Intra-Industry Trade, Factor Proportions, and Aggregation*

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Introduction

In the literature on the empirical explanation of trade flows, it appears to have become a universally accepted dictum that the existence of so-called "intra-industry trade," i.e., trade between countries of products within the same industrial category, is *prima facie* evidence of the existence of economies of scale or monopolistic competition, or both, and in particular is incompatible with the "factor-proportions theory," by which is meant the theory developed by Heckscher and Ohlin according to which trade flows among countries are explained by differences in their relative factor endowments, it being assumed that production functions among countries are the same.

In this paper I shall argue, both on empirical and theoretical grounds, that there is nothing in the empirical observations of international trade statistics that cannot be explained perfectly easily by the "Heckscher-Ohlin theory" as formulated by Lerner (1952) and Samuelson (1953). (I shall refer to this as the HOLS model.) I do not wish to contend that alternatives to this theory are not worth exploring and developing; but if so, the reason for doing so should not be that the more conventional theory is unable to explain the observed facts. In the last analysis, that theory should be accepted which is able to explain them best; but before this can be done, a clearer understanding is needed of the extent to which the conventional theory is or is not able to do so. The belief that it is not able to do so is, in my opinion, based on a misunderstanding of the nature of that theory and a lack of appreciation of its rich potentialities.

In Section 1, I examine some empirical evidence which points to the conclusion that if commodity classification systems were to carry the disaggregation process sufficiently far, two-way trade could be expected to disappear from international trade statistics. In Section 2, I consider the pure theory of international trade in

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the textbook case of two countries, two commodities, and two factors, and come to the conclusion that if (as is alleged to be the case in the literature on intra-industry trade) production processes are very similar as between the two commodities and factor endowments are very similar as between the two countries, it is possible for each country to export up to 50 percent of the output of its export good, the percentage *increasing* as the production processes become more similar. Finally, in Section 3, I consider the frequently repeated assertion that, when there are three or more countries, one would under the "factor-proportions theory" expect to find less trade between countries with similar than with dissimilar endowments; I find instead that, assuming identical and homothetic preferences within and across countries, if the two countries with similar endowments export goods with similar production processes, they may be expected to trade more with each other than with the third country, this effect being further accentuated when (1) there is a strong world preference for the goods which they export, or (2) when their absolute endowment levels are greater than those of the third country, or both.

1 Some empirical evidence

Grubel and Lloyd (1975, pp. 86–9) distinguish between two types of intra-industry trade: trade in goods which are close substitutes in use but produced by different production processes (e.g., wood and metal furniture); and trade in goods which are not close substitutes in use but are produced by very similar or identical production processes (e.g., steel bars and sheets). They also consider a third type of goods, such as automobiles, which are close substitutes and are also produced by similar processes. They allow that the first type of intra-industry trade is compatible with the HOLS model, and then introduce a second issue, (p. 87): "For this group of goods the intra-industry trade phenomenon is simply the result of statistical aggregation." It is implied but not stated explicitly that intra-industry trade in the other two cases is not a result of statistical aggregation.

Let me now take up this second issue. If it is granted in all the above cases that the goods being traded are physically distinct (and indeed, Grubel and Lloyd themselves emphasize that this is so, pp. 125–6), then a fine enough classification system will recognize the distinction. The fact that trade statistics using existing, cruder classification systems exhibit two-way trade within categories is as much a result of statistical aggregation in the case of one kind of good as in the other; for why should our description (as opposed to explanation) of an empirical phenomenon depend on our theories about it? The only question, then, is how much disaggregation would be necessary in order for two-way trade to disappear from international trade statistics. In this section I try to form a rough estimate, based on data presented by Grubel & Lloyd (1975) and Gray (1978); and on a new set of data for Sweden.

1.1 The Grubel-Lloyd data

Grubel and Lloyd (1975, p. 50) have presented a table showing the percentage of Australian intra-industry trade in its total trade with various countries and country groups, at various SITC digit levels, using a 7-digit refinement of the 5-digit Stan-

Table 1a

Australian Intra-Industry Trade, 1968–69
as a Proportion of Total Trade, with Major Trading Partners

Country or Country Group	SITC level of disaggregation				
	1	2	3	5	7
United States	0.397	0.250	0.146	0.100	0.032
United Kingdom	0.315	0.125	0.077	0.042	0.013
Japan	0.180	0.106	0.048	0.022	0.002
European Community	0.153	0.063	0.049	0.032	0.010
Canada	0.386	0.275	0.176	0.072	0.008
New Zealand	0.798	0.475	0.305	0.195	0.044
Hong Kong	0.505	0.173	0.133	0.065	0.014
India	0.495	0.095	0.055	0.018	0.002
South Africa	0.654	0.303	0.163	0.073	0.007
Southeast Asia	0.174	0.098	0.087	0.044	0.015
Rest of the world	0.520	0.270	0.189	0.106	0.031
All countries	0.429	0.259	0.202	0.149	0.062

dard International Trade classification. For Australia's leading trading partners in 1968–69 (the European Community consisted then of West Germany, France, Italy, Belgium and the Netherlands) their figures for the proportion of intra-industry trade in total trade (adjusted for global bilateral trade imbalances) are reproduced in Table 1a, ranked in order of decreasing value of exports to Australia in 1968–69 (except for the last two groups).

These figures show a very clear downward trend. It is tempting, therefore, to extrapolate. We do not, of course, possess a theory of how compilers of commodity classification systems decide to associate the number of digits in the classification code with the degree of fineness of the disaggregation. The best we can do at this stage is to fit a reasonably-shaped curve. Accordingly, I have chosen to fit the reciprocal power function[1]

$$P = 1 - aS^b \tag{1.1}$$

where S is the SITC level of disaggregation and P is the proportion of intra-industry trade in total trade. This function has the desirable property that at the null (0-digit) level of disaggregation, when the index has been adjusted for trade imbalances, 100% of trade is intra-industry trade. The curve (1.1) has been fitted to the data of Table 1a by the Fletcher-Powell (1963) method.[2] With two parameters and five data points, there are only three degrees of freedom; but given the limitations of data, presumably this is the best one can do. Table 1b gives the parameter estimates and

[1] I had originally chosen the form $P = a + bS^c$ for both intra- and inter-industry trade, but in the latter case this resulted in highly correlated estimates of a and b. The forms (1.1) and (1.2) were suggested by Joan Rodgers.

[2] Calculations were performed on the HP-71B handheld computer, equipped with mathematics and curve-fitting modules and additional 100K RAM.

the coefficient of determination (R^2), where $1 - R^2$ is defined as in Theil (1971, p. 164) as the ratio of the sum of squares of the residuals $P - 1 + aS^b$ to the sum of squares of the observations on the dependent variable P. The three panels of Figure 1 show the curves and the corresponding data points.

Table 1b

Australian Intra-Industry Trade, 1968–69

Parameters (a, b) of the reciprocal power function, coefficient of determination (R^2), and zero cutoff point (S^*), for the data of Table 1a.

Country or Country Group	a	b	R^2	S^*
United States	0.632151	0.226733	0.987579	7.56
United Kingdom	0.738451	0.164275	0.938998	6.33
Japan	0.832555	0.099745	0.981524	6.28
European Community	0.867818	0.072194	0.958070	7.13
Canada	0.617048	0.248759	0.998534	6.96
New Zealand	0.318901	0.581939	0.974533	7.13
Hong Kong	0.601277	0.277028	0.916313	6.27
India	0.651323	0.254798	0.806576	5.38
South Africa	0.472589	0.411547	0.939699	6.18
Southeast Asia	0.834666	0.085760	0.992714	8.23
Rest of the world	0.547025	0.308483	0.974598	7.07
All countries	0.603694	0.228339	0.990105	9.12

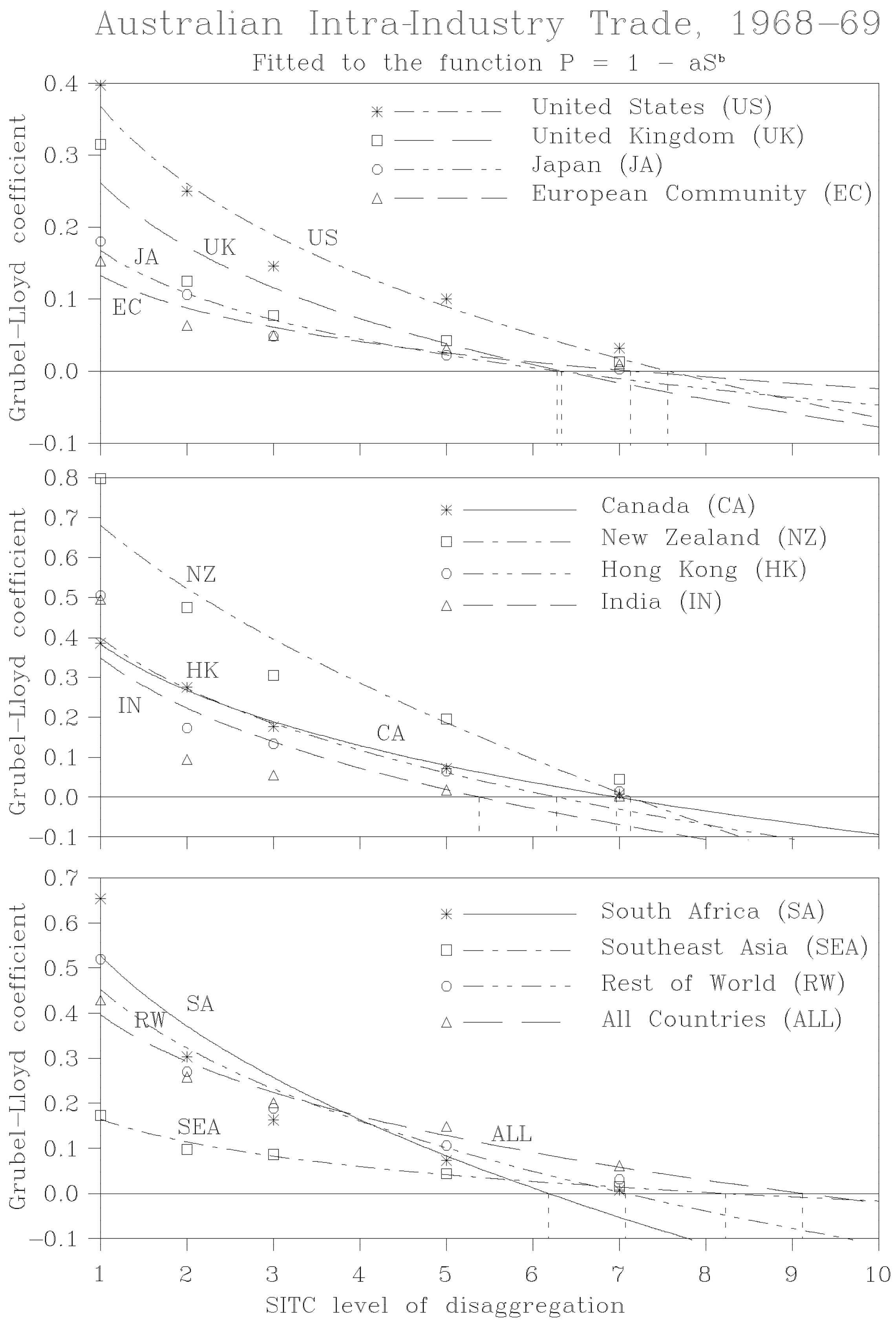
The last column of Table 1b provides the solution S^* of equation (1.1) for $P = 0$. Thus, for all of the individual countries and groups, the curves predict that intra-industry trade will cease to be observed if the SITC is refined to the ninth level of disaggregation; and for all countries together, to the tenth level.

Taken as a whole, these results support the hypothesis that intra-industry trade is a statistical phenomenon in the sense that it would cease to be observed if sufficiently disaggregated data were obtained, and that this would be achieved with a not unreasonable degree of refinement of existing classification systems.

1.2 Gray's data

The second data set I consider is taken from a study by Gray (1978) of West German and French trade with specified partners, in 1-digit SITC categories at each of the five SITC levels of aggregation.

In the case of West German trade, only for trade with Belgium and France—in SITC categories 6 (manufactured goods classified chiefly by material) and 7 (machinery and transport equipment)—are data furnished by Gray for all five levels of aggregation (cf. Gray, 1978, p. 105). These are therefore the only series from his table that I have attempted to analyze. The measure of intra-industry trade used



Australian Intra-Industry Trade, 1968–69
Fitted to the function P = 1 − aS^b
Grubel–Lloyd coefficient
United States (US)
United Kingdom (UK)
Japan (JA)
European Community (EC)
US
UK
JA
EC
Canada (CA)
New Zealand (NZ)
Hong Kong (HK)
India (IN)
NZ
HK
IN
CA
South Africa (SA)
Southeast Asia (SEA)
Rest of World (RW)
All Countries (ALL)
SA
RW
SEA
ALL
SITC level of disaggregation

Figure 1

by Gray is the Balassa index (cf. Balassa, 1966), defined as

$$B = \frac{1}{n}\sum_{i=1}^{n}\frac{|X_i - M_i|}{X_i + M_i}, \tag{1.2}$$

where X_i andM_i are the values of exports and imports in category i, and n is the number of categories at the given level of aggregation. The index actually measures *inter-* rather than *intra*-industry trade; it has the value zero when all trade is intra-industry trade, and unity when none of it is.

Table 2a

West German Inter-Industry Trade, 1973

Balassa coefficient for trade with Belgium & France in two SITC categories, with number of categories at each SITC level

Country	SITC level of disaggregation				
	1	2	3	4	5
1. SITC 6—manufactured goods classified chiefly by material					
Number of categories	*1*	*8*	*50*	*53*	*26*
Belgium	0.31	0.33	0.40	0.49	0.58
France	0.09	0.17	0.32	0.42	0.48
2. SITC 7—machinery and transport equipment					
Number of categories	*1*	*3*	*18*	*46*	*52*
Belgium	0.38	0.41	0.53	0.57	0.61
France	0.18	0.27	0.36	0.50	0.52

Table 2b

West German Inter-Industry Trade, 1973

Parameters (a,b) of the power function, coefficient of determination (R^2), and unit cutoff point (S^*), for the data of Table 2a.

Country	a	b	R^2	S^*
1. SITC 6—manufactured goods classified chiefly by material				
Belgium	0.265177	0.450626	0.994306	19.02
France	0.096109	1.025047	0.996071	9.83
2. SITC 7—machinery and transport equipment				
Belgium	0.358752	0.328844	0.998182	22.59
France	0.172730	0.707112	0.996856	11.98

As explained by Grubel & Lloyd (1975, p. 26), the Balassa measure has the disadvantage of being an unweighted average of the ratios $|X_i - M_i|/(X_i + M_i)$; if instead we weight industry i by its importance,$(X_i+M_i)/\sum_{j=1}^{n}(X_j+M_j)$, we obtain

$$C = \frac{\sum_{i=1}^{n}|X_i - M_i|}{\sum_{i=1}^{n}(X_i + M_i)}. \tag{1.3}$$

The Grubel-Lloyd measure P corresponds to $1-C$. The latter has the advantageous property of being necessarily nonincreasing as the degree of disaggregation becomes finer (cf. Grubel & Lloyd, 1975, p. 23). This monotonicity property is lost with the unweighted Balassa measure. Balassa himself has since (1986) adopted the Grubel-Lloyd measure.

The Balassa coefficient (1.2) has been fitted to the SITC data on inter-industry trade by the power function

(1.4) $$B = aS^b.$$

This has the property that inter-industry trade, corrected for trade imbalance, has the value 0 at the null ($S = 0$) level of disaggregation.

Table 2a reproduces the relevant figures from Gray's table for West German inter-industry trade for the Balassa coefficient (1.2) and for the number of aggregated subcategories at each SITC level (the parameter n of (1.2)). Table 2b provides the estimates of the parameters a and b of (1.4), the coefficient of determination (R^2), and the unit cutoff point (S^*), i.e., the value of S that solves $B = 1$ in (1.4). The fitted curves are shown in Figure 2.

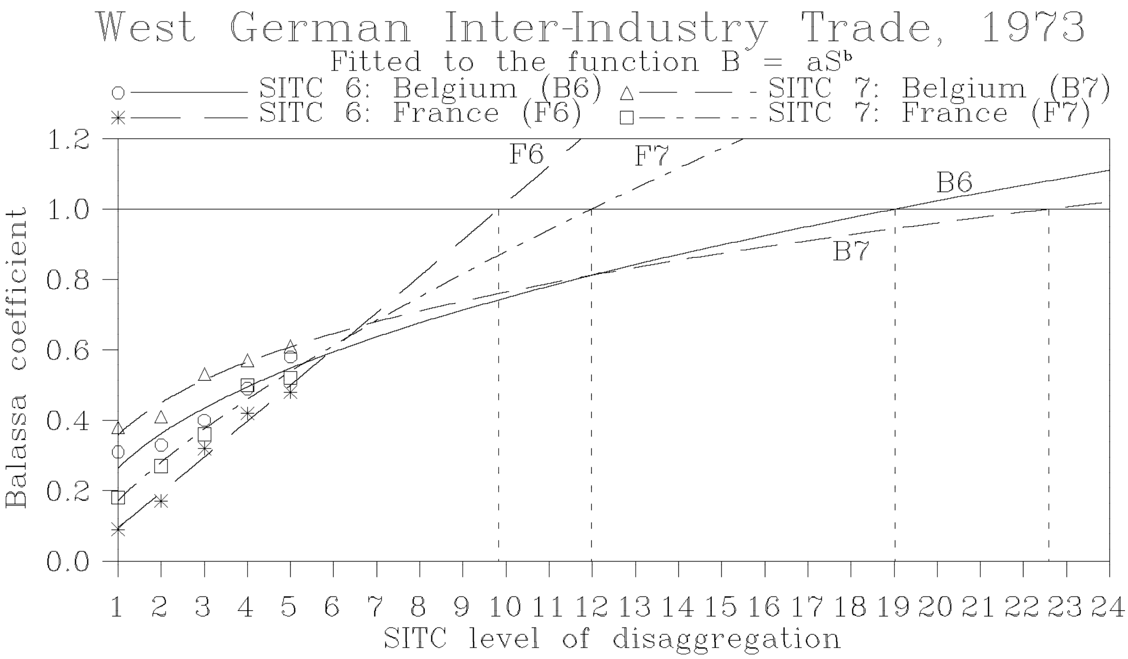


Figure 2

The curves for West German inter-industry trade with Belgium and France lend some support to the hypothesis that intra-industry trade in basic materials is more of a "statistical phenomenon" than is intra-industry trade in finished products, since they predict that intra-industry trade in the former will cease to be observed at the twentieth and tenth levels of disaggregation respectively, whereas it will continue to be observed in the latter up to the twenty-third and twelfth levels respectively. But this conclusion depends upon a willingness to extrapolate curves far from the observations used to fit them.

In the case of French inter-industry trade, Gray (1978, p. 106) provides calculations of the Balassa coefficients for French trade with Belgium, West Germany,

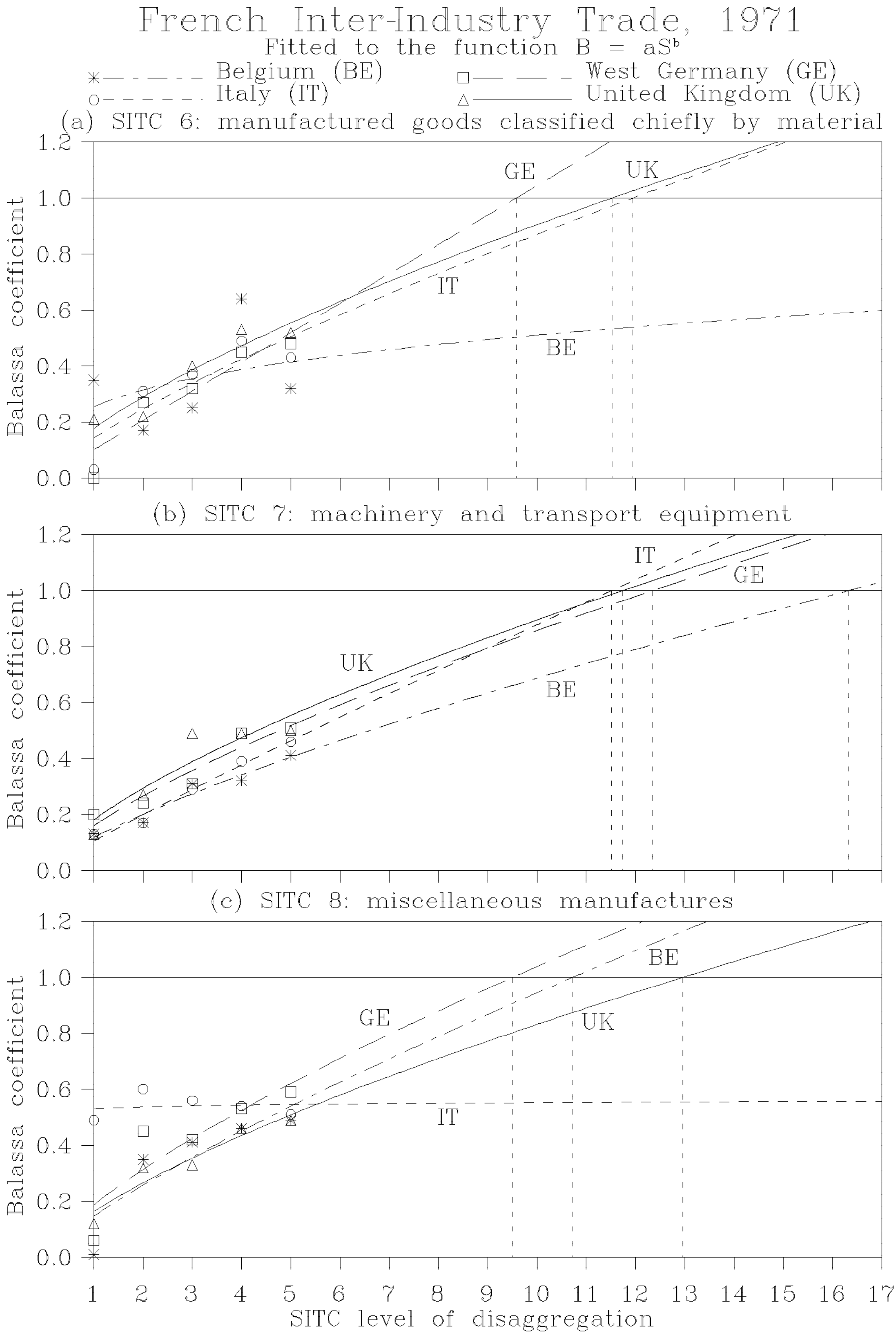

French Inter-Industry Trade, 1971
Fitted to the function B = aS^b
Belgium (BE)
West Germany (GE)
Italy (IT)
United Kingdom (UK)
(a) SITC 6: manufactured goods classified chiefly by material
(b) SITC 7: machinery and transport equipment
(c) SITC 8: miscellaneous manufactures
Balassa coefficient
SITC level of disaggregation
GE
UK
IT
BE

Figure 3

Italy, and the United Kingdom in SITC categories 6, 7, and 8. These figures are reproduced in Table 3a. Table 3b furnishes the estimates of the parameters a and b of (1.4), the coefficient of determination (R^2), and the unit cutoff point (S^*). The fitted curves are displayed in Figure 3.

Table 3a

French Inter-Industry Trade, 1971

Balassa coefficient for trade with selected partners and SITC categories, with number of categories at each SITC level

Country	SITC level of disaggregation				
	1	2	3	4	5
1. SITC 6—manufactured goods classified chiefly by material					
Number of categories	*2*	*7*	*32*	*129*	*21*
Belgium	0.35	0.17	0.25	0.64	0.32
West Germany	0.00	0.27	0.32	0.45	0.48
Italy	0.03	0.31	0.37	0.49	0.43
United Kingdom	0.21	0.22	0.40	0.53	0.52
2. SITC 7—machinery and transport equipment					
Number of categories	*1*	*3*	*18*	*73*	*75*
Belgium	0.13	0.17	0.31	0.32	0.41
West Germany	0.20	0.24	0.31	0.49	0.51
Italy	0.13	0.17	0.29	0.39	0.46
United Kingdom	0.13	0.27	0.49	0.49	0.50
3. SITC 8—miscellaneous manufactures					
Number of categories	*1*	*7*	*18*	*57*	*21*
Belgium	0.01	0.35	0.41	0.46	0.49
West Germany	0.06	0.45	0.42	0.53	0.59
Italy	0.49	0.60	0.56	0.54	0.51
United Kingdom	0.12	0.32	0.33	0.46	0.49

It is apparent from Table 3b and the graphs of Figure 3 that, except for two anomalous cases, the fitted curves predict the disappearance of intra-industry trade at reasonable levels of disaggregation. The two exceptional cases are those of trade with Belgium in SITC 6 (for which the curve fit is very poor—0.84 being a very low coefficient of determination given that there are only three degrees of freedom) and trade with Italy in SITC 8 (for which the Balassa coefficient is, beyond SITC level 2, monotone decreasing in the SITC level of disaggregation). In the cases of SITC 6 and 8 it is apparent from Table 3a that there are far fewer subcategories in the Balassa coefficient at the 5-digit than at the 4-digit level, indicating that numerous probably redundant 5-digit categories (categories for which the fifth digit is zero and no subdivision actually takes place) have been excluded from the calculations. This adds to the non-monotonicity of the Balassa coefficient itself. Taking account of these considerations, it seems a fair inference that one could expect intra-industry trade to disappear at or slightly above the sixteenth level of disaggregation.

Table 3b

French Inter-Industry Trade, 1971

Parameters (a,b) of the power function, coefficient of determination (R^2), and unit cutoff point (S^*), for the data of Table 3a

Country	a	b	R^2	S^*
1. SITC 6—manufactured goods classifed chiefly by material				
Belgium	0.255230	0.301232	0.843465	93.07
West Germany	0.102449	1.008442	0.971280	9.58
Italy	0.143065	0.784057	0.957771	11.94
United Kingdom	0.176951	0.708637	0.986922	11.52
2. SITC 7—machinery and transport equipment				
Belgium	0.118237	0.764512	0.993027	16.33
West Germany	0.160754	0.727198	0.989941	12.35
Italy	0.105163	0.921781	0.996672	11.51
United Kingdom	0.181954	0.691905	0.979750	11.74
3. SITC 8—miscellaneous manufactures				
Belgium	0.147025	0.807923	0.955960	10.73
West Germany	0.187742	0.742835	0.964668	9.50
Italy	0.531450	0.016625	0.995044	3.2+E16
United Kingdom	0.162729	0.708720	0.990619	12.96

1.3 The Swedish data

Unpublished monthly data on Swedish imports and exports have been supplied to the author by the Statistiska centralbyrån, Stockholm, covering the period 1977–1984.[3] These data are classified according to the Svensk standard för näringsgrensindelning (SNI), which is a 6-digit refinement of the 4-digit International Standard Industrial Classification of All Economic Activities (ISIC) constructed by the United Nations. The Grubel-Lloyd coefficients have been computed by Joan R. Rodgers, adjusted for trade imbalances by Aquino's (1978) method (cf. Chipman, 1987, p. 940), and are presented in Tables 4a and 4b for two different modes of calculation. In the first, when a k-digit category is trivially refined to a $(k+1)$-digit category by adding a zero to the code but not subdividing the category, this $(k+1)$-digit category is included in the calculations; in the second mode it is excluded.

Tables 4c and 4d furnish the coefficients of the fitted curves, the coefficient of determination, and the unit cutoff point, corresponding to Tables 4a and 4b respectively. The fitted curves for the pooled data in these two cases are displayed in Figure 4. As the tables and figure indicate, the two modes of calculation provide

[3]I wish to thank Professor Sten Johansson, General Director of the Statistiska centralbyrån (SCB), Mr. Gunnar Stolpe, Head of the Foreign Trade and Prices Division of the SCB, Dr. Edward Palmer and Mr. Randall Bowie of the Konjunkturinstitutet (National Institute of Economic Research), and Ms. Anna Odhner, of the Sveriges Riksbank, for all their efforts and cooperation in helping me acquire this data set at an affordable cost. These data have been further analyzed by Rodgers (1987, 1988).

Table 4a

Swedish Intra-Industry Trade 1977–1984

Grubel-Lloyd coefficients for SNI categories (with repetition of trivial subcategories)

Year	SNI level of disaggregation					
	1	2	3	4	5	6
1977	0.9108	0.6459	0.6300	0.5970	0.5911	0.5865
1978	0.8570	0.6344	0.6198	0.5895	0.5837	0.5761
1979	0.9037	0.6406	0.6238	0.5901	0.5825	0.5707
1980	0.8717	0.6511	0.6337	0.6020	0.5946	0.5821
1981	0.8401	0.6338	0.6222	0.5873	0.5840	0.5752
1982	0.8658	0.6558	0.6405	0.5967	0.5924	0.5855
1983	0.8297	0.6699	0.6598	0.6184	0.6154	0.6085
1984	0.8185	0.6819	0.6723	0.6307	0.6273	0.6199

Table 4b

Swedish Intra-Industry Trade 1977–1984

Grubel-Lloyd coefficients for SNI categories (without repetition of trivial subcategories)

Year	SNI level of disaggregation					
	1	2	3	4	5	6
1977	0.9108	0.6459	0.6604	0.6488	0.6174	0.4615
1978	0.8570	0.6344	0.6491	0.6261	0.5929	0.4425
1979	0.9037	0.6406	0.6536	0.6405	0.6052	0.4420
1980	0.8717	0.6511	0.6826	0.6600	0.6205	0.4606
1981	0.8401	0.6338	0.6728	0.6405	0.6047	0.4633
1982	0.8658	0.6558	0.6862	0.6437	0.6066	0.4727
1983	0.8297	0.6699	0.7059	0.6480	0.6159	0.4746
1984	0.8185	0.6819	0.7150	0.6543	0.6136	0.4759

Table 4c

Swedish Intra-Industry Trade 1977–1984

Parameters (a,b) of the power function, coefficient of determination (R^2), and zero cutoff point (S^*) for data of Table 4a.

Year	a	b	R^2	S^*
1977	0.195211	0.475726	0.991660	31.00
1978	0.224189	0.400879	0.994390	41.68
1979	0.199229	0.478432	0.991958	29.14
1980	0.208818	0.429991	0.994833	38.19
1981	0.231913	0.379543	0.995366	47.01
1982	0.209289	0.427409	0.995397	38.84
1983	0.221710	0.353183	0.997731	71.18
1984	0.221639	0.332268	0.998537	93.19
Pooled	0.213379	0.411247	0.994496	42.78

Table 4d

Swedish Intra-Industry Trade 1977–1984

Parameters (a,b) of the power function, coefficient of determination (R^2), and zero cutoff point (S^*) for data of Table 4b.

Year	a	b	R^2	S^*
1977	0.163248	0.619753	0.991455	18.59
1978	0.191203	0.547961	0.993529	20.48
1979	0.166576	0.626580	0.991254	17.47
1980	0.171347	0.579884	0.993090	20.95
1981	0.197004	0.504332	0.993971	25.06
1982	0.175331	0.568340	0.994906	21.40
1983	0.182338	0.530882	0.995756	24.68
1984	0.180543	0.532134	0.996123	24.95
Pooled	0.178127	0.564111	0.993473	21.29

a striking difference. The very high cutoff points in the first calculation can be interpreted by saying that if disaggregation merely consists in adding zeros to the code and not actually subdividing the categories, it may take a long time or even forever to eliminate statistical observations of two-way trade from international trade statistics.

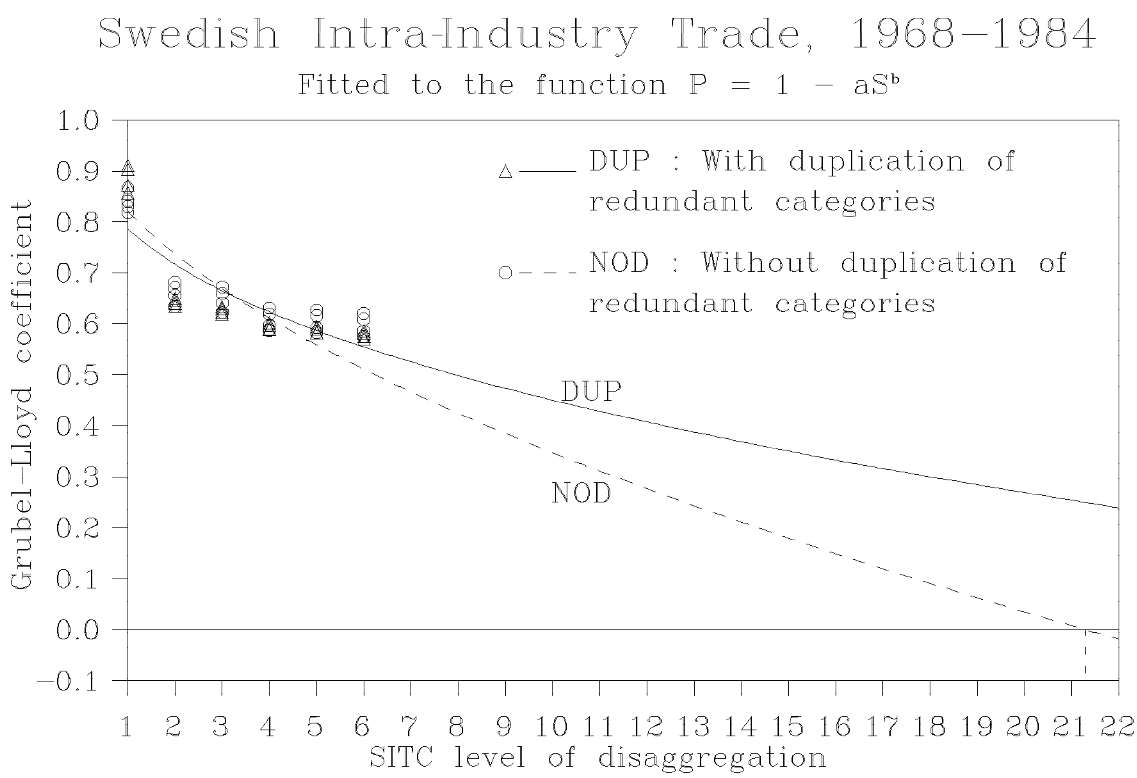


Figure 4

2 The two-commodity, two-factor, two-country case

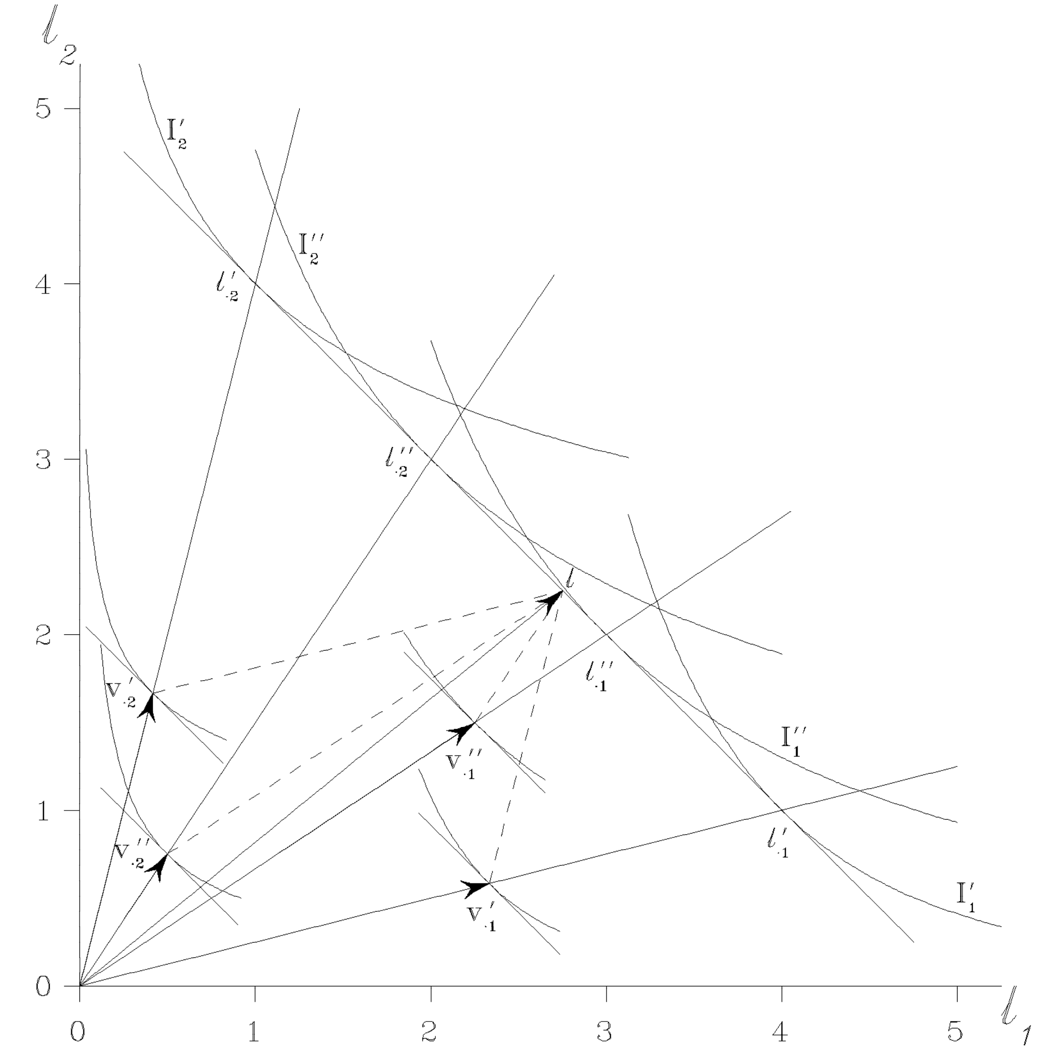
The thesis was presented by Grubel and Lloyd (1975, pp. 88–91) that (1) a preponderant amount of intra-industry trade takes place in industries within which production functions are very similar to one another, between countries with very similar factor endowments, and (2) this fact is inconsistent with the Heckscher-Ohlin theory with its assumptions of perfect competition and constant returns to scale. In this section I take issue with the second of these propositions, and show that this type of intra-industry trade is readily explained in terms of the HOLS model.

In their words (Grubel & Lloyd, 1975, pp. 88–9):

> ...in certain industries, developed countries tend to produce large numbers of substitute products with input requirements ...so similar that they may be considered identical. The Heckscher-Ohlin model also assumes the identity of production functions across countries. ...therefore [*sic*] the constant rates of transformation between these products and their relative prices must be the same across countries. As a result the exchange of these commodities with identical input requirements for each other is not profitable, because profits arise from exploitation of differences in relative prices among countries. Yet we observe the exchange of such products. The inconsistency between the theory and reality can be explained by relaxing either the assumption that the production functions are identical across countries or the assumption that there are no economies of scale.

The assumption that the rates of transformation will be the same across countries requires the additional assumption of identical factor endowment ratios; however, this seems to be implicitly assumed, since the authors subsequently state (p. 92) that "countries trading in these products have similar endowments with human, knowledge, and real capital relative to labour and land."

In terms of the standard two-commodity, two-factor, two-country model, the case discussed by Grubel and Lloyd may easily be analyzed in terms of the well-known "Lerner diagram" (cf. Lerner, 1952; Chipman, 1966). In Figure 5 a case is shown in which the production isoquants (which are the same for the two countries) are extremely close and the diversification cone very narrow. It is assumed that commodity i uses factor i relatively intensively ($i = 1, 2$). Then country 1 will specialize in the production of commodity 1, and country 2 will specialize in the production of commodity 2; both countries will be on the verge of diversifying. Now suppose the two production functions are completely symmetric to each other in their arguments, and suppose further that consumers in both countries have identical and homothetic, and also symmetric, preferences. Then it is clear that in world equilibrium the prices of the two commodities will be the same. To specialize the assumptions still further (for simplicity), suppose consumers have Mill-Cobb-Douglas preferences generated by a utility function



A country with resource endowments $l = (l_1, l_2)$ has technology characterized by isoquants I'_1, I'_2 in situation 1 and I''_1, I''_2 in situation 2. In situation 1, the endowment vector l is allocated between industries according to $v'_{\cdot 1} + v'_{\cdot 2} = l$, where $v'_{\cdot j} = (v'_{1j}, v'_{2j})$; the diversification cone is $l'_{\cdot 1} 0 l'_{\cdot 2}$. In situation 2, this cone shrinks to $l''_{\cdot 1} 0 l''_{\cdot 2}$ in such a way that l is close to the lower edge of the cone; the endowment vector l is now allocated between the two industries according to $v''_{\cdot 1} + v''_{\cdot 2} = l$. A much larger fraction of resources is devoted to industry 1 (the export industry) in the second situation than in the first. In the limit, as $l''_{\cdot 1}$ approaches l, all the country's resources are allocated to industry 1, and since (with unit prices) half of the export good is consumed, the other half is exported.

Figure 5

$$U(x_1, x_2) = x_1^{\theta_1} x_2^{\theta_2} \quad (\theta_j > 0, \theta_1 + \theta_2 = 1) \tag{2.1}$$

where (by symmetry) $\theta_1 = \theta_2 = \frac{1}{2}$, x_j being the consumption of commodity j. Then in each country, with prices of the two commodities being equal to each other, one-half of the output of the good it specializes in is consumed; hence one-half is exported. This is true no matter how wide or narrow the diversification cone.

Now suppose we make the following construction. Let production functions be quite disparate (but still symmetric to each other), as indicated by the outside isoquants in Figure 5, but let the factor endowments remain close as before. Then the respective countries' resource allocations will be very close, and there will be very little trade. Now let the isoquants become closer and closer, until they reach the narrow diversification cone, that is, until the diversification cone is bounded by the given endowment vectors. Then each country will export a larger and larger proportion of the good in which it has a comparative advantage, up to the limit of one-half of its export good.

Let us now make this argument more precise. Using the model of Chipman (1985), suppose that there are two Cobb-Douglas production functions (identical between countries)

$$f_j(v_{1j}, v_{2j}) = \mu_j v_{1j}^{\beta_{1j}} v_{2j}^{\beta_{2j}} \quad (j = 1, 2), \tag{2.2}$$

where $\beta_{ij} > 0$ and $\beta_{1j} + \beta_{2j} = 1$ and v_{ij} denotes the allocation of factor i to industry j (a country superscript is omitted for notational convenience), and let the resource-allocation constraints (in each country)

$$v_{11} + v_{12} = l_1^k, \quad v_{21} + v_{22} = l_2^k \tag{2.3}$$

be satisfied, where l_i^k denotes the endowment of country k in factor i $(i, k = 1, 2)$. Let preferences be given by (2.1) for both countries. World equilibrium may then be computed as follows. First, denoting the rental of factor i by w_i, and assuming diversification of production in both countries, factor rentals are solved for by setting prices equal to minimum unit costs. The cost functions dual to (2.2) are

$$g_j(w_1, w_2) = \nu_j w_1^{\beta_{1j}} w_2^{\beta_{2j}} \text{ where } \nu_j = \frac{1}{f_j(\beta_{1j}, \beta_{2j})} \quad (j = 1, 2) \tag{2.4}$$

(cf. Chipman, 1985, p. 293); equating each of these to p_j we obtain

$$\begin{bmatrix} \log w_1 \\ \log w_2 \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{bmatrix}^{-1} \begin{bmatrix} \log(\frac{p_1}{\nu_1}) \\ \log(\frac{p_2}{\nu_2}) \end{bmatrix}. \tag{2.5}$$

The cost-minimizing factor-output matrix is

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix}^{-1} \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix}. \tag{2.6}$$

Denoting its inverse and that of $[\beta_{ij}]$ by

$$B^{-1} = \begin{bmatrix} b^{11} & b^{12} \\ b^{21} & b^{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}^{-1}, \quad \begin{bmatrix} \beta^{11} & \beta^{12} \\ \beta^{21} & \beta^{22} \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}^{-1} \tag{2.7}$$

respectively, we find from (2.4), (2.5), and (2.7) that

$$w_i = \prod_{h=1}^{2} (\mu_h \beta_{1h}^{\beta_{1h}} \beta_{2h}^{\beta_{2h}} p_h)^{\beta^{hi}}, \tag{2.8}$$

hence the elements of the inverse factor-output matrix are

$$b^{ji} = \frac{w_i \beta^{ji}}{p_j} = \frac{[\prod_{h=1}^{2}(\mu_h \beta_{1h}^{\beta_{1h}} \beta_{2h}^{\beta_{2h}} p_h)^{\beta^{hi}}]\beta^{ji}}{p_j}. \tag{2.9}$$

Now, country k's Rybczynski function for commodity j is, in the region of diversification, given by

$$y_j^k = \hat{y}_j(p_1, p_2, l_1^k, l_2^k) = b^{j1}(p_1, p_2)l_1^k + b^{j2}(p_1, p_2)l_2^k \tag{2.10}$$

(as is seen by substituting $v_{ij} = b_{ij}y_j$ in (2.3) and inverting), where the functions $b^{ji}(p_1, p_2)$ are given by (2.9). (The dependence of the b^{ji} on the parameters μ_j, β_{ij} is not explicitly indicated but should be kept in mind). Let world outputs, world consumption, and world factor endowments be denoted

$$y_j = \sum_{k=1}^{2} y_j^k, \; x_j = \sum_{k=1}^{2} x_j^k, \; l_i = \sum_{k=1}^{2} l_i^k. \tag{2.11}$$

Given that both countries are assumed to be diversifying, and that the production functions (2.2) satisfy the hypothesis of absence of factor-intensity reversal, we obtain from (2.10) and (2.11) the world Rybczynski functions

$$\hat{y}_j(p_1, p_2, l_1, l_2) = b^{j1}(p_1, p_2)l_1 + b^{j2}(p_1, p_2)l_2. \tag{2.12}$$

Since preferences in both countries are assumed to be generated by the utility function (2.1), which can be aggregated, equilibrium world consumption must satisfy

$$\frac{x_2}{x_1} = \frac{\theta_2}{\theta_1}\frac{p_1}{p_2}. \tag{2.13}$$

Setting $x_j = y_j$ for world equilibrium, and taking account of the homogeneity of degree 0 of the funtions b^{ji}, we obtain from (2.12) and (2.14) the equation

$$\frac{b^{21}(p_1/p_2, 1)l_1 + b^{22}(p_1/p_2, 1)l_2}{b^{11}(p_1/p_2, 1)l_1 + b^{12}(p_1/p_2, 1)l_2} - \frac{\theta_2}{\theta_1}\frac{p_1}{p_2} = 0. \tag{2.14}$$

Standard methods, such as Newton's method or the secant method, may be used to solve this equation for the world price ratio p_1/p_2.

Once the equilibrium world price ratio has been obtained, each country's export-output ratio is readily computed. Assuming that commodity j uses factor j relatively intensively (i.e., that $\beta_{11} > \beta_{12}$), the output of each country's export good is computed from the Rybczynski function (2.10), for $j = k$ (say, setting $p_2 = 1$). The value of the national-product function

$$\Pi(p_1, p_2, l_1^k, l_2^k) = \sum_{j=1}^{2} p_j \hat{y}_j(p_1, p_2, l_1^k, l_2^k) \tag{2.15}$$

is then computed, and the consumption of commodity j in country k is

$$x_j^k = \frac{\theta_j \Pi(p_1, p_2, l_1^k, l_2^k)}{p_j}. \tag{2.16}$$

For each country, the export-output ratio is then

$$\frac{y_k^k - x_k^k}{y_k^k} = 1 - \theta_k\left(1 + \frac{p_j y_j^k}{p_k y_k^k}\right) \quad (j \neq k). \tag{2.17}$$

Our object is now to show that, at least under certain additional hypotheses, these export-output ratios will increase as production functions become more similar. The additional hypotheses will impose complete symmetry as between the two commodities, factors, and countries. Specifically, I shall assume that $\mu_1 = \mu_2 = 1$ and $\beta_{12} = \beta_{21}$; the latter of course implies $\beta_{11} = \beta_{22}$, providing complete symmetry as between production functions. To obtain symmetry in consumption we set $\theta_1 = \theta_2 = \frac{1}{2}$, and to have symmetry as between the countries' factor endowments we set $l_1^1 = l_2^2$ and $l_2^1 = l_1^2$, where $l_1^1 > l_2^1$ (country k is relatively well endowed in factor k). With these symmetry assumptions, the solution of (2.14) may be bypassed, since clearly $p_1 = p_2$. Without loss of generality it will be assumed that $p_1 = p_2 = 1$, and for convenience we shall denote $\beta = \beta_{12} = \beta_{21}$ and $1 - \beta = \beta_{11} = \beta_{22}$. Since we assume $\beta_{11} > \beta_{21}$, this is equivalent to $\beta < \frac{1}{2}$.

With these symmetry assumptions, (2.8) reduces to

$$w_1 = w_2 = (1-\beta)^{1-\beta}\beta^{\beta}. \tag{2.18}$$

Thus, it is interesting to note that for $\beta < \frac{1}{2}$ both factor rentals are decreasing functions of β; that is, factor rentals decrease as production functions become more similar. The inverse factor-output matrix now becomes

$$B^{-1} = \frac{(1-\beta)^{1-\beta}\beta^{\beta}}{1-2\beta}\begin{bmatrix} 1-\beta & -\beta \\ -\beta & 1-\beta \end{bmatrix}, \tag{2.19}$$

and we find that

$$\frac{dB^{-1}}{d\beta} = \frac{(1-\beta)^{1-\beta}\beta^{\beta}}{1-2\beta}\begin{bmatrix} (1-\beta)\psi(\beta)-1 & -\beta\psi(\beta)+1 \\ -\beta\psi(\beta)+1 & (1-\beta)\psi(\beta)-1 \end{bmatrix}, \tag{2.20}$$

where

$$\psi(\beta) = \log\beta - \log(1-\beta) + 2(1-2\beta)^{-1}. \tag{2.21}$$

The Rybczynski functions (2.10) reduce to

$$y_j^k = \frac{(1-\beta)^{1-\beta}\beta^{\beta}}{1-2\beta}\left[l_j^k - \beta(l_1^k + l_2^k)\right] \; (j = 1, 2). \tag{2.22}$$

From (2.22) and (2.17) (with $\theta = \frac{1}{2}$ and $p_1 = p_2 = 1$), it follows that country k's export-output ratio is equal to

$$\frac{y_k^k - x_k^k}{y_k^k} = \frac{l_k^k - l_j^k}{2\left[l_k^k - \beta(l_k^k + l_j^k)\right]} = \frac{L-1}{2[L - \beta(L+1)]} \; (j \neq k), \tag{2.23}$$

(where $L = l_k^k / l_j^k$, $j \neq k$) which is an increasing function of β since $l_k^k > l_j^k$ for $j \neq k$. This proves the

Theorem. *If the symmetry conditions $\mu_1 = \mu_2 = 1$, $\beta_{12} = \beta_{21} = \beta$, $\theta_1 = \theta_2 = \frac{1}{2}$, $l_1^1 = l_2^2$, $l_2^1 = l_1^2$ hold in the model (2.1), (2.2), (2.3), if commodity j uses factor j relatively intensively ($\beta < \frac{1}{2}$) and country k is relatively will endowed in factor k ($l_k^k > l_k^j$ for $j \neq k$), and if both commodities continue to be produced in both countries, then as the production functions become more similar (i.e., β increases), each country's exports increase as a proportion of the output of the export good.*

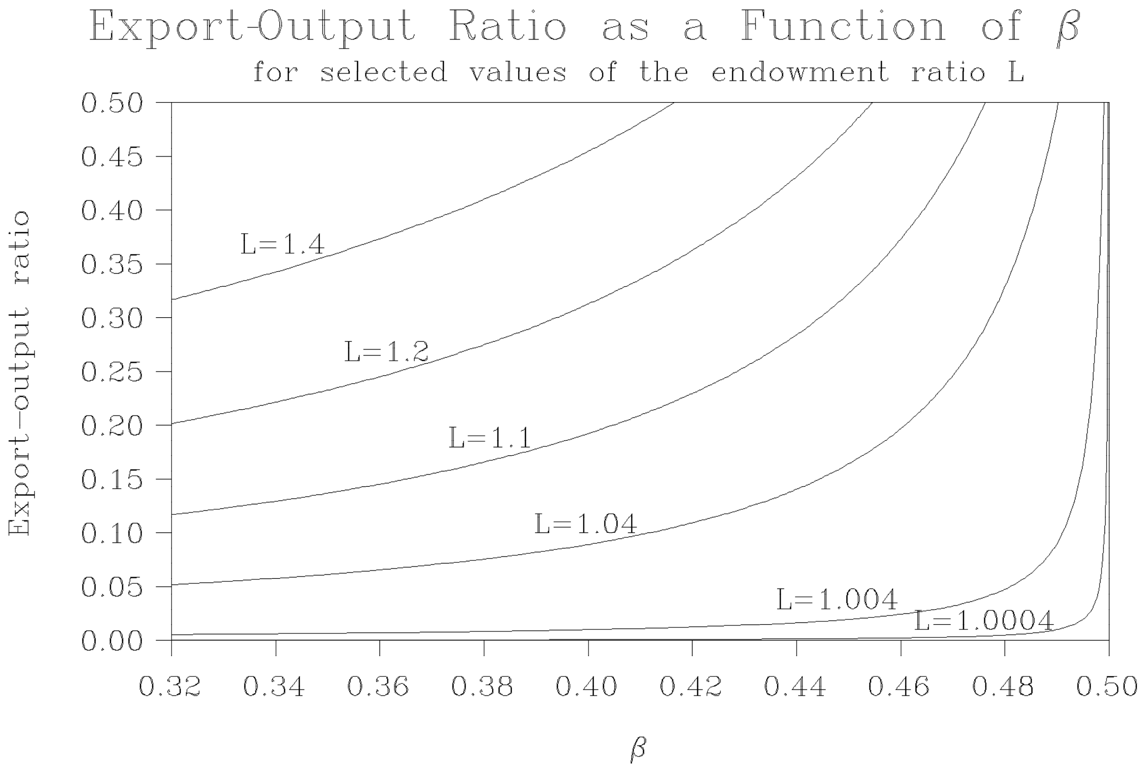


Figure 6

Figure 6 depicts the export-output ratio (2.23) as a function of β ($= \beta_{12} = \beta_{21}$) for selected values of the endowment ratio $L = l_k^k / l_j^k$ ($j \neq k$). This diagram brings out an interesting aspect of the situation: not only does the proportion exported increase as the production functions become closer, but when production functions are already very close, the proportion exported increases very dramatically when the distance between the production functions decreases only slightly. It is stated by Grubel and Lloyd (1975, p. 91) that "it is highly unlikely that the minor differences in input requirements between goods within . . . industries could lead to the large observed trade if production were subject to constant returns to scale." But, as the above theorem and the figure show, *the more minor* the difference in input requirements, *the greater* is the percentage of output exported, and *the more sensitive* is the percentage exported to narrowing of the differences between production functions.

What happens in the limit as both the production functions for the two commodities and the factor endowments of the two countries become identical? In particular, what happens when $\beta_{12} = \beta_{21} = .5$ and $l_1^1 = l_2^1 = 1$? It is clear that the curves of Figure 6 approach a discontinuous correspondence in the shape of a backward L. The diversification cone shrinks to a ray on which lie both countries' endowment vectors; the countries' production-possibility frontiers become parallel straight lines. World trade becomes indeterminate.

The issue of indeterminacy was raised by Corden (1978, p. 4):

> Suppose that all goods in an "industry" used factors in identical proportions—that is, had identical factor intensities. In this case factor proportions theory could not explain intra-industry trade. For this purpose an industry must be defined in terms of the statistical classification used the intra-industry calculations. Usually the SITC three-digit classification is used though, with even narrower definitions, substantial intra-industry trade apparently remains. It follows that, in such a case, other theories must explain intra-industry trade.

One could argue by analogy with the indeterminacy in the size distibution of firms in an industry under constant returns and competitive equilibrium, that either the constant-returns or the competitive assumption (or both) must be relaxed in order to yield a determinate theory.

But the indeterminateness involved in the present case results from the totally improbable assumption that production functions are exactly the same; let them differ in only the slightest degree, as in the above example, and the determinacy is restored.

The indeterminacy of this limiting case points to what is a fallacy in Grubel and Lloyd's argument when they state (p. 89) that "exchange of these commodities for each other is not profitable, because profits arise from the exploitation of differences in relative prices among countries." It is a confusion between conditions for existence and conditions for uniqueness of equilibrium. Zero profits, and equal prices in the same market, are part of the defining conditions of competitive equilibrium in general and the HOLS model in particular. A competitive equilibrium certainly exists in this limiting case, but it is a neutral equilibrium, with each country's exports indeterminate—anywhere between zero and fifty percent of either good may be exported. Far from being inconsistent with the observation that there is a large haphazard amount of trade between similar countries, the model is entirely in conformity with it.

3 The three-commodity, three-factor, three-country case

The following statement by Hufbauer and Chilas (1974, p. 3) appears to express a widely-held and unchallenged point of view:

> Neoclassical trade theory once predicted that trade would wither between similar nations. After all, trade supposedly compensates for factor endowment disparities or differences in tastes, and if these disparities or taste differences are modest, the need for trade is small.

No references to "neoclassical trade theory" were cited to support this contention, and I doubt whether they could be easily found. Ricardo discussed the question of the *direction* of trade between two countries, but as far as I know he never concerned himself with explaining the *amount* of trade. The same appears to be true of neoclassical and, in particular, the Heckscher-Ohlin theory. But whether or not sources could be found in support of such a proposition, the result of the previous section shows it to be incorrect.

Somewhat similar contentions are frequently made in contexts that are relevant only if trade among more than two countries is involved. Fairly typical is the following argument put forward by Gray (1980, p. 447):

> ...a preponderant amount of international trade takes place among industrial nations with relatively similar resource endowments. ...this pattern of international trade cannot be readily accounted for by the orthodox, factor-proportions theory of international trade even in its multiple-factor version. The standard theory would suggest that the larger trade flows would take place among nations with markedly different factor endowments.

The only proposition known to me that relates trade patterns to relative factor endowments is the "Heckscher-Ohlin theorem," which states that if there are two countries, two factors, and two commodities produced competitively under constant returns to scale and freely traded with zero transport costs, and if (1) preferences are identical and homothetic within and between countries, (2) production functions are identical as between countries, (3) there are no factor-intensity reversals, and (4) trade is balanced, then each country will export the commodity in which its relatively abundant factor is used relatively intensively. If any of the above four conditions is removed, it is easy to construct a counterexample to the proposition; in fact, much of the literature on the "Leontief paradox" was devoted precisely to such exercises. Thus, very special assumptions are required even in the simple $2 \times 2 \times 2$ case to obtain unequivocal results; and even then, the results concern only the *direction* of trade, not the *amount* of trade.

Let us consider now a model of three countries, each producing three commodities with three factors. Countries 1 and 2 will be "similar" in their factor endowments, and they will have a comparative advantage in producing commodities 1 and 2, which have "similar" production functions. The object is to show that countries 1 and 2 may trade more, even much more, with each other than with the third country.

To produce some definite examples. I shall as in the previous section assume that production functions in each country have the Cobb-Douglas form

$$(3.1) \quad y_j^k = f_j(v_{1j}^k, v_{2j}^k, v_{3j}^k) = \mu_j v_{1j}^{\beta_{1j}} v_{2j}^{\beta_{2j}} v_{3j}^{\beta_{3j}} \quad (\beta_{ij} > 0, \sum_{i=1}^{3} \beta_{ij} = 1) \; (j = 1, 2, 3)$$

and that the resource-allocation constraints

$$(3.2) \qquad v_{i1}^k + v_{i2}^k + v_{i3}^k = l_i^k \quad (i = 1, 2, 3)$$

are satisfied. Here, $v_{ij} = v_{ij}^k$ denotes the input of factor i into the production of commodity j in country k (the superscript is omitted in (3.1) for notational simplicity); y_j^k is the output of commodity j in country k, and l_i^k is the endowment of factor i in country k. I shall finally assume that preferences in each country are identical and homothetic, generated by the Mill-Cobb-Douglas utility function

$$U(x_1, x_2, x_3) = x_1^{\theta_1} x_2^{\theta_2} x_3^{\theta_3} \quad (\theta_j > 0, \sum_{j=1}^{3} \theta_j = 1). \tag{3.3}$$

Now we must define "similarity". Since in the examples to follow I shall assume $\mu_1 = \mu_2 = \mu_3 = 1$, the dissimilarity between two production functions f^j and $f^{j'}$ may be defined simply as the Euclidean distance betweeen the vectors of exponents β_j and $\beta_{j'}$, where $\beta_j = (\beta_{1j}, \beta_{2j}, \beta_{3j})$, i.e.,

$$\sqrt{\sum_{i=1}^{3} (\beta_{ij} - \beta_{ij'})^2}. \tag{3.4}$$

In the case of factor-endowment vectors $l^k = (l_1^k, l_2^k, l_3^k)$ and $l^{k'} = (l_1^{k'}, l_2^{k'}, l_3^{k'})$, I shall define the *relative dissimilarity* between them as the normalized distance

$$\sqrt{\sum_{i=1}^{3} \left(\frac{l_i^k}{\|l^k\|} - \frac{l_i^{k'}}{\|l^{k'}\|} \right)}, \text{ where } \|l^k\| = \sqrt{\sum_{i=1}^{3} (l_i^k)^2}, \tag{3.5}$$

i.e., the distance between them after they have both been normalized to unit length.

World equilibrium is solved for in the following manner. It is assumed that the equilibrium is one in which each country produces all three commodities, hence in view of the assumption that the countries have identical production functions, and of the form (3.1) (ruling out factor-intensity reversal), factor rentals are equalized; therefore, each country produces according to the same technical coefficients. World national product is then determined by the world national-product function

$$Y = \Pi(p_1, p_2, p_3, l_1, l_2, l_3) = \sum_{k=1}^{3} \Pi(p_1, p_2, p_3, l_1^k, l_2^k, l_3^k), \tag{3.6}$$

where $l_i = \sum_{k=1}^{3} l_i^k$, and world output of commodity j is determined according to the world Rybczynski function

$$y_j = \hat{y}_j(p_1, p_2, p_3, l_1, l_2, l_3) = \sum_{k=1}^{3} \hat{y}_j^k(p_1, p_2, p_3, l_1^k, l_2^k, l_3^k), \tag{3.7}$$

where $\hat{y} = \partial\Pi/\partial p_j$. To compute (3.6) and (3.7), we first obtain the factor rentals; this is done by solving from the system of minimum-unit-cost functions

$$p_j = g_j(w_1, w_2, w_3) = \nu_j w_1^{\beta_{1j}} w_2^{\beta_{2j}} w_3^{\beta_{3j}} \ (j = 1, 2, 3) \tag{3.8}$$

where $\nu_j = 1/f_j(\beta_{1j}, \beta_{2j}, \beta_{3j})$, given that this system is linear in the logarithms of the p_j/ν_j and w_i.

From these factor rentals, the matrix of factor-output coefficients $b_{ij} = \beta_{ij}p_j/w_i$ is obtained, and the world outputs (3.7) are obtained by solving the system of equations

$$\sum_{j=1}^{3} b_{ij}y_j = l_i \quad (i = 1, 2, 3). \tag{3.9}$$

World national product, (3.6), is then obtained from

$$Y = \sum_{j=1}^{3} p_j y_j. \tag{3.10}$$

From (3.10) and the assumption that world preferences are identical and homothetic and are generated by (3.3), world consumption is given by

$$x_j = \frac{\theta_j Y}{p_j}. \tag{3.11}$$

The system of equations to be solved to obtain world equilibrium prices is obtained by setting demand equal to supply for two out of the three commodities (since the third equality will follow by Walras' law); we may write this in the form

$$p_j y_j = \theta_j Y \quad (j = 1, 2) \tag{3.12}$$

where y_j and Y are obtained from (3.9) and(3.10), and x_j from (3.11).

To solve the equations (3.12), Wolfe's (1959) algorithm has been used. Setting $p_3 = 1$, three trial solutions p^1, p^2, p^3 are chosen, to form the matrix

$$P = \begin{bmatrix} p_1^1 & p_1^2 & p_1^3 \\ p_2^1 & p_2^2 & p_2^3 \end{bmatrix}. \tag{3.13}$$

The matrix

$$A = \begin{bmatrix} p_1^1 y_1^1 - \theta_1 Y^1 & p_1^2 y_1^2 - \theta_1 Y^2 & p_3^3 y_1^3 - \theta_1 Y^3 \\ p_2^1 y_2^1 - \theta_2 Y^1 & p_2^2 y_2^2 - \theta_2 Y^2 & p_2^3 y_2^3 - \theta_2 Y^3 \\ 1 & 1 & 1 \end{bmatrix} \tag{3.14}$$

is formed, where y_j^k and Y^k are obtained from (3.9) and (3.10) using the factor-output coefficients $b_{ij} = \beta_{ij} p_j^k / w_i^k$, the w_i^k being obtained from the p_k^k via (3.8). The norm of $p^k = (p_1^k, p_2^k)$ is defined as

$$\|p^k\| = \sum_{j=1}^{2} \left(p_j^k y_j^k - \theta_j Y^k \right)^2. \tag{3.15}$$

The Wolfe algorithm then proceeds as follows. A new average price vector $\bar{p}$ is obtained from the formula

$$\begin{bmatrix} \bar{p}_1 \\ \bar{p}_2 \end{bmatrix} = \tag{3.16}$$
$$\begin{bmatrix} p_1^1 & p_1^2 & p_1^3 \\ p_2^1 & p_2^2 & p_2^3 \end{bmatrix} \begin{bmatrix} p_1^1 y_1^1 - \theta_1 Y^1 & p_1^2 y_1^2 - \theta_1 Y^2 & p_3^3 y_1^3 - \theta_1 Y^3 \\ p_2^1 y_2^1 - \theta_2 Y^1 & p_2^2 y_2^2 - \theta_2 Y^2 & p_2^3 y_2^3 - \theta_2 Y^3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

One of the columns, p^k of (3.13) of maximal norm (3.15) is then dropped and replaced by $\bar{p}$; from the new price vector, the factor rentals w_i, then factor-output coefficients b_{ij}, the outputs y_j, and the national product Y are recomputed and the new column of coefficients of A is substituted for the dropped one. When the maximal norm (3.15) has reached a prescribed small number, the process converges.

This algorithm assumes diversification by all three countries, but this assumption may be incorrect. That is, for a particular choice of matrices of exponents β_{ij} in (3.1) and factor endowments l_i^k $(i, j, k = 1, 2, 3)$, it could happen that the algorithm yields negative outputs for some commodities. This solution would have to be rejected, of course. In the examples to be give below, positive production has been verified for all cases.

I shall choose initially the matrix of exponents

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} = \begin{bmatrix} .49 & .46 & .1 \\ .46 & .49 & .1 \\ .05 & .05 & .8 \end{bmatrix}, \tag{3.17}$$

the matrix of factor endowments

$$\begin{bmatrix} l_1^1 & l_1^2 & l_1^3 \\ l_2^1 & l_2^2 & l_2^3 \\ l_3^1 & l_3^2 & l_3^3 \end{bmatrix} = \begin{bmatrix} 1180 & 1120 & 725/t \\ 1120 & 1180 & 725/t \\ 300 & 300 & 1300/t \end{bmatrix} \tag{3.18}$$

(where t is a positive parameter), and the vector of constant expenditure shares

$$(\theta_1, \theta_2, \theta_3) = \left(\frac{r}{2r+1}, \frac{r}{2r+1}, \frac{1}{2r+1}\right) \tag{3.19}$$

where $0 < r \leq 1$. The countries 1 and 2, as well as commodities 1 and 2, are completely symmetric in their differences, and the prices of commodities 1 and 2 in world equilibrium will necessarily be equal to one another. If $t = 1$ in (3.8), the length of country 3's endowment vector (1655.7) is just slightly larger than that of countries 1 and 2 (1654.3)—giving this country the benefit of the doubt. For $t = 1$ and $r = 1$ we find that the prices are[4]

$$p_1 = p_2 = .98441846365, \quad p_3 = 1. \tag{3.20}$$

The pattern of world outputs is given by

$$\begin{bmatrix} y_1^1 & y_1^2 & y_1^3 \\ y_2^1 & y_2^2 & y_2^3 \\ y_3^1 & y_3^2 & y_3^3 \end{bmatrix} = \begin{bmatrix} 908.11 & 71.94 & 224.42 \\ 71.24 & 908.11 & 224.42 \\ 150.32 & 150.32 & 885.09 \end{bmatrix}; \tag{3.21}$$

and the pattern of world consumption is

$$\begin{bmatrix} x_1^1 & x_1^2 & x_1^3 \\ x_2^1 & x_2^2 & x_2^3 \\ x_3^1 & x_3^2 & x_3^3 \end{bmatrix} = \begin{bmatrix} 377.58 & 377.58 & 449.31 \\ 377.58 & 377.58 & 449.31 \\ 371.71 & 371.71 & 442.32 \end{bmatrix}. \tag{3.22}$$

[4]Computations were carried out on the HP-71B handheld computer. A listing of the BASIC program is available from the author on request.

The row sums of the output and consumption matrices are equal to one another (except for rounding error). The matrix of world trade values is

$$(3.23)\qquad \begin{bmatrix} p_1 z_1^1 & p_1 z_1^2 & p_1 z_1^3 \\ p_2 z_2^1 & p_2 z_2^2 & p_2 z_2^3 \\ p_3 z_3^1 & p_3 z_3^2 & p_3 z_3^3 \end{bmatrix} = \begin{bmatrix} -522.27 & 300.89 & 221.39 \\ 300.89 & -522.27 & 221.39 \\ 221.39 & 221.39 & -442.77 \end{bmatrix},$$

where $z_j^k = x_j^k - y_j^k$. Thus we see that countries 1 and 2 trade more with each other than with country 3. Nevertheless, their factor endowments are closer to one another than to those of country 3 as measured by the indices of relative dissimilarity (3.5), yielding the matrix

$$(3.24)\qquad \begin{bmatrix} 0 & .0513 & .7054 \\ .0513 & 0 & .7054 \\ .7054 & .7054 & 0 \end{bmatrix}.$$

Likewise, the production coefficients of commodities 1 and 2 are much closer to each other than to that of commodity 3, as measured by the dissimilarity criterion (3.4), yielding the matrix

$$(3.25)\qquad \begin{bmatrix} 0 & .0424 & .9188 \\ .0424 & 0 & .9188 \\ .9188 & .9188 & 0 \end{bmatrix}.$$

As r increases, i.e., as the relative share of world expenditure devoted to commodities 1 and 2 increases, the ratio of trade between countries 1 and 2 to trade between countries 1 and 3 increases. The same is true as t increases, i.e., as countries 1 and 2's absolute endowments increase relatively to country 3's. Table 5 gives the ratio of trade between dissimilar countries as a function of t and r; note that the relationship between this ratio and the scale factor t is linear, for each r.

Table 5

Ratios of trade between similar and dissimilar countries
(for the model (3.17)–(3.19)—countries 1 & 2 versus countries 1 & 3 and countries 2 & 3)
for various values of the endowment ratio t and expenditure ratio r

Endow-ment Ratio	Expenditure Ratio									
	1	2	3	4	5	6	7	8	9	10
1	1.359	2.289	3.049	3.683	4.219	4.679	5.077	5.426	5.733	6.007
2	1.946	3.169	4.170	5.004	5.710	6.314	6.839	7.297	7.702	8.062
3	2.533	4.050	5.291	6.325	7.200	7.950	8.600	9.169	9.670	10.116
4	3.120	4.931	6.412	7.646	8.690	9.585	10.361	11.040	11.639	12.171
5	3.707	5.811	7.532	8.967	10.180	11.221	12.122	12.911	13.607	14.226
6	4.295	6.692	8.653	10.288	11.671	12.856	13.884	14.783	15.576	16.281
7	4.882	7.572	9.774	11.609	13.161	14.492	15.645	16.654	17.544	18.338
8	5.469	8.453	10.895	12.930	14.651	16.127	17.406	18.525	19.513	20.390
9	6.056	9.334	12.016	14.250	16.142	17.763	19.167	20.397	21.481	22.445
10	6.643	10.214	13.136	15.571	17.632	19.398	20.929	22.268	23.450	24.500

It seems quite reasonable to assume that there is a preponderance of world expenditure on the products of industrial countries which are similar to each other in both their endowments and in the production processes on which they concentrate; and that these countries have higher absolute productivity (whether measured by t in (3.18) or by the μ_j in (3.1)—which we have assumed = 1). Thus, there are three forces all of which lead to greater trade between similar countries: (1) similarity in the production functions for the goods which they export—a circumstance which (as we saw in the previous section) makes for *more* intra-industry trade; (2) greater world demand for their products; and (3) greater absolute productivity of the similar industrial countries compared to the dissimilar one.

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