Marshallian Offer Curves

John S. Chipman

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Let it be assumed that there are two countries trading two commodities. In an initial equilibrium, country 1 is exporting commodity 1 to country 2, and trade is assumed to be balanced. The balanced-trade condition for country 1 is

$$p_1 z_1^1 + p_2 z_2^1 = 0, \tag{1}$$

which of course implies

$$\frac{p_1}{p_2} = -\frac{z_2^1}{z_1^1} = \frac{z_2^1}{z_1^2}, \tag{2}$$

where we have used the material-balance condition $z_1^1 + z_1^2 = 0$ (world excess demand for commodity 1 is zero). Equation (2) states that the *terms of trade* of country 1 (defined by the term on the left in prices) are equal to its *barter terms of trade* given by either of the other two expressions. Defining

$$r_1 = p_1/p_2, \quad r_2 = p_2/p_1, \tag{3}$$

where p_j is the world price of commodity j, let country 1's trade-demand for its import good (commodity 2) be expressed as

$$z_2^1 = \hat{h}_2^1(1, r_2, 0, l^1). \tag{4}$$

Provided $\partial\hat{h}_2^1/\partial p_2 \neq 0$ in a neighborhood of the initial equilibrium, (4) implicitly defines country 1's *inverse trade-demand function*

$$r_2 = \hat{r}_2^1(z_2^1), \tag{5}$$

and it satisfies the property

$$\frac{d\hat{r}_2^1}{dz_2^1} = \frac{1}{\partial\hat{h}_2^1/\partial p_2|_{p_1=1}}. \tag{6}$$

The Marshallian *reciprocal demand function* F^1, or *offer function*, of country 1 is defined by

$$-z_1^1 = F^1(z_2^1) \equiv \hat{r}_2^1(z_2^1) z_2^1. \tag{7}$$

This is just a restatement of the condition (1) of balanced trade, expressed in terms of country 1's inverse trade-demand function $\hat{r}_2^1(z_2^1)$.

An exactly similar development holds for country 2. Its balanced-trade condition is

$$p_1 z_1^2 + p_2 z_2^2 = 0, \tag{8}$$

which of course implies

$$\frac{p_2}{p_1} = -\frac{z_1^2}{z_2^2} = \frac{z_1^2}{z_2^1}, \tag{9}$$

where we have used the material-balance condition $z_2^1 + z_2^2 = 0$ (world excess demand for commodity 2 is zero). Equation (9) states that the *terms of trade* of country 2 (defined by the term on the left in prices) are equal to its *barter terms of trade* given by either of the other two expressions. Defining r_1 as above, country 2's trade-demand for its import good (commodity 1) be expressed as

$$z_1^2 = \hat{h}_1^2(r_1, 1, 0, l^1). \tag{10}$$

Provided $\partial\hat{h}_1^2/\partial p_2 \neq 0$ in a neighborhood of the initial equilibrium, (10) implicitly defines country 2's *inverse trade-demand function*

$$r_1 = \hat{r}_1^2(z_1^2), \tag{11}$$

and it satisfies the property

$$\frac{d\hat{r}_1^2}{dz_1^2} = \frac{1}{\partial\hat{h}_1^2/\partial p_1|_{p_2=1}}. \tag{12}$$

The Marshallian *reciprocal demand* or *offer function* F^2 of country 2 is defined by

$$-z_2^2 = F^2(z_1^2) \equiv \hat{r}_1^2(z_1^2)z_1^2. \tag{13}$$

This is just a restatement of the condition (8) of balanced trade, expressed in terms of country 2's inverse trade-demand function $\hat{r}_1^2(z_1^2)$.

Now let us define the elasticities of these offer functions, also called *elasticities of trade*, by

$$\alpha^1 = \frac{z_2^1}{F^1}\frac{dF^1}{dz_2^1} \quad \text{and} \quad \alpha^2 = \frac{z_1^2}{F^2}\frac{dF^2}{dz_1^2}. \tag{14}$$

Likewise we define the Marshallian *elasticities of demand for imports* of the two countries by

$$\eta^1 = -\frac{p_2}{\hat{h}_2^1}\frac{\partial\hat{h}_2^1}{\partial p_2} \quad \text{and} \quad \eta^2 = -\frac{p_1}{\hat{h}_1^2}\frac{\partial\hat{h}_1^2}{\partial p_1}. \tag{15}$$

Now we show that

$$\alpha^1 = 1 - \frac{1}{\eta^1} \quad \text{and} \quad \alpha^2 = 1 - \frac{1}{\eta^2}. \tag{16}$$

It is enough to derive the second of these formulas (recall that superscripts are country suffixes, not exponents, and that we use the normalization $p_2 = 1$):

$$\begin{aligned}
\alpha^2 &= \frac{z_1^2}{z_2^1}\left(r_1 + z_1^2 \frac{d\hat{r}_1^2}{dz_1^2}\right) && \text{from (14) and (13)} \\
&= \frac{z_1^2}{z_2^1}\left(\frac{z_2^1}{z_1^2} + \frac{z_1^2}{\partial \hat{h}_1^2/\partial p_1}\right) && \text{from (3) and (2)} \\
&= 1 + \frac{z_1^2}{z_2^1}\frac{1}{\dfrac{1}{\hat{h}_1^2}\dfrac{\partial \hat{h}_1^2}{\partial p_1}} && \text{from (10)} \qquad (17)\\
&= 1 + \frac{1}{\dfrac{p_1}{\hat{h}_1^2}\dfrac{\partial \hat{h}_1^2}{\partial p_1}} && \text{from (2) and } p_2 = 1 \\
&= 1 - \frac{1}{\eta^2} && \text{from (15).}
\end{aligned}$$

Thus we may write

$$\eta^2 = \frac{1}{1-\alpha^2} = \frac{z_2^1}{z_2^1 - z_1^2 \dfrac{dF^2}{dz_2^1}}. \qquad (18)$$

The last expression enables us to read off from the Marshallian offer-curve diagram the elasticity of country 2's offer curve corresponding to any point z_2^1 on the abscissa.