Notes on the Theory of Quotas

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1 Aggregable preferences and terms of trade

We suppose that there are two factors of production, each with aggregable but different preferences. Assume that commodities and factors are so labelled that the production of commodity 1 uses a relatively higher ratio of factor 1 to factor 2 than the production of commodity 2, and that country 1 exports commodity 1 and imports commodity 2. We examine the consequences on country 1's terms of trade and potential welfare of the imposition by country 1 of a quota on its import of commodity 2 from country 2. Superscripts refer to countries; in particular, p_j^k denotes the price of commodity j on country k's markets; commodity 1 will be taken as numéraire, and unrestricted, hence $p_1^1 = p_1^2$.

Imposition by country 1 of a quota of q_2 on its import of commodity 2 leads to the following excess-demand function for commodity 2:

$$(1.1) \qquad \hat{z}_2^1(p_1^2, p_2^2, q_2; l^1) = \min\{\hat{h}_2^1(p_1^2, p_2^2, 0; l^1), q_2\},$$

where $\hat{h}_2^1(p_1^1, p_2^1, D^1; l^1)$ is country 1's trade-demand function for commodity 2. The question naturally arises: if $q_2 < \hat{h}_2^1(p_1^2, p_2^2, 0; l^1)$, how is this reduced excess demand brought about in a market economy?

If $q_2 < \hat{h}_2^1(p_1^2, p_2^2, 0; l^1)$, holders of import licenses can import commodity 2 at the price p_2^2 and sell it on the home market at a price $p_2^1 > p_2^2$, making a profit of $(p_2^1 - p_2^2)q_2$; accordingly, the trade-demand for commodity 2 must satisfy

$$(1.2) \qquad \hat{h}_2^1\big(p_1^1, p_2^1, (p_2^1 - p_2^2)z_2^1; l^1\big) = z_2^1.$$

If $q_2 \geq \hat{h}_2^1(p_1^2, p_2^2, 0; l^1)$, then $p_2^1 = p_2^2$ and (1.2) is satisfied automatically. If on the other hand $q_2 < \hat{h}_2^1(p_1^2, p_2^2, 0; l^1)$, then setting $z_2^1 = q_2$ in (1.2) defines implicitly the function
$$(1.3) \qquad p_2^1 = \hat{p}_2^1(p_2^2, q_2)$$

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(the arguments p_1^2, l^1 being suppressed since these are held constant). Substituting (1.3) in (1.2) and differentiating with respect to p_2^2, we obtain

$$(1.4) \qquad \left[\frac{\partial \hat{h}_2^1}{\partial p_2^1} + \frac{\partial \hat{h}_2^1}{\partial D^1} q_2\right] \frac{\partial \hat{p}_2^1}{\partial p_2^2} = q_2 \frac{\partial \hat{h}_2^1}{\partial D^1} \quad \text{hence} \quad \frac{\partial \hat{p}_2^1}{\partial p_2^2} = \frac{q_2 \partial \hat{h}_2^1 / \partial D^1}{\hat{s}_{22}^1},$$

the bracketed expression in (1.4) being the own-trade-Slutsky term since (1.2) must be satisfied for $z_2^1 = q_2$. Thus, if there is an exogenous fall in the world price of commodity 2, leading to more profits to owners of import licenses in country 1 and therefore greater demand for imports (if commodity 2 is a trade-normal good), this can only be choked off by a higher domestic price of commodity 2. The two prices must therefore move in opposite directions—a situation that cannot occur with tariffs.

Likewise, substituting (1.3) in (1.2) and differentiating with respect to q_2, we obtain

$$(1.5) \qquad \left[\frac{\partial \hat{h}_2^1}{\partial p_2^1} + \frac{\partial \hat{h}_2^1}{\partial D^1} q_2\right] \frac{\partial \hat{p}_2^1}{\partial q_2} = 1 - (p_2^1 - p_2^2) \frac{\partial \hat{h}_2^1}{\partial D^1} = 1 - \left(1 - \frac{1}{T_2}\right) \hat{m}_2^1$$

where $T_2 = p_2^1 / p_2^2$ is the implicit tariff factor and $\hat{m}_2^1$ is country 1's marginal trade-propensity to import commodity 2. As long as this propensity is between 0 and 1, since $0 \leq 1 - 1/T_2 < 1$ it follows that the term on the right in (1.5) is positive. Thus, if world prices are given, a tightening of the quota (a fall in q_2) will lead to a rise in the domestic price p_2^1.

Now we must look at world equilibrium. If the quota is binding, the equation for world equilibrium is simply

$$(1.6) \qquad q_2 + \hat{h}_2^2(p_1^2, p_2^2, 0; l^2) = 0.$$

Holding p_1^2 and l^2 constant, this equation implictly defines the function $p_2^2 = \bar{p}_2^2(q_2)$. Substituting this in (1.6) and differentiating with respect to q_2 we obtain

$$1 + \frac{\partial \hat{h}_2^2}{\partial p_2^2} \frac{d\bar{p}_2^2}{dq_2} = 0, \quad \text{or} \quad \frac{d\bar{p}_2^2}{dq_2} = -\frac{1}{\dfrac{\partial \hat{h}_2^2}{\partial p_2^2}}.$$

Expressing this as an elasticity, we have

$$(1.7) \qquad \frac{q_2}{p_2^2} \frac{d\bar{p}_2^2}{dq_2} = -\frac{1}{\dfrac{p_2^2}{\hat{h}_2^2} \dfrac{\partial \hat{h}_2^2}{\partial p_2^2}} = \frac{1}{\eta^2 - 1},$$

the last equation following from "Notes on the Theory of Tariffs," equation (3.10).

This result may be compared with equation (4.3) of "Notes on the Theory of Tariffs." Since by definition an increase in the quota (as long as it remains binding) leads to an equal increase in imports, the numerator is 1 instead of $-\zeta^1$; and since the

quota paralyzes country 1's demand for imports, its elasticity of demand for imports, η^1, is zero, hence the denominator is $\eta^2 - 1$ instead of $\eta^1 + \eta^2 - 1$. But since under a tariff, $\eta^1 > 0$, the stability condition $\eta^2 > 1$ is now much more stringent. As long as it holds, a tightening of the quota leads to a fall in the world price of commodity 2, i.e., an improvement in country 1's terms of trade. But the meaning of the more stringent stability condition will now be discussed.

2 Marshallian offer curves

As long as Giffen's paradox does not hold with respect to trade-demand functions, i.e., as long as $\partial \hat{h}_2^1/\partial p_2^1 < 0$ and $\partial \hat{h}_1^2/\partial p_1^2 < 0$, we may define the countries' inverse trade-demand functions, as follows. Let country 1's inverse trade-demand function $\hat{r}_2(z_2^1)$ be defined implicitly by

$$(2.1) \qquad\qquad \hat{h}_2^1(1, \hat{r}_2(z_2^1), 0; l^1) = z_2^1.$$

Given an amount z_2^1 of commodity 2 imported into country 1, $\hat{r}_2^1(z_2^1)$ is that world relative price ratio p_2^2/p_1^2 that will induce country 1 to import z_2^1. Now from the balance-of-trade identity

$$p_1^2 z_1^1 + p_2^2 z_2^1 = 0, \quad \text{or} \quad z_1^1 + \hat{r}_2^1(z_2^1)z_2^1 = 0,$$

we may define country 1's Marshallian offer function as the quantity of its exports expressed as a function of the quantity of its imports, as follows:

$$(2.2) \qquad\qquad - z_1^1 = \hat{r}_2^1(z_2^1)z_2^1 \equiv F^1(z_2^1).$$

Similarly for country 2, we define its inverse trade-demand function $\hat{r}_1(z_1^2)$ implicitly by

$$(2.3) \qquad\qquad \hat{h}_1^2(\hat{r}_1(z_1^2), 1, 0; l^2) = z_1^2.$$

Given an amount z_1^2 of commodity 1 imported into country 2, $\hat{r}_1^2(z_1^2)$ is that world relative price ratio p_1^2/p_2^2 that will induce country 2 to import z_1^2. From the balance-of-trade identity

$$p_1^2 z_1^2 + p_2^2 z_2^1 = 0, \quad \text{or} \quad \hat{r}_1^2(z_1^2)z_1^2 + z_2^2 = 0,$$

we may define country 2's Marshallian offer function as the quantity of its exports expressed as a function of the quantity of its imports, as follows:

$$(2.4) \qquad\qquad - z_2^2 = \hat{r}_1^2(z_1^2)z_1^2 \equiv F^2(z_1^2).$$

From the material-balance conditions

$$z_1^1 + z_1^2 = 0 \quad \text{and} \quad z_2^1 + z_2^2 = 0$$

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we then obtain the conditions for world equilibrium:

$$(2.5) \qquad\qquad F^1(z_2^1) = z_1^2 \quad \text{and} \quad F^2(z_1^2) = z_2^1.$$

Let us now define the elasticities of these functions, or *elasticities of trade* in Alexander's (1951) terminology;

$$(2.6) \qquad\qquad \alpha^1 = \frac{z_2^1}{F^1}\frac{dF^1}{dz_2^1}, \quad \alpha^2 = \frac{z_1^2}{F^2}\frac{dF^2}{dz_1^2}.$$

Let us relate these elasticities to the Marshallian elasticities of demand for imports defined by (3.3) in the "Notes on the Theory of Tariffs." For country 2, differentiating (2.3) with respect to z_1^2 we obtain

$$\frac{\partial \hat{h}_1^2}{\partial p_1^2}\frac{d\hat{r}_1}{dz_1^2} = 1 \quad \text{or} \quad \frac{d\hat{r}_1}{dz_1^2} = \frac{1}{\dfrac{\partial \hat{h}_1^2}{\partial p_1^2}},$$

hence, in terms of elasticities,

$$(2.7) \qquad\qquad \frac{z_1^2}{r_1}\frac{d\hat{r}_1}{dz_1^2} = \frac{1}{\dfrac{r_1}{z_1^2}\dfrac{\partial \hat{h}_1^2}{\partial p_1^2}} = \frac{1}{\dfrac{p_1^2}{\hat{h}_1^2}\dfrac{\partial \hat{h}_1^2}{\partial p_1^2}} = -\frac{1}{\eta^2}.$$

Consequently,

$$(2.8) \qquad \alpha^2 = \frac{z_1^2}{F^2}\frac{dF^2}{dz_1^2} = \frac{z_1^2}{\hat{r}_1(z_1^2)z_1^2}\left(\hat{r}_1(z_1^2) + z_1^2\frac{d\hat{r}_1}{dz_1^2}\right) = 1 + \frac{z_1^2}{r_1}\frac{d\hat{r}_1}{dz_1^2} = 1 - \frac{1}{\eta^2}.$$

Thus,

$$(2.9) \qquad\qquad \eta^2 = \frac{1}{1-\alpha^2} = \frac{1}{1 - \dfrac{z_1^2}{F^2}\dfrac{dF^2}{dz_1^2}} = \frac{z_2^1}{z_2^1 - z_1^2\dfrac{dF^2}{dz_1^2}}.$$

Country 2's Offer Curve and Geometrical
Determination of Its Elasticity

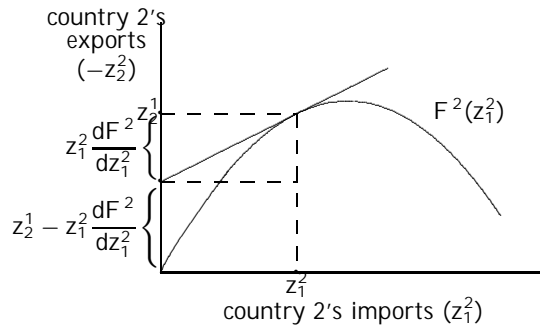country 2's exports $(-z_2^2)$

country 2's imports (z_1^2)

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Figure 1

Figure 1 dispays an offer curve $z_2^1 = F^2(z_1^2)$ for country 2. At the point (z_1^2, z_2^1), the amount $z_1^2 dF^2/dz_1^2$ is indicated on the y-axis, as is the amount $z_2^1 - z_1^2 dF^2/dz_1^2$. Since the numerator in the last term of (2.9), z_2^1, exceeds the denominator, $z_2^1 - z_1^2 dF^2/dz_1^2$, $\eta^2 > 1$. However, it is easy to see that where $F^2(z_1^2)$ attains its maximum, i.e., $dF^2/dz_1^2 = 0$, we have $\eta^2 = 1$; and to the right of this maximum point, $\eta^2 < 1$. (Cf. Edgeworth, 1894.)

3   Comparison of a tariff and a quota

Let us apply the above concepts to country 1's tariff-modified and quota-modified excess-demand functions for its import good, commodity 2. These are defined respectively by equation (1.2) of "Notes on the Theory of Tariffs" and equation (1.1) above.

Starting with the tariff case, let the inverse trade-demand function $\hat{r}(z_2^1, T_2)$ be defined implicitly by

(3.1) $$\hat{z}_2^1\left(1, \hat{r}_2(z_2^1, T_2), T_2; l^1\right) = z_2^1.$$

This is possible if the Jacobian $\partial \hat{z}_2^1/\partial p_2 \neq 0$. We shall assume that

(3.2) $$\frac{\partial \hat{z}_2^1}{\partial p_2^2} < 0.$$

Differentiating (3.1) with respect to T_2 we obtain

(3.3) $$\frac{\partial \hat{r}_2}{\partial T_2} = -\frac{\partial \hat{z}_2^1/\partial T_2}{\partial \hat{z}_2^1/\partial p_2^2}.$$

The numerator is negative by equation (1.8) of "Notes on the Theory of Tariffs," and the denominator is negative by assumption (3.2) above, hence (3.3) is negative. Thus,

(3.4) $$\frac{\partial F^1}{\partial T_2} = -z_2^1 \frac{\partial \hat{z}_2^1/\partial T_2}{\partial \hat{z}_2^1/\partial p_2^2} < 0.$$

This means that country 1's tariff-modified offer curve will shift leftward (see Figure 2).
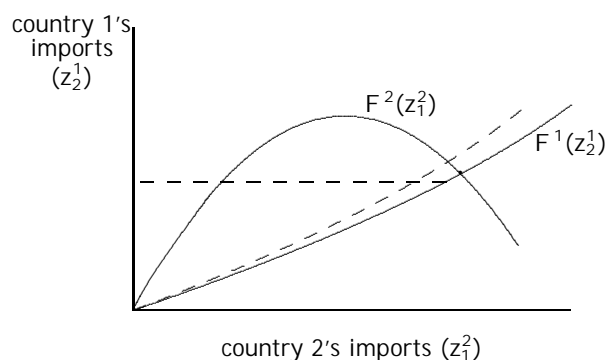
Figure 2

Now consider the case of a quota. This is also illustrated in Figure 2, which shows that country 1's quota-modified offer function cannot be defined analytically as above since it is no longer single-valued; but it may be considered as defined parametrically, as a vector-valued function of the price ratio. Figure 2 illustrates a case in which the initial equilibrium occurs at a point where $\eta^2 < 1$; a quota less than the quantity of imports at the initial equilibrium then leads to a catastrophic shift to a new equilibrium at the left, entailing a catastrophic decline in exports; this follows because the initial equilibrium is no longer stable.

We may therefore conclude: for every quota there is an equivalent tariff; howver, the converse is not true unless $\eta^2 > 1$ at the initial equilibrium. (Cf. Falvey, 1975).

References

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