

Capital Movement in an Open Economy*

JOHN S. CHIPMAN
University of Minnesota

November 25, 2005

Let an open economy be assumed to produce n_1 tradable goods, import another n_2 tradable goods which it does not produce, and produce n_3 nontradable goods, all with m factors of production. The system of equations determining its equilibrium is given by

$$(1) \quad \begin{aligned} h^3(p^1, p^2, p^3, p^1 \cdot y^1 + p^3 \cdot y^3 + D) &= y^3 \\ B^1(w)y^1 + B^3(w)y^3 &= l \\ g^1(w) &= p^1 \\ g^3(w) &= p^3 \end{aligned}$$

where p^k is the n_k -vector of prices in category k , w is the m -vector of factor rentals, y^k is the $n_k \times 1$ vector of outputs in category $k = 1, 3$, l is the $m \times 1$ vector of factor endowments, D is the deficit in the open economy's balance of payments on current account, $g^k(w)$ is the $n_k \times 1$ vector of minimum-unit-cost functions for the products in category $k = 1, 3$, $B^k(w) = [b_{ij}^k(w)]$ is the matrix of input-output ratios $v_{ij}^k/y_j^k = \partial g_j^k(w)/\partial w_i$ of the amount of i th factor employed per unit of output in industry j in category k , and h^3 is the $n_3 \times 1$ vector-valued aggregate demand function for nontradables (say, generated by identical homothetic preferences), with value x^3 . (Cf. Chipman 1980, pp. 166–172.) We note that (1) is a system of $n_3 + m + n_1 + n_3$ equations in the $n_3 + m + n_1 + n_3$ unknowns p^3, w, y^1, y^3 .

We treat the special case in which our country starts out in an initial equilibrium in which $m \geq n_1 + n_3$, so that an $n_3 \times 1$ -valued Rybczynski function $\hat{y}^3(p^1, p^3, l)$ exists and is differentiable (cf. Chipman 1987, Theorem 9). Then the above system reduces to a system of n_3 equations

$$(2) \quad h^3(p^1, p^2, p^3, \Pi(p^1, p^3, l), l) + D = \hat{y}^3(p^1, p^3, l),$$

where Π is the domestic-product function. The exogenous variables are the world prices of tradables p^1, p^2 , the factor endowments l , and the capital inflow D . Then (2) implicitly defines the function

$$(3) \quad p^3 = \tilde{p}^3(p^1, p^2, D, l).$$

*Revised version.

We wish to evaluate $\partial\bar{p}^3/\partial D$.

Totally differentiating (2) with respect to D and taking account of the fact that $\partial\Pi/\partial p_j^3 = \hat{y}_j^3$ and that $x_j^3 = y_j^3$ by definition of nontradables, we obtain

$$(4) \quad \sum_{j=1}^{n_3} \left(\frac{\partial h_i^3}{\partial p_j^3} + \frac{\partial h_i^3}{\partial Y} x_j^3 - \frac{\partial \hat{y}_i^3}{\partial p_j^3} \right) \frac{\partial \bar{p}_j^3}{\partial D} = -\frac{\partial h_i^3}{\partial Y},$$

where $Y = \Pi(p^1, p^3, l) + D$ (disposable national income) is the last argument of h^3 . In matrix notation, (4) is written

$$(5) \quad (S^{33} - T^{33}) \frac{\partial \bar{p}^3}{\partial D} = -c^3,$$

where S^{33} is the sub-Slutsky matrix for nontradables (which is negative-definite), T^{33} is the sub-transformation matrix for nontradables (which is positive-definite, since $\Pi(p, l)$ is convex in p), and

$$(6) \quad c^3 \equiv \frac{\partial h^3}{\partial Y} = \frac{x^3}{Y}.$$

The last equality of (6) follows from homotheticity of preferences. Our desired solution of (5) is

$$(7) \quad \frac{\partial \bar{p}^3}{\partial D} = -(S^{33} - T^{33})^{-1} c^3.$$

From the last equality in (6) one can in principle obtain data on the components of c^3 in the initial equilibrium; let this define \bar{c}^3 . Then defining a price index of nontradables as $\bar{c}^3 \cdot p^3$, it follows from the negative-definiteness of $S^{33} - T^{33}$ that

$$(8) \quad \frac{\partial(\bar{c}^3 \cdot p^3)}{\partial D} = -c^{3'}(S^{33} - T^{33})^{-1} c^3 > 0$$

(cf. Chipman 1980, p. 172).

Now, suppose that our open economy adopts a policy of flexible exchange rates, accompanied by a policy on the part of the monetary authority to stabilize a general price index of all commodities (expressed in domestic currency)

$$(9) \quad p \equiv \bar{c}^1 \cdot p^1 + \bar{c}^2 \cdot p^2 + \bar{c}^3 \cdot p^3 = \bar{p}$$

at the initial level \bar{p} , where \bar{c}^k denotes the value of x^k/Y in the initial equilibrium. Let world (foreign) prices in categories $k = 1, 2$ (the tradables) now be denoted p^{k*} , and be related to domestic prices p^k by the first pair of equations in

$$(10) \quad p^{k*} = \chi p^k \quad \text{for } k = 1, 2; \quad p^{3*} = \chi p^3; \quad D^* = \chi D.$$

The first pair of equations defines the exchange rate χ as the price of the open economy's currency relative to the world's; the second equation simply expresses the prices of the open economy's nontradables in terms of foreign currency; and the third expresses the capital inflow (the open economy's balance-of-payments deficit)

in terms of foreign currency. It is of course the external prices p^{1*}, p^{2*} which must be taken to be exogenous. We wish to analyze the effect of a capital inflow on our open economy's exchange rate. However, it makes a difference whether we measure the capital movement in foreign or domestic currency. If our country is, say, a developing country receiving a loan from the IMF, it is best to denominate the capital inflow in foreign currency. On the other hand, if our country is a developed country which has generated a budget deficit which is to be financed by borrowing from abroad, then it is best to denominate the capital inflow in domestic currency.

We first take up the case in which the capital inflow is denominated in foreign currency. We need to define our country's exchange rate χ as a function of the relevant exogenous variables p^{1*}, p^{2*} , and D^* . Substituting (10) and (3) in (9) we obtain

$$(11) \quad \bar{c}^1 \cdot p^{1*}/\chi + \bar{c}^2 \cdot p^{2*}/\chi + \bar{c}^3 \cdot \tilde{p}^3(p^{1*}/\chi, p^{2*}/\chi, D^*/\chi, l) = \bar{p}.$$

At this point we need to take account of an important property of the function (3), namely that it is homogeneous of degree 1 in p^1, p^2, D . This may be proved by verifying the equivalent property by Euler's theorem (cf. Chipman 2003). Then we have

$$(12) \quad p^3 = \tilde{p}^3(p^{1*}/\chi, p^{2*}/\chi, D^*/\chi, l) = \tilde{p}^3(p^{1*}, p^{2*}, D^*, l)/\chi,$$

hence (11) yields

$$(13) \quad \chi = \tilde{\chi}(p^{1*}, p^{2*}, D^*, l) \equiv \frac{1}{\bar{p}} \left[\bar{c}^1 \cdot p^{1*} + \bar{c}^2 \cdot p^{2*} + \bar{c}^3 \cdot \tilde{p}^3(p^{1*}, p^{2*}, D^*, l) \right].$$

From this and the fact that $\partial \tilde{p}^3 / \partial D$ is homogeneous of degree 0 in p^1, p^2 , and D it follows immediately from (7) that

$$(14) \quad \frac{\partial \tilde{\chi}}{\partial D^*} = -\frac{1}{\bar{p}} \bar{c}^{3'} (S^{33} - T^{33})^{-1} \bar{c}^3 > 0.$$

That is, a capital inflow raises the value of our country's currency.

Now from (12) and (13) we see that the domestic price index of nontradables as a function of the exogenous variables p^{1*}, p^{2*}, D^*, l is given by

$$(15) \quad \begin{aligned} \bar{c}^3 \cdot p^3 &= \frac{\bar{p} \bar{c}^2 \cdot \tilde{p}^3(p^{1*}, p^{2*}, D^*, l)}{\bar{c}^1 \cdot p^{1*} + \bar{c}^2 \cdot p^{2*} + \bar{c}^3 \cdot \tilde{p}^3(p^{1*}, p^{2*}, D^*, l)} \\ &= \frac{\bar{p}}{\frac{\bar{c}^1 \cdot p^{1*} + \bar{c}^2 \cdot p^{2*}}{\bar{c}^3 \cdot \tilde{p}^3(p^{1*}, p^{2*}, D^*, l)} + 1}. \end{aligned}$$

From this it is easy to see that $\partial(\bar{c}^3 \cdot p^3) / \partial D^* > 0$, i.e., the index of nominal domestic prices of nontradables still rises, and consequently the index $\bar{c}^1 \cdot p^1 + \bar{c}^2 \cdot p^2$ of nominal domestic prices of tradables must fall, in response to a capital inflow.

It remains to consider the case in which the capital inflow is denominated in domestic currency. From (13), the function $\hat{\chi}(p^{1*}, p^{2*}, D, l)$ is defined implicitly by

$$(16) \quad \chi = \frac{1}{\bar{p}} \left[\bar{c}^1 \cdot p^{1*} + \bar{c}^2 \cdot p^{2*} + \bar{c}^3 \cdot \tilde{p}^3(p^{1*}, p^{2*}, \chi D, l) \right],$$

hence from (7)

$$(17) \quad \frac{\partial \hat{\chi}}{\partial D} = -\frac{\chi}{\bar{p}} \bar{c}' (S^{33} - T^{33})^{-1} \bar{c}^3 > 0.$$

This agrees with (14) if $\chi = 1$ in the initial equilibrium.

References

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