Voluntary Export Restraints

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October 20, 2006

Let country k's trade-demand function for good i be defined by

$$z_i^k = \hat{h}_i^2(p_1^k, p_2^k, D^k; l^k) = h_i^k(p_1^k, p_2^k, \Pi^k(p_1^k, p_2^k; l^k) + D^k) - \hat{y}_i^k(p_1^k, p_2^k, l^k),$$

where p_j^k denotes the price of commodity j on country k's markets, $l^k = (l_1^k, l_2^k)$ is the vector of country k's factor endowments,

$$x_i^k = h_i^k(p_1^k, p_2^k, Y^k) \quad \text{and} \quad y_i^k = \hat{y}_i^k(p_1^k, p_2^k, l_1^k, l_2^k) = \partial\Pi^k/\partial p_i^k$$

are respectively its demand and supply (Rybczynski) functions for good i, and where in turn

$$Y^k = \Pi^k(p_1^k, p_2^k; l_1^k, l_2^k) + D^k$$

is country k's disposable national income, equal to the sum of its domestic product Π^k and its current-account deficit D^k. It will be assumed that the demand functions h_i^k are generated by a homothetic utility function. We will take commodity 1 to the numeraire and assume that it is uncontrolled by policy measures, hence $p_1^1 = p_1^2 = p_1$.

Since country 2's export of good 2 is the negative of its excess demand $z_2^2 = x_2^2 - y_2^2$ for good 2, denoting by q_2^2 the *negative* of the export quota, the assumption that this quota falls short of the amounted exported under free trade may be written

$$q_2^2 > \hat{h}_2^2(p_1, p_2, 0; l^2), \tag{1}$$

where p_j is the price of good j on the world market. Since the quota will cause an increase in the supply of good 2 on country 2's markets—and from the assumption that preferences are homothetic it follows that both goods are superior—other things being equal the domestic price p_2^2 of good 2 will necessarily fall below the world price p_2. However, the holders of the export licenses (assumed to be the members of factor 2) will have the opportunity to purchase $-q_2^2$ units of good 2 on the domestic market and sell them on the world market at the price $p_2 > p_2^2$, to make a profit of $-(p_2 - p_2^2)q_2^2 > 0$. This profit will necessarily counteract the effect of the export quota on lowering the domestic price of good 2. We must show that it will only partially do so, that is, that the trade-demand for commodity 2 will satisfy

$$\hat{h}_2^2(p_1, p_2^2, -(p_2 - p_2^2)q_2^2; l^2) = q_2^2 \tag{2}$$

for some $p_2 < p_2^2$. From the implicit-function theorem, existence of a solution to (2) requires that the Jacobian satisfy

$$\frac{\partial \hat{h}_2^2}{\partial D^2}(p_2^2 - p_2) - 1 \neq 0. \tag{3}$$

In fact, we shall require (3) to be negative.

(a) Assuming the price $p_1 = p_1^2 = p_1^2$ of commodity 1 to be fixed (as numéraire), country 2's factor endowments to be fixed, and (3) to be satisfied, equation (2) implicitly defines the function

$$p_2^2 = \hat{p}_2^2(p_2, q_2^2). \tag{4}$$

Substituting (4) in (2) and differentiating the result with respect to q_2^2, we obtain

$$\left(\frac{\partial \hat{h}_2^2}{\partial p_2^2} + \frac{\partial \hat{h}_2^2}{\partial D^2} q_2^2\right)\frac{\partial \hat{p}_2^2}{\partial q_2^2} + (p_2^2 - p_2)\frac{\partial \hat{h}_2^2}{\partial D^2} = 1. \tag{5}$$

Note that the term on the left in parentheses is the own trade-Slutsky term $\hat{s}_{22}^2 < 0$. Thus we may write (5) as

$$\hat{s}_{22}^2 \frac{\partial \hat{p}_2^2}{\partial q_2^2} = 1 - \left(1 - \frac{p_2}{p_2^2}\right) p_2^2 \frac{\partial \hat{h}_2^2}{\partial D^2} = 1 - \left(1 - \frac{1}{T_2^2}\right)\hat{m}_2^2, \tag{6}$$

where T_2^2 is the implicit export-tax factor (from $p_2^2 = T_2^2 p_2$) and $\hat{m}_2^2$ is country 2's marginal trade-propensity to consume its export good, which by our homotheticity assumption is positive and less than 1. Likewise, $1 - 1/T_2^2$ lies in the interval $[0, 1)$. Consequently, the expression (6) is positive, hence $\partial \hat{p}_2^2 / \partial q_2^2 < 0$, verifying that with the world price p_2^2 constant, a tightening of the export quota results in a fall in the domestic price p_2^2.[1]

In similar fashion, substituting (4) in (2) and differentiating the result with respect to q_2^2, we obtain

$$\left(\frac{\partial \hat{h}_2^2}{\partial p_2^2} + \frac{\partial \hat{h}_2^2}{\partial D^2} q_2^2\right)\frac{\partial \hat{p}_2^2}{\partial p_2} = \frac{\partial \hat{h}_2^2}{\partial D^2} q_2^2, \tag{7}$$

so that

$$\frac{\partial \hat{p}_2^2}{\partial p_2} = \frac{q_2^2}{\hat{s}_{22}^2}\frac{\partial \hat{h}_2^2}{\partial D^2} > 0. \tag{8}$$

Thus, the function (4) is well defined, and satisfies $\partial \hat{p}_2^2 / \partial q_2^2 < 0$ and $\partial \hat{p}_2^2 / \partial p_2 > 0$.

(b) The equation of world equilibrium is

$$\hat{h}_2^1(p_1, p_2, 0; l^1) + q_2^2 = 0. \tag{9}$$

[1] A tightening of the quota means a fall in $|q_2^2|$ and thus a rise in q_2^2,

With p_1 and l^1 constant, and $\partial \hat{h}_2^1/\partial p_2 < 0$ from our assumptions, this implicitly defines the function $p_2 = \bar{p}_2(q_2^2)$. Substituting this function in (9) and differentiating the resulting composed function with respect to q_2^2, we obtain

$$\text{(10)} \qquad \frac{\partial \hat{h}_2^1}{\partial p_2}\frac{\partial \bar{p}_2}{\partial q_2^2} + 1 = 0, \quad \text{or} \quad \frac{d\bar{p}_2}{dq_2^2} = -1 \Big/ \frac{\partial \hat{h}_2^1}{\partial p_2}.$$

Since $\partial \hat{h}_2^1/\partial p_2 < 0$ by the law of demand, $d\bar{p}_2/dq_2^2 > 0$, i.e., a tightening of country 2's export quota improves country 2's (and worsens country 1's) terms of trade.

Stability of the equilibrium (9) may be analyzed as follows, according to the "tâtonnement" process. Dynamic stability is defined by the condition that

$$\text{(11)} \qquad \frac{dp_2}{dt} \propto \bar{p}_2(q_2^2) - p_2,$$

where the symbol $\propto$ means "is proportional to" but could be interpreted more broadly to mean "is a sign-preserving function of". What (11) asserts is that if the world price of commodity 2 is below its equilibrium price, it must rise; and that if it is above its equilibrium value, it must fall. This is simply the definition of dynamic stability of the tâtonnement (price-adjustment) process. It may be combined with the empirical (*ad hoc*) assumption

$$\text{(12)} \qquad \frac{dp_2}{dt} \propto \hat{h}_2^1(p_1, p_2, D^1; l^1) + q_2^2,$$

which states that if the world excess demand for commodity 2 is positive (resp. negative), the price of commodity 2 will rise (resp. fall). (Theoretical justification of this assumption would require a model of speculative behavior.) Combining (12) with (11) we of course obtain (since the relation $\propto$ is transitive)

$$\text{(13)} \qquad \hat{h}_2^1(p_1, p_2, D^1; l^1) + q_2^2 \propto \bar{p}_2(q_2^2) - p_2$$

for given p_1, D^1, l^1. Now reverting to the narrower interpretation of $\propto$, the condition (13) implies that $\hat{h}_2^1$ is a decreasing function of p_2. From the assumed differentiability of $\hat{h}_2^1$ it follows that $\partial \hat{h}_2^1/\partial p_2 < 0$, hence from (10) it follows that $d\bar{p}_2/dq_2^2 > 0$, i.e., that the export quota must improve country 2's terms of trade (recall that q_2^2 is negative, so a rise in q_2^2 means that a smaller amount $-q_2^2$ is allowed to be exported.) Of course, this result also ensues from the original assumption of homothetic preferences in country 1.

Expressing the second formula of (10) as an elasticity we have from (9) at the equilibrium point

$$\text{(14)} \qquad \frac{q_2^2}{p_2}\frac{d\bar{p}_2}{dq_2^2} = -1 \Big/ \frac{p_2}{q_2^2}\frac{\partial \hat{h}_2^1}{\partial p_2} = 1 \Big/ \frac{p_2}{\hat{h}_2^1}\frac{\partial \hat{h}_2^1}{\partial p_2} = -\frac{1}{\eta^1},$$

where η^k is country k's elasticity of demand for imports (see "Notes on the Theory of Tariffs", formula (3.3)).

(c) By the Stolper-Samuelson theorem, in country 1 labor necessarily gains and land necesarily loses; in country 2, the rent of land necessarily rises and the wage rate of labor necessarily falls, and the question left to consider is whether and under what conditions this fall in wages will be offset by the income from the export licenses granted to labor.

Denoting by w_i^k the rental of factor i in country k, and letting $\hat{w}_i^2(p_1^2, p_2^2)$ denote the Stolper-Samuelson function for factor i in country 2, from (4) the domestic price of good 2 and the rental of factor 2 are given by

$$\bar{\bar{p}}_2^2(q_2^2) = \hat{p}_2^2(\bar{p}_2(q_2^2), q_2^2) \quad \text{and} \quad \bar{w}_2^2(q_2^2) = \hat{w}_2^2(\bar{p}_1, \bar{\bar{p}}_2^2(q_2^2)), \tag{15}$$

the world-equilibrium price of good 2 having been defined implicitly from (9). Consequently, factor 2 (labor) in country 2 receives an income of

$$\bar{Y}_2^2(q_2^2) = \bar{w}_2^2(q_2^2) l_2^2 - [\bar{p}_2(q_2^2) - \bar{\bar{p}}_2^2(q_2^2)] q_2^2. \tag{16}$$

Differentiating (16) with respect to q_2^2 and evaluating the result at $p_2^2 = p_2$ we obtain

$$\left.\frac{d\bar{Y}_2^2}{dq_2^2}\right|_{p_2^2=p_2} = \frac{d\bar{\bar{p}}_2^2}{dq_2^2}\left[\frac{\partial \hat{w}_2^2}{\partial p_2^2} l_2^2 - q_2^2\left(\frac{d\bar{p}_2/dq_2^2}{d\bar{\bar{p}}_2^2/dq_2^2} - 1\right)\right]. \tag{17}$$

Factor 2's *real income*, as a function of the quota, is

$$\frac{\bar{Y}_2^2(q_2^2)}{\bar{\bar{p}}_2^2(q_2^2)} = \frac{\bar{w}_2^2(q_2^2)}{\bar{\bar{p}}_2^2(q_2^2)} l_2^2 - \left(\frac{\bar{p}_2(q_2^2)}{\bar{\bar{p}}_2^2(q_2^2)} - 1\right) q_2^2. \tag{18}$$

Correspondingly, the effect of a small export quota on factor 2's real income is given by

$$\frac{d}{q_2^2}\left.\left(\frac{\bar{Y}_2^2(q_2^2)}{\bar{\bar{p}}_2^2(q_2^2)}\right)\right|_{p_2^2=p_2} = (p_2^2)^{-2}\frac{d\bar{\bar{p}}_2^2}{dq_2^2}\left[\frac{\partial \hat{w}_2^2}{\partial p_2^2} p_2^2 l_2^2 - q_2^2\left(\frac{d\bar{p}_2/dq_2^2}{d\bar{\bar{p}}_2^2/dq_2^2} - 1\right) - Y_2^2\right]. \tag{19}$$

Consequently, factor 2's real income improves as a result of imposition of a small export quota if and only if

$$Y_2^2 + q_2^2\left(\frac{d\bar{p}_2/dq_2^2}{d\bar{\bar{p}}_2^2/dq_2^2} - 1\right) > p_2^2\frac{\partial \hat{w}_2^2}{\partial p_2^2} l_2^2. \tag{20}$$

Both terms on the left are positive. If l_2^2 is sufficiently small, it is possible for factor 2's real income to improve even without compensation from factor 1.

References

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