3) that the country's trade is balanced (in external prices), conditions are obtained for an import tariff or an export subsidy to cause an appreciation of the country's exchange rate (one of these must always do so). In the two-factor case these conditions have to do with the relative factor-output ratios as between tradables and the nontradable; in the case of three or more factors, they concern the "net substitutabilities" (Slutsky terms minus production transformation terms) between these sectors. In both cases, either an import tariff or an export subsidy necessarily causes a currency appreciation, and possibly each may do so.

In the case of three or more factors, section 4 deals with a situation in which our country introduces a Tinbergen-type policy function which expresses a tariff or subsidy factor as a function of the foreign balance—as an alternative to relying on a gold-standard mechanism or a flexible exchange rate—to adjust resource allocation to an exogenous capital outflow. Recent history has shown that some countries (e.g., Argentina in the period 2000–2003) have employed such tools to cope with capital flight during an exchange crisis. It is shown that, in principle, such a mechanism can work if the net-substitutability conditions are satisfied.

2   The Two-Factor Case

We start by considering the effect on the price of our country's nontradable good of (*a*) a tariff imposed on its import good, or (*b*) a subsidy conferred on its export

good, when there is a uniform world currency and the world prices of our country's import and export goods are beyond its control. We assume that the country has neoclassical production functions $y_j = f_j(v_{1j}, v_{2j})$ for the three commodities $j = 1, 2, 3$, concave and exhibiting constant returns to scale, and that the factors are perfectly mobile between industries with resource-allocation constraints $v_{i1} + v_{i2} = l_i$ for the two factor endowments l_1, l_2 The country's two factor rentals are determined as the solution of the equations $g_j(w_1, w_2) = p_j$ $(j = 1, 2)$ setting minimum-unit costs equal to the domestic prices of the two tradables, where the g_j are the dual minimum-unit-cost functions, the p_j are the domestic prices of the three commodities (the domestic prices of the tradables being manipulable by tariffs and subsidies), and the w_i are the rentals of the two factors. Denoting the solutions to these two cost equations by $\hat{w}_i(p_1, p_2)$, they may be substituted into the minimum-unit-cost function for the nontradable good to obtain:

$$p_3 = \tilde{p}_3(p_1, p_2) \equiv g_3\Big(\hat{w}_1(p_1, p_2), \hat{w}_2(p_1, p_2)\Big). \tag{1}$$

The problem is simply to compute the partial derivatives of this function $\tilde{p}_3$. We note for future reference that since the minimum-unit-cost functions $g_j(w_1, w_2)$ are homogeneous of degree 1, so are the inverse cost functions $\hat{w}_i(p_1, p_2)$, and therefore so is the function $\tilde{p}_3(p_1, p_2)$.

We assume that our country exports good 1 and imports good 2 in the initial equilibrium. Adopting the convention that commodity 2 requires a larger proportion of factor 2 to factor 1 than commodity 1, we have

$$|B| \equiv \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = b_{11}b_{22} - b_{21}b_{12} = b_{11}b_{12}\left[\frac{b_{22}}{b_{12}} - \frac{b_{21}}{b_{11}}\right] > 0, \tag{2}$$

where by Shephard's theorem the ratio of the input of factor i to the output of commodity j is $b_{ij}(w_1, w_2) = \partial g_j(w_1, w_2)/\partial w_i$. Differentiating (1) we obtain

$$
\begin{aligned}
\left(\frac{\partial \tilde{p}_3}{\partial p_1}, \frac{\partial \tilde{p}_3}{\partial p_2}\right) &= (b_{13}, b_{23}) \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix}^{-1} \\
&= |B|^{-1} (b_{13}, b_{23}) \begin{bmatrix} b_{22} & -b_{21} \\ -b_{12} & b_{11} \end{bmatrix} \\
&= |B|^{-1} \left(b_{12}b_{13}\left[\frac{b_{22}}{b_{12}} - \frac{b_{23}}{b_{13}}\right], b_{11}b_{13}\left[\frac{b_{23}}{b_{13}} - \frac{b_{21}}{b_{11}}\right]\right). \tag{3}
\end{aligned}
$$

Given the assumption (2), it is obviously impossible for *both* bracketed expressions in (3) to be nonpositive, hence at least one of them must be positive. This means that, necessarily, *either* a rise in the export price, *or* a rise in the import price, will cause a rise in the price of the nontradable. In order for *both* partial derivatives $\partial \tilde{p}_3/\partial p_j$ to be positive we must have

$$\frac{b_{22}}{b_{12}} > \frac{b_{23}}{b_{13}} > \frac{b_{21}}{b_{11}}, \tag{4}$$

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i.e., the factor-intensity ratio of the nontradables sector must lie strictly in between those of the two tradables sectors.
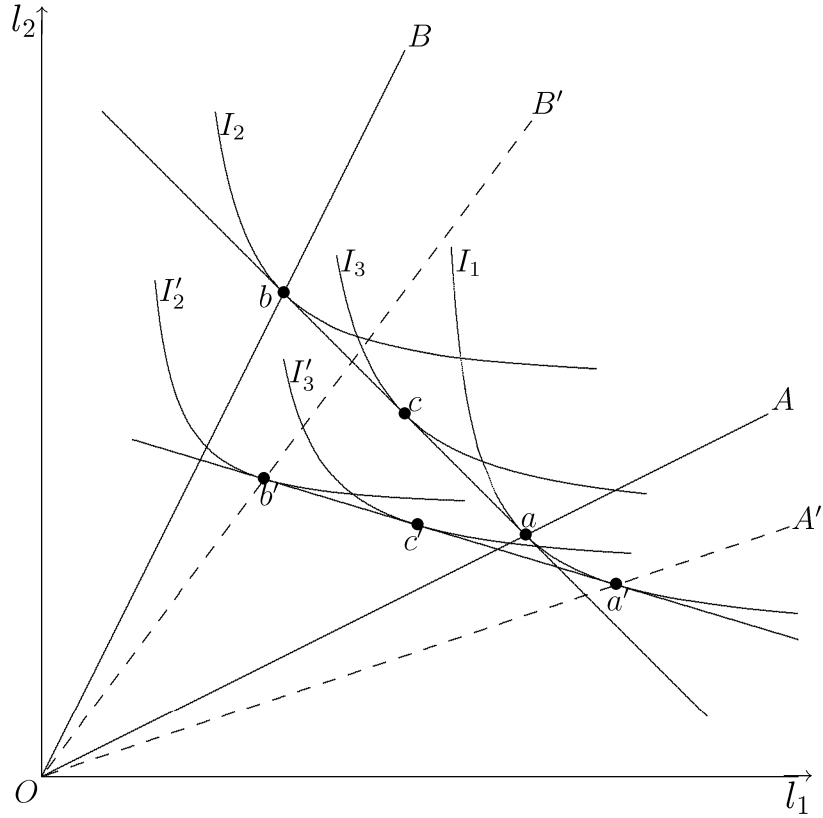

Figure 1

Now we introduce trade restrictions which create a wedge between the world prices p_1^*, p_2^* of the two tradables and their corresponding domestic prices p_1, p_2.

(a) In the case of a tariff at the rate $\tau_2 > 0$ imposed by our country on its import of commodity 2, and expressed as a proportion of its world price p_2^*, we have $p_2 = (1 + \tau_2)p_2^* = T_2 p_2^*$, where $T_2 = 1 + \tau_2 > 1$ is the *tariff factor*. Thus, a rise in the tariff rate τ_2 brings about a rise in p_2, given that p_2^* is assumed to be given on the world market. A necessary and sufficient condition for this to cause a rise in the price of the nontradable, p_3, is then, from (3), that $b_{23}/b_{13} > b_{21}/b_{11}$, i.e., that the nontradables sector should use a larger proportion of factor 2 to factor 1 than the export sector.

The result is illustrated in Figure 1, where the inequalities (4) are assumed to hold. I_1 is the isoquant for the export good (commodity 1), defined by $f_1(l_1, l_2) = 1/p_1$, showing the combinations of inputs of the two factors l_1, l_2 that will yield an output of an amount of commodity 1 whose value (at the initial domestic price p_1) is equal to one unit of our country's currency. I_2 is the isoquant for the import good (commodity

4

2) defined by $f_2(l_1, l_2) = 1/p_2$. The common tangent to these two isoquants goes through points a and b. Since it is assumed that commodity 3 is also produced, its isoquant I_3 (defined in the same way as the other two) must be tangential to the line through a and b, at the point c. Now suppose the domestic price of the import good rises (say as a result of imposition of a tariff); then the isoquant I_2 must shift inward, say to I'_2. The diversification cone swings clockwise from AOB to $A'OB'$, and the new common tangent to I'_2 and I_1 now goes through points a' and b'. Since it is assumed that commodity 3 continues to be produced, the isoquant I_3 must also shift inward, to I'_3, forming the new tangency point c'. This means that the price of the nontradable good must rise.

Figure 2 illustrates an opposite case, in which instead of (4) holding, we have $b_{22}/b_{12} > b_{21}/b_{11} > b_{23}/b_{13}$. In this case, the nontradables sector uses an even lower ratio of factor 2 to factor 1 than the export sector. Starting from the initial situation in which the isoquants I_2, I_1, and I_3 line up with common tangents at b, a, and c, an increase in the domestic price of the import good (commodity 2) will (as before) result in an inward shift of I_2 to I'_2, and the new common tangent to I'_2 and I_1 goes through b' and a'. This time, however, the isoquant for the nontradable must shift outward from I_3 at c to I'_3 at c'; this means that the price of the nontradable must fall. However, the obverse implication of the above set of inequalities is that a rise in the export price must lead to a rise in the price of the nontradable.

(b) In the case of a subsidy at the rate $\bar{\sigma}_1 > 0$ imposed by country 1 on its export of commodity 1, expressed as a proportion of the *home* price (to which the bar refers), we have $p_1^* = (1 - \bar{\sigma}_1)p_1 = \bar{T}_1 p_1$, where $\bar{T}_1 = 1 - \bar{\sigma}_1 < 1$. Equivalently, defining the *subsidy factor*

$$T_1 = 1 + \sigma_1 = 1/(1 - \bar{\sigma}_1) = 1/\bar{T}_1, \tag{5}$$

where $\sigma_1 > 0$, we have $p_1 = T_1 p_1^* = p_1^*/\bar{T}_1$ where $T_1 > 1$. This expresses the subsidy rate σ_1 as a proportion of the *foreign* price. Thus, a rise in either rate of subsidy σ_1 or $\bar{\sigma}_1$ brings about a rise in the domestic price p_1, given that p_1^* is assumed to be given on the world market. From (3), a necessary and sufficient condition for this to cause a rise in the price of the nontradable is that $b_{22}/b_{12} > b_{23}/b_{13}$, i.e., that the import-competing sector should use a larger ratio of factor 2 to country 1 than the nontradables sector.

(c) Let χ denote the value of the home country's currency in amounts of the foreign currency. Then the nominal prices of the tradable goods on the home country's markets will be related to those on the world market by

$$p_1 = T_1 p_1^*/\chi = (1 + \sigma_1)p_1^*/\chi \quad \text{and} \quad p_2 = T_2 p_2^*/\chi = (1 + \tau_2)p_2^*/\chi. \tag{6}$$

We may note from (6) that if $T_1 = T_2 = T$, then a rise in this uniform rate T of subsidy and tariff has exactly the same effect as a fall in χ, i.e., as a devaluation of country 1's currency. This was first observed by Keynes (1931); we shall call this *Keynes's equivalence theorem.* Under the same conditions, the domestic price ratio
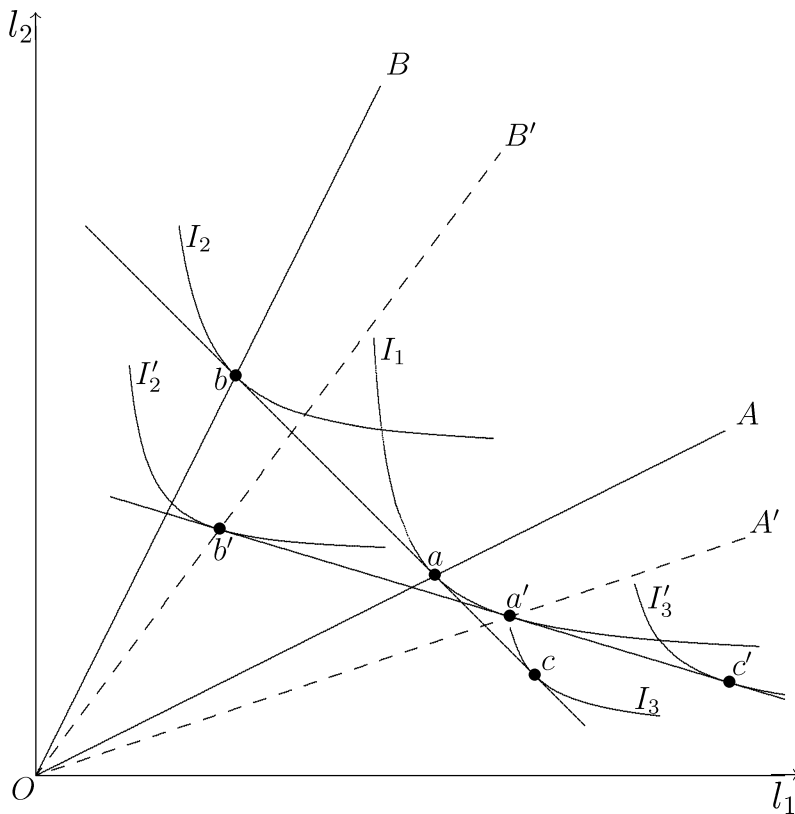
Figure 2

is equal to the foreign price ratio, hence there is no distortion in relative prices—an observation first made by Hicks (1951). These two observations together may be described as the Keynes-Hicks equivalence theorem.[3]

(d) Let us now assume that the monetary authority acts so as to stabilize a *price index*

$$p \equiv \bar{c}_1 p_1 + \bar{c}_2 p_2 + \bar{c}_3 p_3, \tag{7}$$

where the $\bar{c}_i$ are nonnegative weights and $\bar{c}_3 > 0$. For definiteness we may think of these weights as defined by the partial derivatives of the demand functions with respect to income, $c_i = \partial h_i(p_1, p_2, p_3, Y)/\partial Y$, where the prices p_j and disposable national income Y are evaluated at their initial levels. Assuming preferences in the country to be identical and homothetic, of course $\partial h_i/\partial Y = x_i/Y$ where $x_i = h_i(p_1, p_2, Y)$ is the amount of commodity i consumed in the home country. We will assume that $c_i \geq 0$ for $i = 1, 2, 3$. Now substituting (6) into (7) and fixing the price level at $\bar{p}$, we obtain, using the homogeneity of degree 1 of the function $\tilde{p}_3(p_1, p_2)$, $\bar{p} = [\bar{c}_1 T_1 p_1^* + \bar{c}_2 T_2 p_2^* + \bar{c}_3 \tilde{p}_3(T_1 p_1^*, T_2 p_2^*)]/\chi$. The exchange rate is then determined from $\chi = [\bar{c}_1 T_1 p_1^* + \bar{c}_2 T_2 p_2^* + \bar{c}_3 \tilde{p}_3(T_1 p_1^*, T_2 p_2^*)]/\bar{p} \equiv \check{\chi}(T_1, T_2, p_1^*, p_2^*, \bar{p}, \bar{c}_1, \bar{c}_2, \bar{c}_3)$, which

defines the function $\check{\chi}$. We see then that

$$
\begin{aligned}
\frac{T_1}{\chi}\frac{\partial\check{\chi}}{\partial T_1} &= \frac{p_1}{\bar{p}}\left\{\bar{c}_1 + \bar{c}_3\frac{b_{12}b_{13}}{|B|}\left[\frac{b_{22}}{b_{12}} - \frac{b_{23}}{b_{13}}\right]\right\}, \\
\frac{T_2}{\chi}\frac{\partial\check{\chi}}{\partial T_2} &= \frac{p_2}{\bar{p}}\left\{\bar{c}_2 + \bar{c}_3\frac{b_{11}b_{13}}{|B|}\left[\frac{b_{23}}{b_{13}} - \frac{b_{21}}{b_{11}}\right]\right\}.
\end{aligned}
\tag{8}
$$

Thus, as long as $\bar{c}_j \geq 0$ $(j = 1, 2)$ and $\bar{c}_3 > 0$, as assumed in (7), the inequalities (4) are *sufficient* for an increase in the rate of subsidy on exports, or an increase in the rate of tariff on imports, or both, to cause an appreciation of the exchange rate. More generally, assuming only the factor-endowment inequality (2) to hold, it is necessarily the case that *either* an increase in the rate of subsidy on exports, *or* an increase in the rate of tariff on imports, and possibly but not necessarily each, will cause an appreciation of our country's exchange rate. That is, one of the tools is always available to cause a currency appreciation.

(f) Finally, let us suppose that the subsidy and tariff factors are required to be uniform, i.e., $T_1 = T_2 = T$. Then once again using the homogeneity of degree 1 of $\tilde{p}_3$ we see that $\partial\check{\chi}/\partial T = 1$. This is Keynes's original proposition (1931).

3   The Three-Factor Case

We now consider the case of three (or more) factors of production l_i. As in the preceding section we assume that trade is balanced when denominated in world prices.

In the previous section, no information on consumer demand was needed to obtain the results. This is no longer the case when there are three or more factors. Letting $y_j = \hat{y}_j(p_1, p_2, p_3, l) = \partial\Pi(p_1, p_2, p_3, l)/\partial p_j$ denote the country's supply or Rybczynski function, where l is the vector of its factor endowments (of dimension at least 3), the equality between demand and supply of the nontradable,

$$
h_3\big(p_1, p_2, p_3, \Pi(p_1, p_2, p_3, l) + D\big) = \hat{y}_3(p_1, p_2, p_3, l),
\tag{9}
$$

where Π is the country's domestic-product function and D is the deficit in its trade balance (in home prices), implicitly defines the function

$$
p_3 = \tilde{p}_3(p_1, p_2, D; l).
\tag{10}
$$

We verify the following

LEMMA. The function $\tilde{p}_3(p_1, p_2, D; l)$ is homogeneous of degree 1 in (p_1, p_2, D).

PROOF: By Euler's theorem, the assertion is equivalent to the identity

$$
\frac{\partial\tilde{p}_3}{\partial p_1}p_1 + \frac{\partial\tilde{p}_3}{\partial p_2}p_2 + \frac{\partial\tilde{p}_3}{\partial D}D = \tilde{p}_3.
$$

Substituting (10) in (9) and then differentiating the resulting identity totally with respect to p_j $(j = 1, 2)$ and D, we obtain

$$\frac{\partial \tilde{p}_3}{\partial p_j} = -\frac{s_{3j} - t_{3j} - c_3 z_j}{s_{33} - t_{33}} \ (j = 1, 2) \quad \text{and} \quad \frac{\partial \tilde{p}_3}{\partial D} = -\frac{c_3}{s_{33} - t_{33}}, \qquad (11)$$

where

$$s_{ij} = \frac{\partial h_i}{\partial p_j} + \frac{\partial h_i}{\partial Y} h_j, \quad t_{ij} = \frac{\partial \hat{y}_i}{\partial p_j}, \quad c_i = \frac{\partial h_i}{\partial Y}, \quad \text{and} \quad z_i = x_i - y_i. \qquad (12)$$

Now denoting the Slutsky and transformation matrices $S = [s_{ij}]_{i,j=1,2,3}$ and $T = [t_{ij}]_{i,j=1,2,3}$, since the Slutsky matrix S is negative semi-definite and (from the homogeneity of degree zero of the demand functions in (p_1, p_2, p_3, Y)) satisfies $Sp = 0$ (where $p = (p_1, p_2, p_3)'$), and likewise the transformation matrix T is positive semi-definite (from the convexity of the domestic-product function Π in the prices) and satisfies $Tp = 0$ (from the homogeneity of degree 1 of the domestic-product function in the prices), we have

$$(s_{31} - t_{31})p_1 + (s_{32} - t_{32})p_2 + (s_{33} - t_{33})p_3 = 0. \qquad (13)$$

Finally we use our country's budget equation

$$p_1 z_1 + p_2 z_2 = D, \qquad (14)$$

where the $z_i = x_i - y_i$ are the *trades* (excess demands) defined in (12). Combining (11), (13), and (14) we see that

$$\begin{aligned} \frac{\partial \tilde{p}_3}{\partial p_1} p_1 + \frac{\partial \tilde{p}_3}{\partial p_2} p_2 + \frac{\partial \tilde{p}_3}{\partial D} D &= -\frac{(s_{31} - t_{31} - c_3 z_1)p_1 + (s_{32} - t_{32} - c_3 z_2)p_2 + c_3 D}{s_{33} - t_{33}} \\ &= -\frac{-(s_{33} - t_{33})p_3 - c_3(p_1 z_1 + p_2 z_2 - D)}{s_{33} - t_{33}} = p_3, \end{aligned}$$

as desired. QED

In this case it is necessary to consider income effects. Assuming balanced trade in world prices $p_1^* z_1 + p_2^* z_2 = 0$, the country's government's net revenue from the two instruments T_1 and T_2, expressed in its own currency, is

$$R = (T_1 - 1)(p_1^*/\chi)z_1 + (T_2 - 1)(p_2^*/\chi)z_2 = T_1(p_1^*/\chi)z_1 + T_2(p_2^*/\chi)z_2, \qquad (15)$$

It is assumed that any positive net revenue is distributed in a lump sum to the population, and that any negative net revenue is financed by lump-sum taxation. Since this net revenue constitutes (if positive) a part of total domestic expenditure over and above the domestic product, and by assumption trade is balanced when denominated in world prices, we have $R = D$. Noting that the price and deficit terms

(6) and (15) all contain the factor $1/\chi$, and using the fact that for $i = 1, 2$ the home country's trade-demand functions

$$z_i = \hat{h}_i(p_1, p_2, D, l) \equiv h_i\big(p_1, p_2, \tilde{p}_3, \Pi(p_1, p_2, \tilde{p}_3, l) + D\big) - \hat{y}_i(p_1, p_2, \tilde{p}_3, l) \qquad (16)$$

—where the function $\tilde{p}_3(\cdot)$ of (10) is substituted for p_3 in our country's consumer-demand and Rybczinski functions—are homogeneous of degree 0 in (p_1, p_2, D), our country's *trades* z_1, z_2 may be obtained by solving simultaneously the two equations $\hat{z}_i(\cdot) = \hat{h}_i\big(T_1 p_1^*, T_2 p_2^*, T_1 p_1^* \hat{z}_1(\cdot) + T_2 p_2^* \hat{z}_2(\cdot), l\big)$ $(i = 1, 2)$, which implicitly define the excess-demand functions

$$\hat{z}_i(\cdot) \equiv \hat{z}_i(p_1^*, p_2^*, T_1, T_2, l) \quad (i = 1, 2). \qquad (17)$$

Note that these functions are independent of χ, and homogeneous of degree 0 in (T_1, T_2) and in (p_1^*, p_2^*). Moreover, since the trade-balance constraint implies

$$p_1^* \partial \hat{z}_1 / \partial T_j + p_2^* \partial \hat{z}_2 / \partial T_j = 0, \qquad (18)$$

the functions (17) are of course not independent.

We now consider the effect of an export subsidy or import tariff on the price of the nontradable, when the exchange rate is fixed. It will further be assumed that the subsidy and tariff factors T_1 and T_2 are initially equal to one another (which of course includes the special case in which they are both $= 1$, i.e., in which there is initially free trade). Substituting the functions (17) in the expression (15) for our country's net revenues from the tariff and subsidy, and then substituting (6) and (15) in turn in (10), we see that the dependence of the nominal price of our country's nontradable on the tax factors and the exchange rate is given by (using the Lemma)

$$p_3 = \tilde{p}_3\big(T_1 p_1^*, T_2 p_2^*, T_1 p_1^* \hat{z}_1(\cdot) + T_2 p_2^* \hat{z}_2(\cdot), l\big) \Big/ \chi \equiv \breve{p}_3(p_1^*, p_2^*, T_1, T_2, \chi, l). \qquad (19)$$

Differentiating $\breve{p}_3$ with respect to T_j we have

$$\frac{\partial \breve{p}_3}{\partial T_j} = \frac{1}{\chi} \left[\frac{\partial \tilde{p}_3}{\partial p_j} p_j^* + \frac{\partial \tilde{p}_3}{\partial D} \left(p_j^* z_j + T_1 p_1^* \frac{\partial \hat{z}_1}{\partial T_j} + T_2 p_2^* \frac{\partial \hat{z}_2}{\partial T_j} \right) \right].$$

Now using the assumption that $T_1 = T_2$ initially, combined with (18), (11) and (6), this becomes (expressed in elasticity form)

$$\frac{T_j}{p_3} \frac{\partial \breve{p}_3}{\partial T_j} = \frac{p_j}{p_3} \left(\frac{\partial \tilde{p}_3}{\partial p_j} + \frac{\partial \tilde{p}_3}{\partial D} z_j \right) = -\frac{p_1(s_{3j} - t_{3j})}{p_3(s_{33} - t_{33})} \quad (j = 1, 2). \qquad (20)$$

We shall say that tradable commodity $j = 1, 2$ is a *net substitute* of the non-tradable commodity 3 if and only if $s_{3j} - t_{3j} > 0$, that is, if and only if the Slutsky substitution term less the production transformation term between commodities j

and 3, is positive. We see then that: *If the tariff rate on imports is initially equal to the subsidy rate on exports (in particular, if they are both initially equal to zero), and if the exchange rate is fixed, then an export tax will lead to a rise in the price of the nontradable if and only if the export good is a net substitute of the nontradable good, and an import tariff will lead to a rise in the price of the nontradable if and only if the import good is a net substitute of the nontradable good.*

Now assuming that the exchange rate is flexible and that the monetary authority stabilizes the price index (7) at the level $\bar{p}$, the exchange rate is determined by

$$
\begin{aligned}
\chi &= \left[\bar{c}_1 T_1 p_1^* + \bar{c}_2 T_2 p_2^* + \bar{c}_3 \tilde{p}_3 \left(T_1 p_1^*, T_2 p_2^*, T_1 p_1^* z_1(\cdot) + T_2 p_2^* z_2(\cdot), l \right) \right] \Big/ \bar{p} \\
&\equiv \breve{\chi}(T_1, T_2, p_1^*, p_2^*, \bar{p}, l; \bar{c}_1, \bar{c}_2, \bar{c}_3),
\end{aligned}
\tag{21}
$$

which defines the function $\breve{\chi}$. We find then without difficulty that

$$
\frac{T_1}{\chi} \frac{\partial \breve{\chi}}{\partial T_1} = \frac{p_1}{\bar{p}} \left[\bar{c}_1 - \bar{c}_3 \frac{s_{31} - t_{31}}{s_{33} - t_{33}} \right] \quad \text{and} \quad \frac{T_2}{\chi} \frac{\partial \breve{\chi}}{\partial T_2} = \frac{p_2}{\bar{p}} \left[\bar{c}_2 - \bar{c}_3 \frac{s_{32} - t_{32}}{s_{33} - t_{33}} \right].
\tag{22}
$$

Thus, given our assumption from (7) that $\bar{c}_3 > 0$ and $\bar{c}_i \geq 0$ for $i = 1, 2$, a *sufficient* condition for an export subsidy to cause an appreciation of our country's currency is that $s_{31} - t_{31} > 0$, i.e., that the export good be a net substitute of the nontradable good; and a *sufficient* condition for an import tariff to cause an appreciation of our country's currency is that $s_{32} - t_{32} > 0$, i.e., that the import good be a net substitute of the nontradable good. Since necessarily $s_{33} - t_{33} < 0$, it follows from (13) that *one* of the two tradables must be a net substitute of the nontradable, and possibly both. Thus it is always the case that *either* a subsidy on exports, *or* a tariff on imports, and possibly each, will lead to an appreciation of our country's currency.

It is clear that given our "small-country" assumption, an optimal subsidy-tariff combination (σ_1, τ_2) is any combination satisfying $\sigma_1 = \tau_2$, of which $\sigma_1 = \tau_2 = 0$ is a special case. This agrees with Hicks (1951) and Haberler (1967).

4   Protection as a Means of Adjusting to Imbalance in Payments

In this section I consider protective measures that might be taken by a country to adjust to an exogenous capital outflow. This is a case in which total expenditure ("absorption") falls short of total domestic product, and as long as the nontradable is a superior good, the demand for it falls short of the supply. In order to adjust to the outflow, the country's resources must move out of the nontradable into the tradables, and such a resource reallocation usually calls for a change in relative prices.[4] In the following subsections, I consider how these necessary adjustments can be brought about under (1) a gold-standard regime, (2) a flexible-exchange-rate regime, and (3) a regime in which protective measures are used.

4.1 Specie-flow adjustment

Under a gold standard with uniform international currencies, the effect of an exogenous capital inflow $I = D$ on the price of the nontradable is given by (11). Disposable income as a function of this inflow is given by $Y = \Pi(p_1, p_2, \tilde{p}_3(p_1, p_2, D, l), l) + D$, whence

$$\frac{dY}{dD} = \frac{\partial \Pi}{\partial p_3} \frac{\partial \tilde{p}_3}{\partial D} + 1 = 1 - \frac{c_3 y_3}{s_{33} - t_{33}} \geq 1, \tag{23}$$

the inequality being strict as long as the nontradable is a superior good ($c_3 > 0$). Assuming the demand for money to be proportionate to disposable income, gold must move in, not only to cover the capital inflow itself, but also to cover the increased production of the nontradable, which draws resources out of the nontradables industries. The same analysis of course applies to a capital outflow $O = -D$.

4.2 Exchange-rate adjustment

Now we introduce the exchange rate χ, which in the absence of trade restrictions satisfies $\chi = p_j^*/p_j$ for $j = 1, 2$, where also $D^* = \chi D$. Expressing (9) in terms of the exogenous variables p_1^*, p_2^*, and D^*, and using the homogeneity properties of the functions Π, $\hat{y}_3$, and h_3, we have

$$h_3(p_1^*, p_2^*, \chi p_3, \Pi(p_1^*, p_2^*, \chi p_3, l) + D^*) = \hat{y}_3(p_1^*, p_2^*, \chi p_3, l). \tag{24}$$

Likewise, the condition for stabilizing the price level (7) becomes

$$\bar{c}_1 p_1^* + \bar{c}_2 p_2^* + \bar{c}_3 \chi p_3 = \chi \bar{p}. \tag{25}$$

Equations (24) and (25) together define implicitly the functions $\chi = \hat{\chi}(p_1^*, p_2^*, D^*, l)$ and $p_3 = \hat{p}_3(p_1^*, p_2^*, D^*, l)$, whose partial derivatives are found to be

$$\frac{\partial \hat{\chi}}{\partial D^*} = \frac{\bar{c}_3}{\bar{p}} \frac{c_3}{-(s_{33} - t_{33})} \geq 0 \quad \text{and} \quad \frac{\partial \hat{p}_3}{\partial D^*} = \frac{\bar{p} - \bar{c}_3 p_3}{\bar{p}\chi} \frac{c_3}{-(s_{33} - t_{33})} \geq 0. \tag{26}$$

Thus, as long as the nontradable is a superior good ($c_3 > 0$), a capital outflow must lead to a devaluation, as well as to a decline in the nominal price of the nontradable, but (if $\chi = 1$ initially) a smaller decline than that indicated by (11). Obviously, the devaluation in that case is accompanied by a *rise* in the nominal domestic prices of the two tradables. In the extreme case $\bar{c}_1 = \bar{c}_2 = 0$ in which $\bar{p} = \bar{c}_3 p_3$, the price of the nontradable remains constant and the burden of the adjustment falls entirely on the rise in the prices of the two tradables.

4.3 Adjustment by trade restrictions

Now we suppose that in response to a capital outflow our country's government decides to keep the exchange rate fixed, and either confers a subsidy $\sigma_1 > 0$ on

its exports, or imposes a tariff $\tau_2 > 0$ on its imports, or both. Now our country's current-account balance, denominated in foreign prices, must satisfy

$$p_1^* z_1 + p_2^* z_2 = D^*, \qquad (27)$$

where $D^* = I^*$ denotes the exogenous capital inflow (outflow if negative) denominated in foreign prices, and z_j denotes the net import of commodity j, it being assumed that $z_1 < 0$ (export) and $z_2 > 0$ (import). Unlike the situation in subsection 4.2, however, it is no longer the case that $D = D^*/\chi$; this is because while there is only an exogenous inflow I^* denominated in world prices, in terms of domestic prices there are two components to the current-account deficit D, namely the exogenous component $I^*/\chi = D^*/\chi$, and the endogenous one, R, consisting of the net revenue from the trade restrictions (which are by assumption distributed to or taxed from consumers in a lump sum). This net revenue is (compare (15), where $I^* = 0$)

$$
\begin{aligned}
R &= \sigma_1(p_1^*/\chi)z_1 + \tau_2(p_2^*/\chi)z_2 = (T_1 - 1)p_1^* z_1/\chi + (T_2 - 1)p_2^* z_2/\chi \\
&= T_1(p_1^*/\chi)z_1 + T_2(p_2^*/\chi)z_2 - I^*/\chi = p_1 z_1 + p_2 z_2 - I^*/\chi.
\end{aligned}
\qquad (28)
$$

Our country's balance-of-payments deficit (denominated in its own currency) is then

$$D = I + R = I^*/\chi + R = p_1 z_1 + p_2 z_2. \qquad (29)$$

In the special case (to be considered below) in which $T_1 = T_2 = T$, the government's net revenue reduces to $R = (T - 1)D^*/\chi$ which is negative in the case of a capital outflow $O^* = -D^*$. The total deficit (in domestic currency) in this special case is

$$D = (T - 1)D^*/\chi + D^*/\chi = TD^*/\chi. \qquad (30)$$

We can now proceed with the analysis of the problem. We adopt the approach of Tinbergen (1952, p. 7) who pursued the theory of economic policy in terms of *targets* and *instruments*, and noted (p. 27) that in order to solve the policy problem, the number of instruments must be equal to the number of targets. In the present case we have a single target: the balance of payments; so we are limited to one instrument: either the subsidy factor, or the tariff factor, or a combination of both (constraining them, say, to be equal). The required policy variables can be defined by the action on the part of the central bank to stabilize the general price level, which results (with $D^* = -O^*$) in the following equation (using the Lemma):

$$\chi\bar{p} = \bar{c}_1 T_1 p_1^* + \bar{c}_2 T_2 p_2^* + \bar{c}_3 \tilde{p}_3\Big(T_1 p_1^*, T_2 p_2^*, (T_1 - 1)p_1^* \hat{z}_1(\cdot) + (T_2 - 1)p_2^* \hat{z}_2(\cdot) - O^*, l\Big). \quad (31)$$

If one of the tariff factors (say T_j) is given, and the other (say T_i) is the policy variable, then (31) implicitly defines the policy function $\hat{T}_i(O^*; T_j)$ where $j \neq i$. If $T_1 = T_2 = T$, then (31) implicitly defines the function $\hat{T}(O^*)$. Our object is to evaluate the derivatives of these functions.

12

Let us first evaluate $d\hat{T}_1/dO^*\big|_{T_1=T_2=1}$. Differentiating (31) with respect to O^*:

$$\left[p_1^*\left(\bar{c}_1 + \bar{c}_3\frac{\partial \tilde{p}_3}{\partial p_1}\right) + \bar{c}_3\frac{\partial \tilde{p}_3}{\partial D}\left(p_1^* z_1 + (T_1-1)p_1^*\frac{\partial \hat{z}_1}{\partial T_1} + (T_2-1)p_2^*\frac{\partial \hat{z}_2}{\partial T_1}\right)\right]\frac{\partial \hat{T}_1}{\partial O^*} = \bar{c}_3\frac{\partial \tilde{p}_3}{\partial D}.$$

At the values $T_1 = T_2 = 1$ in the initial equilibrium, this becomes

$$p_1^*\left[\bar{c}_1 + \bar{c}_3\left(\frac{\partial \tilde{p}_3}{\partial p_1} + \frac{\partial \tilde{p}_3}{\partial D}z_1\right)\right]\frac{\partial \hat{T}_1}{\partial O^*} = \bar{c}_3\frac{\partial \tilde{p}_3}{\partial D}. \tag{32}$$

But from the formulas (11) we have

$$\frac{\partial \tilde{p}_3}{\partial D} = -\frac{c_3}{s_{33}-t_{33}} \quad \text{and} \quad \frac{\partial \tilde{p}_3}{\partial p_1} + \frac{\partial \tilde{p}_3}{\partial D}z_1 = -\frac{s_{31}-t_{31}}{s_{33}-t_{33}}. \tag{33}$$

Substituting the two equations (33) in (32) we obtain

$$\partial \hat{T}_1/\partial O^* = -\bar{c}_3 c_3 / \{p_1^*[\bar{c}_1(s_{33}-t_{33}) - \bar{c}_3(s_{31}-t_{31})]\}. \tag{34}$$

This expression is positive, i.e., a capital outflow will call for an export subsidy, *as long as* (i) the nontradable is a superior good ($c_3 > 0$), and (ii) the export good and the nontradable good are *net substitutes*, i.e., $s_{31} - t_{31} > 0$.

Exactly the same steps provide the formula for the import tariff that would be required in order to adjust to the capital outflow. The corresponding formula is

$$\partial \hat{T}_2/\partial O^* = -\bar{c}_3 c_3 / \{p_2^*[\bar{c}_2(s_{33}-t_{33}) - \bar{c}_3(s_{32}-t_{32})]\}. \tag{35}$$

This expression is positive, i.e., a capital outflow will call for an import tariff, as long as (i) the nontradable is a superior good ($c_3 > 0$) and (ii) the import good and the nontradable good are *net substitutes*, i.e., $s_{32} - t_{32} > 0$.

Of course, these formulas provide no information on the required magnitudes of the subsidy or tariff, even if the assumptions of net substitutability are met.

Now let us consider the case in which $T_1 = T_2 = T$. Then differentiation of (31) with respect to O^* gives

$$0 = \left\{\bar{c}_1 p_1^* + \bar{c}_2 p_2^* + \bar{c}_3\left[\frac{\partial \tilde{p}_3}{\partial p_1}p_1^* + \frac{\partial \tilde{p}_3}{\partial p_2}p_2^* + \right.\right.$$
$$\left.\left.\frac{\partial \tilde{p}_3}{\partial D}\left(p_1^* z_1 + p_2^* z_2 + (T-1)p_1^*\frac{\partial \hat{z}_1}{\partial T} + (T-1)p_2^*\frac{\partial \hat{z}_2}{\partial T}\right)\right]\right\}\frac{d\hat{T}}{dO^*} - \bar{c}_3\frac{\partial \tilde{p}_3}{\partial D}. \tag{36}$$

Using (27) and (30), defining $p_3^* = \chi p_3/T$, and assuming $T = 1$ in the initial equilibrium, we may write the bracketed expression in (36) as

$$\frac{\chi}{T}\left(\frac{\partial \tilde{p}_3}{\partial p_1}p_1 + \frac{\partial \tilde{p}_3}{\partial p_2}p_2 + \frac{\partial \tilde{p}_3}{\partial D}D\right) = \frac{\chi}{T}p_3 = p_3^*,$$

13

using the Lemma in the first equality Now evaluating the derivative at the initial equilibrium where $\chi = T = 1$, (36) becomes

$$\frac{d\hat{T}}{dO^*} = \frac{\bar{c}_3 \partial \tilde{p}_3 / \partial D}{\sum_{i=1}^3 \bar{c}_i p_i^*} = \frac{\bar{c}_3}{\bar{p}} \cdot \frac{-c_3}{s_{33} - t_{33}} \geq 0, \tag{37}$$

the inequality being strict as long as $c_3 > 0$ (the nontradable is a superior good). This of course is just Keynes's equivalence theorem, which states that the effect (37) of a uniform export subsidy and import tariff is the same as the effect (26) of a currency devaluation.

In view of (13), *one* of the two terms $s_{31} - t_{31}$ and $s_{32} - t_{32}$ must be positive, so one of the two policy tools operating alone would (despite its adverse welfare effects) enable the country to adjust to the capital outflow. If both are positive, any combination (σ_1, τ_2) would work, but of course only the combinations with $\sigma_1 = \tau_2$ would avoid adverse welfare effects. Such tools, necessarily clumsy and inaccurate, would nevertheless have the advantage over devaluation (stressed by Keynes) of leaving long-term debt contracts unaffected.

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Notes

1. In a footnote (1931, p. 192; 1981, p. 286) Keynes stated that "to restore our position and to make full employment possible, we require both to increase our surplus on the balance of trade and to find outlets for our savings at home." It was later made clear in the literature that the increase in employment and income (brought about in this case by the devaluation and fall in real wages) would itself result in increased saving and thus in an increase in the balance of trade.

2. An earlier article (Chipman, 1992) analyzed these relationships in the case of a two-country model, but only incomplete results were obtained. That paper also discussed related contributions in the literature.

3. It is natural to inquire how Keynes's equivalence theorem is related to Lerner's symmetry theorem. The latter (Lerner, 1936) states, in effect, that the combination $(T_1, 1)$ of export and import tax factors is equivalent (in its effect on relative domestic prices) to the combination $(1, T_2)$—we may write this as $(T_1, 1) \sim (1, T_2)$—if and only if $T_1 T_2 = 1$. We may define the relationship between the export-tax factors expressed in terms of domestic and foreign prices respectively by

$$\bar{T}_1 = 1 + \bar{\tau}_1 = 1/(1 - \tau_1) = 1/T_1$$

(compare (5), where $\sigma_1 = -\tau_1$ and $\bar{\sigma}_1 = -\bar{\tau}_1$). Here, τ_1 is the tax rate on the export good when levied on the *foreign* price, i.e., $p_1 = T_1 p_1^* = (1 - \tau_1)p_1^*$, and $\bar{\tau}_1$ is the tax rate on the export good when levied on the *domestic* price, i.e., $p_1^* = \bar{T}_1 p_1 = (1 + \bar{\tau}_1)p_1$. Since $T_1 T_2 = 1$ if and only if $\bar{T}_1 = T_2$, the condition reduces to $\bar{\tau}_1 = \tau_2$, which is the form in which the theorem was stated by Lerner.

By way of contrast, Keynes's equivalence theorem states that the combination (T_1, T_2) of export and import tax factors—*both* levied on the *foreign* price—is equivalent to the combination $(1, 1)$ if and only if $T_1 = T_2$.

In Lerner's case it is the *distortion* in relative prices of the tradables that is preserved, while in Keynes's case it is the *lack* of such distortion which is preserved. Both are special cases of the formula

$$(T_1, T_2) \sim (T_1', T_2') \iff (\exists \lambda > 0) \colon T_i' = \lambda T_i \quad (i = 1, 2), \quad \text{i.e.,} \quad T_1'/T_1 = T_2'/T_2.$$

This formula obviously extends to any number of traded goods. See also the interesting discussion in Kaempfer and Tower (1982).

Note that in Keynes's own application, either a devaluation or a tariff-and-bounty scheme *removes* (or at least *lessens*) the distortion in relative *wages*.

4. An exception is the case considered in section 2 in which there are two factors and three goods, so that the country's production-possibility frontier is a ruled surface, and the price of the nontradable is determined uniquely by the domestic prices of the tradables. While temporary changes in relative prices might be required in this case to facilitate the necessary resource reallocation, the equilibrium relative prices would remain unchanged. With at least three factors of production, however, a permanent change in relative prices would be required (to the extent, of course, that the capital outflow remains "permanent").