Notes on the Theory of Tariff Wars

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Suppose two countries are trading two commodities, country 1 exporting commodity 1 to and importing commodity 2 from country 2. Denoting by z_j^k country k's net import of commodity j (import if positive, export if negative), let the two countries' trade-utility functions (each assumed known by the other) be given by

$$(1)\qquad \begin{aligned} \hat{U}^1(z_1^1, z_2^1) &= z_1^1 + \beta_1 (z_2^1)^{\alpha_1} \\ \hat{U}^2(z_1^2, z_2^2) &= \beta_2 (z_1^2)^{\alpha_2} + z_2^2 \end{aligned}$$

respectively, where $0 < \alpha_j < 1$ and $\beta_j > 0$. Let each country impose a tariff on its import good; for simplicity, denote by τ_2 the tariff rate imposed by country 1 on its import of commodity 2 from country 2, expressed as a proportion of the world-market price of commodity 2, and by τ_1 the tariff rate imposed by country 2 on its import of commodity 1 from country 1, expressed as a proportion of the world-market price of commodity 1. Denote the corresponding tariff factors by $T_1 = 1 + \tau_1$ and $T_2 = 1 + \tau_2$. Then if p_j denotes the price of commodity j on the world market, and p_j^k the price of commodity j on country k's internal market, these prices are related by

$$p_1^1 = p_1, \; p_2^1 = T_2 p_2; \quad p_1^2 = T_1 p_1, \; p_2^2 = p_2,$$

hence

$$p_2^1 = T_2 p_2 = T_2 p_2^2 \quad \text{and} \quad p_1^2 = T_1 p_1 = T_1 p_1^1.$$

For each country, we now determine its optimal tariff factor given the other country's tariff factor. We know that the optimal tariff rates are given by the formula

$$\tau_k = \frac{1}{\eta^k - 1} \quad (k = 1, 2)$$

where η^k is country k's elasticity of demand for imports, defined by

$$\eta^k = -\frac{p_j^k}{\hat{h}_j^k} \frac{\partial \hat{h}_j^k}{\partial p_j^k} \quad (j \neq k = 1, 2),$$

where

$$z_j^k = \hat{h}_j^k(p_1^k, p_2^k, D^k)$$

is country k's trade-demand function for commodity j, p_j^k is the price of commodity j on country k's markets, and D^k in the deficit in country k's balance of payments on current account.

To obtain the expressions for these elasticities, we solve for the trade-demand functions from the trade-utility functions, as follows. The first-order conditions for a maximum of $\hat{U}^1(z_1^1, z_2^1)$ subject to $p_1^1 z_1^1 + p_2^1 z_2^1 = D^1$ are

$$\frac{\partial \hat{U}^1/\partial z_2^1}{\partial \hat{U}^1/\partial z_1^1} = \alpha_1 \beta_1 (z_2^1)^{\alpha_1 - 1} = \frac{p_2^1}{p_1^1}. \tag{2}$$

Solving for z_2^1 we obtain

$$z_2^1 = (\alpha_1\beta_1)^{\frac{1}{1-\alpha_1}} (p_1^1)^{\frac{1}{1-\alpha_1}} (p_2^1)^{-\frac{1}{1-\alpha_1}} \equiv \hat{h}_2^1(p_1^1, p_2^1, D^1)$$

(note the lack of dependence on D^1), hence

$$\log \hat{h}_2^1(p_1^1, p_2^1, D^1) = \frac{1}{1-\alpha_1} \log \alpha_1\beta_1 + \frac{1}{1-\alpha_1} \log p_1^1 - \frac{1}{1-\alpha_1} \log p_2^1.$$

Accordingly,

$$\eta^1 = -\frac{p_2^1}{\hat{h}_2^1} \frac{\partial \hat{h}_2^1}{\partial p_2^1} = -\frac{\partial \log \hat{h}_2^1}{\partial \log p_2^1} = \frac{1}{1-\alpha_1}.$$

Country 2's optimal tariff on its import of commodity 1 from country 1 is therefore

$$\tau_1 = \frac{1}{\eta^1 - 1} = \frac{1}{\alpha_1} - 1,$$

hence its optimal tariff factor is

$$T_1 = \frac{1}{\alpha_1}. \tag{3}$$

Likewise, owing to the complete symmetry of the trade-utility functions (1), country 1's optimal tariff factor on its import of commodity 2 from country 2 is

$$T_2 = \frac{1}{\alpha_2}. \tag{4}$$

Notice that, because of the "parallel" forms of the trade-utility functions (1), the quantity of each country's export good being an additive component of its trade-utility (an increase in the negative quantity z_k^k—i.e., a reduction in exports—releasing more of the export good for consumption and thus adding to utility), the demand for the import good is independent of the deficit D^k.

Still more important, though, is the fact that because of the special forms of these trade-utility functions, each country's optimal tariff is independent of the other country's tariff rate. This means that the tariff war will be over in one step, instead of an infinite sequence of steps.

Given the material-balance conditions $z_1^k + z_2^k = 0$ $(k = 1, 2)$, let us now solve for world equilibrium given any pair of tariff factors (T_1, T_2). The simplest procedure is to consider the inverse trade-demand functions in which each country's trade-demand price ratio is expressed as a function of the quantities of trades. This of course is obtained by setting the ratio of marginal trade-utilities equal to the price ratio. Using the fact that country k's trade must be balanced when reckoned in world prices, i.e.,

$$p_1 z_1^k + p_2 z_2^k = 0 \quad (k = 1, 2),$$

hence $p_2/p_1 = z_1^k/z_2^k$, we obtain from (2) for country 1:

$$\alpha_1\beta_1(z_2^1)^{\alpha_1-1} = \frac{p_2^1}{p_1^1} = T_2\frac{p_2}{p_1} = -T_2\frac{z_1^1}{z_2^1}.$$

Solving this equation we obtain the formula for country 1's offer curve

$$z_1^2 = -z_1^1 = \frac{\alpha_1\beta_1}{T_2}(z_2^1)^{\alpha_1} \equiv F^1(z_2^1, T_2). \tag{5}$$

Similarly for country 2:

$$\frac{\partial\hat{U}^2/\partial z_1^2}{\partial\hat{U}^2/\partial z_2^2} = \alpha_2\beta_2(z_1^2)^{\alpha_2-1} = \frac{p_1^2}{p_2^2} = T_1\frac{p_1}{p_2} = -T_1\frac{z_2^2}{z_1^2},$$

whence we obtain the formula for its offer curve

$$z_2^1 = -z_2^2 = \frac{\alpha_2\beta_2}{T_1}(z_1^2)^{\alpha_2} \equiv F^2(z_1^2, T_1). \tag{6}$$

To obtain world equilibrium we solve (5) and (6) simultaneouly to get

$$\begin{aligned} z_1^2 &= \left(\frac{\alpha_1\beta_1}{T_2}\right)^{\frac{1}{1-\alpha_1\alpha_2}}\left(\frac{\alpha_2\beta_2}{T_1}\right)^{\frac{\alpha_1}{1-\alpha_1\alpha_2}} \\ z_2^1 &= \left(\frac{\alpha_2\beta_2}{T_1}\right)^{\frac{1}{1-\alpha_1\alpha_2}}\left(\frac{\alpha_1\beta_1}{T_2}\right)^{\frac{\alpha_2}{1-\alpha_1\alpha_2}}. \end{aligned} \tag{7}$$

Equations (7) may be substituted into country 1's trade-utility function

$$\hat{U}^1(-z_1^2, z_2^1) = -z_1^2 + \beta_1(z_2^1)^{\alpha_1}$$

to get country 1's potential welfare as a function of the two tariff factors:

$$W^1(T_1, T_2) = (\alpha_1\beta_1)^{\frac{1}{1-\alpha_1\alpha_2}} (\alpha_2\beta_2)^{\frac{\alpha_1}{1-\alpha_1\alpha_2}} T_1^{\frac{-\alpha_1}{1-\alpha_1\alpha_2}} T_2^{\frac{-1}{1-\alpha_1\alpha_2}} \left(\frac{T_2}{\alpha_1} - 1\right). \tag{8}$$

In exactly similar way country 2's potential-welfare function is

$$W^2(T_1, T_2) = (\alpha_2\beta_2)^{\frac{1}{1-\alpha_1\alpha_2}} (\alpha_1\beta_1)^{\frac{\alpha_2}{1-\alpha_1\alpha_2}} T_1^{\frac{-1}{1-\alpha_a\alpha_2}} T_2^{\frac{-\alpha_2}{1-\alpha_1\alpha_2}} \left(\frac{T_1}{\alpha_2} - 1\right). \tag{9}$$

Substituting the optimal tariff factors (3) and (4) in (7) we obtain

$$\begin{aligned} z_1^2 &= (\alpha_1\alpha_2\beta_1)^{\frac{1}{1-\alpha_1\alpha_2}} (\alpha_1\alpha_2\beta_2)^{\frac{\alpha_1}{1-\alpha_1\alpha_2}} \\ z_2^1 &= (\alpha_1\alpha_2\beta_2)^{\frac{1}{1-\alpha_1\alpha_2}} (\alpha_1\alpha_2\beta_1)^{\frac{\alpha_2}{1-\alpha_1\alpha_2}}. \end{aligned} \tag{10}$$

The expressions for the two country's trade utilities at the end of the tariff war are given by substituting the optimal tariff factors in (8) and (9):

$$\begin{aligned} W^1\left(\frac{1}{\alpha_1}, \frac{1}{\alpha_2}\right) &= (\alpha_1\alpha_2\beta_1)^{\frac{1}{1-\alpha_1\alpha_2}} (\alpha_1\alpha_2\beta_2)^{\frac{\alpha_1}{1-\alpha_1\alpha_2}} \left(\frac{1}{\alpha_1\alpha_2} - 1\right) \\ W^2\left(\frac{1}{\alpha_1}, \frac{1}{\alpha_2}\right) &= (\alpha_1\alpha_2\beta_2)^{\frac{1}{1-\alpha_1\alpha_2}} (\alpha_1\alpha_2\beta_1)^{\frac{\alpha_2}{1-\alpha_1\alpha_2}} \left(\frac{1}{\alpha_1\alpha_2} - 1\right). \end{aligned} \tag{11}$$

Likewise, the corresponding expressions under free trade are

$$\begin{aligned} W^1(1,1) &= (\alpha_1\beta_1)^{\frac{1}{1-\alpha_1\alpha_2}} (\alpha_2\beta_2)^{\frac{\alpha_1}{1-\alpha_1\alpha_2}} \left(\frac{1}{\alpha_1} - 1\right) \\ W^2(1,1) &= (\alpha_2\beta_2)^{\frac{1}{1-\alpha_1\alpha_2}} (\alpha_1\beta_1)^{\frac{\alpha_2}{1-\alpha_1\alpha_2}} \left(\frac{1}{\alpha_2} - 1\right). \end{aligned} \tag{12}$$

Note from (8) and (9) that since $0 < \alpha_k < 1$ for $k = 1, 2$, it follows that $W^k(T_1, T_2) > 0$ for all $T_1, T_2 \geqq 1$. For country 1 to be no better off at the end of the tariff war than under free trade we therefore require

$$\frac{W^1\left(\frac{1}{\alpha_1}, \frac{1}{\alpha_2}\right)}{W^1(1,1)} = \frac{1 - \alpha_1\alpha_2}{(1-\alpha_1)\alpha_2} \alpha_2^{\frac{1}{1-\alpha_1\alpha_2}} \alpha_1^{\frac{\alpha_1}{1-\alpha_1\alpha_2}} \leqq 1. \tag{13}$$

(Note that the β_ks cancel out.) Likewise for country 2 to be no better off at the end of the tariff war than under free trade we require

$$\frac{W^2\left(\frac{1}{\alpha_1}, \frac{1}{\alpha_2}\right)}{W^2(1,1)} = \frac{1 - \alpha_1\alpha_2}{(1-\alpha_2)\alpha_1} \alpha_1^{\frac{1}{1-\alpha_1\alpha_2}} \alpha_2^{\frac{\alpha_2}{1-\alpha_1\alpha_2}} \leqq 1. \tag{14}$$

Countries 1 and 2 are no better off in the tariff equilibrium than in the free-trade equilibrium when

$$
(15)\qquad
\begin{aligned}
\frac{W^1\left(\frac{1}{\alpha_1},\frac{1}{\alpha_2}\right)}{W^1(1,1)} &= \frac{1-\alpha_1\alpha_2}{(1-\alpha_1)\alpha_2}\alpha_1^{\frac{\alpha_1}{1-\alpha_1\alpha_2}}\alpha_2^{\frac{1}{1-\alpha_1\alpha_2}} \equiv f_1(\alpha_1,\alpha_2) \leqq 1 \\
\frac{W^2\left(\frac{1}{\alpha_1},\frac{1}{\alpha_2}\right)}{W^2(1,1)} &= \frac{1-\alpha_1\alpha_2}{(1-\alpha_2)\alpha_1}\alpha_2^{\frac{\alpha_2}{1-\alpha_1\alpha_2}}\alpha_1^{\frac{1}{1-\alpha_1\alpha_2}} \equiv f_2(\alpha_1,\alpha_2) \leqq 1.
\end{aligned}
$$

These functions are plotted in Figure 1 and the inequalities of (15) are shown by the hatched areas. It is evident from the figure that the diagonal belongs to the cross-hatched area where both countries lose at the end of the tariff war. It is of interest to observe that the combined area of the two non-cross-hatched regions (where one of the countries gains) is greater than that of the intermediate cross-hatched region in which both countries are worse off at the end of the tariff war.

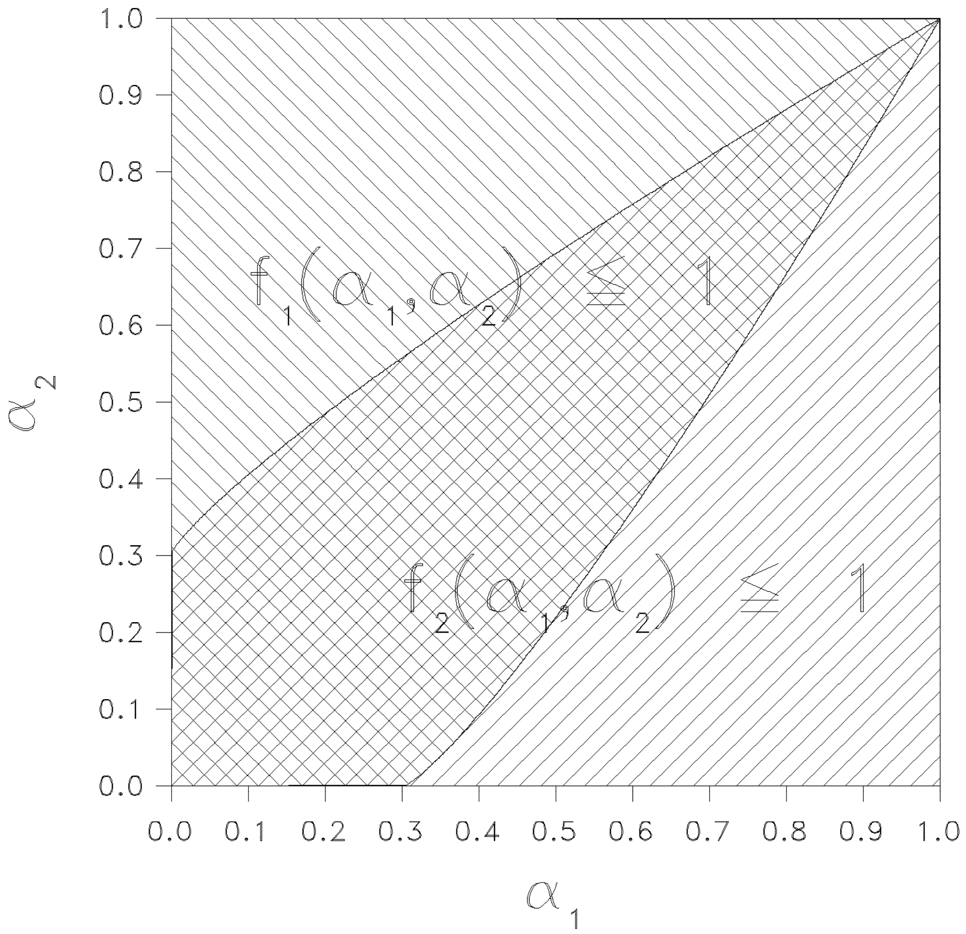

Figure 1

Let us now prove analytically that in the special case $\alpha_1 = \alpha_2 = \alpha$, corresponding to the diagonal of the box in Figure 1, both countries are worse off than they would be under free trade. In this case the inequalities (13) and (14) both reduce to the single condition

$$f(\alpha) \equiv (1+\alpha)\alpha^{\frac{\alpha}{1-\alpha}} \leqq 1. \tag{16}$$

In fact we can show that the inequality (16) is strict over the interval $0 < \alpha < 1$, and indeed that f has the limiting values

$$\lim_{\alpha \to 0} f(\alpha) = 1 \quad \text{and} \quad \lim_{\alpha \to 1} f(\alpha) = \frac{2}{e} = .735758883, \tag{17}$$

and is monotone decreasing over that interval (see Figure 2).

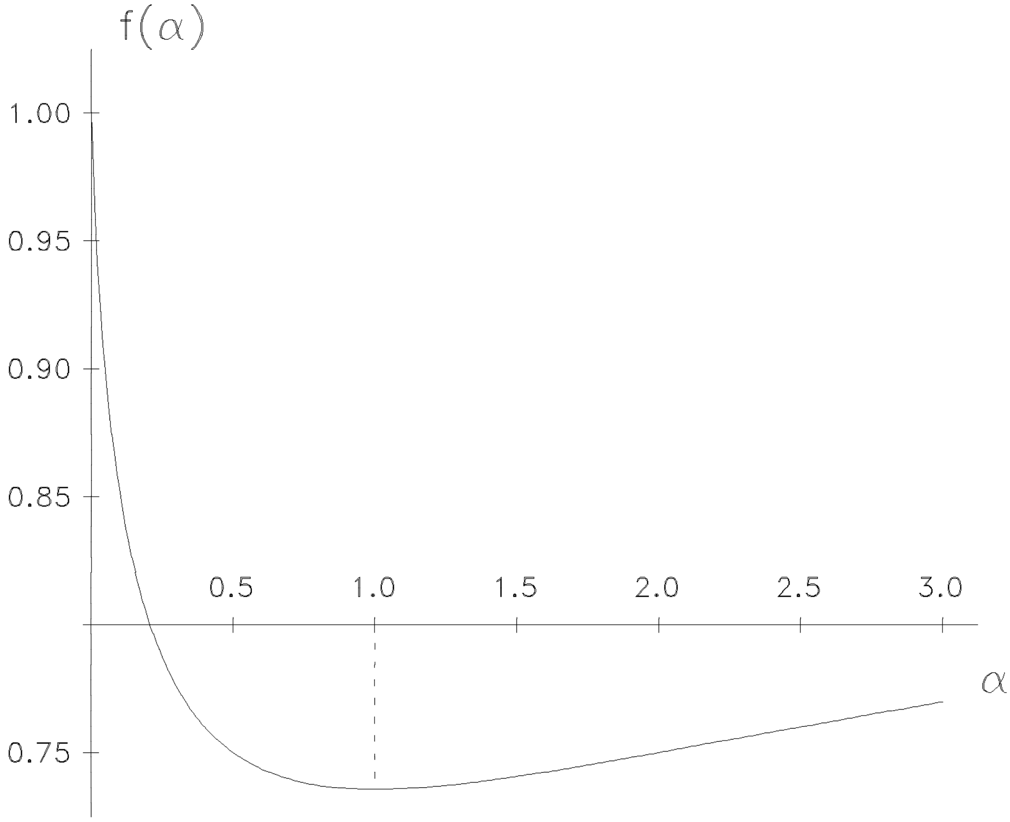

Figure 2

Define

$$\varphi(\alpha) = \log f(\alpha) = \log(1+\alpha) + \frac{\alpha}{1-\alpha}\log(\alpha). \tag{18}$$

Then

$$\lim_{\alpha \to 0} \varphi(\alpha) = \lim_{\alpha \to 0} \frac{\log(\alpha)}{1/\alpha} = 0$$

using l'Hospital's rule, and likewise

$$\lim_{\alpha \to 1} \varphi(\alpha) = \log(2) + \lim_{\alpha \to 1} \frac{\log(\alpha)}{1-\alpha} = \log(2) - 1,$$

from which the limits (17) follow for $f(\alpha) = \exp \varphi(\alpha)$.

To show that f is monotone decreasing over $(0,1)$ we compute the derivative

$$f'(\alpha) = [2(1-\alpha) + (1+\alpha)\log(\alpha)] \frac{\alpha^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)^2},$$

which is negative in $(0,1)$ if and only if

$$\psi(\alpha) \equiv (1+\alpha)\log(\alpha) + 2(1-\alpha) < 0 \quad \text{for} \quad 0 < \alpha < 1.$$

We verify that $\psi(0) = -\infty$ and $\psi(1) = 0$, hence a sufficient condition that $\psi(\alpha) < 0$ for $0 < \alpha < 1$ is that it be monotone increasing there,[1] i.e., that

$$\psi'(\alpha) = \log(\alpha) + \frac{1}{\alpha} - 1 > 0 \quad \text{for} \quad 0 < \alpha < 1.$$

Now,

$$\lim_{\alpha \to 0} \psi'(\alpha) = \lim_{\alpha \to 0} \left(\frac{\frac{\log(\alpha)}{1/\alpha} + 1}{\alpha} - 1 \right) = \infty,$$

and $\lim_{\alpha \to 1} \psi'(\alpha) = 0$. Thus, a sufficient condition that $\psi'(\alpha) > 0$ for $0 < \alpha < 1$ is that $\psi''(\alpha) < 0$ for $0 < \alpha < 1$. But

$$\psi''(\alpha) = \frac{1}{\alpha} - \frac{1}{\alpha^2} = \frac{\alpha - 1}{\alpha^2} < 0 \quad \text{for} \quad 0 < \alpha < 1.$$

This proves that $f(\alpha)$ decreases from 1 to $2/e$ on the interval $0 < \alpha < 1$.

Thus, when $\alpha_1 = \alpha_2$, equilibrium tariffs make both countries worse off than they would be under free trade.

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[1] In fact, when extended to the entire positive real line, the function $f(\alpha)$ reaches an absolute minimum at $\alpha = 1$, as Figure 2 indicates. To see this, repeated application of l'Hospital's rule gives $\lim_{\alpha \to 1} f'(\alpha) = 0$. For $\alpha > 1$, $f'(\alpha) > 0$ if and only if

$$h(\alpha) \equiv (\alpha+1)\log(\alpha) - 2(\alpha - 1) > 0 \quad \text{for} \quad \alpha > 1.$$

We see that $h(1) = 0$ and

$$h(\infty) = \lim_{\alpha \to \infty} (\alpha - 1) \left(\frac{\alpha+1}{\alpha-1} \log(\alpha) + 2 \right) = \infty.$$

Thus, $h(\alpha) > 0$ for $\alpha > 1$ provided it is monotone increasing in that interval. But

$$h'(\alpha) = \log(\alpha) + \frac{1+\alpha}{\alpha} - 2 > 0 \quad \text{for} \quad \alpha > 1.$$

This proves the result.

References

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