Notes on the Theory of Tariffs

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1 The effect of a tariff on the terms of trade

We suppose that there are two countries trading two commodities, country 1 exporting commodity 1 to country 2, and that country 1 imposes a tariff on its import of commodity 2 while country 2 remains passive and does not retaliate. We assume that preferences in country 1 are identical and homothetic. A classic theorem, going back to Mill (1844), Torrens (1844), and Bickerdike (1907), states that under these conditions, starting from free trade the imposition of a tariff by country 1 will improve its terms of trade and therefore its (potential) welfare.

Denote country k's trade-demand function for commodity 2 by

$$z_2^k = \hat{h}_2^k(p_1^k, p_2^k, D^k; l^k),$$

where z_j^k is country k's net trade (import if positive, export if negative) in commodity j, and where p_j^k is the price of commodity j on country k's markets, D^k is the deficit in country k's balance of trade expressed in terms of its domestic prices, and l^k is the vector of country k's factor endowments. Let τ_2 be the ad valorem tariff rate imposed by country 1 on its import of commodity 2, and $T_2 = 1 + \tau_2$ the corresponding tariff factor. Let p_j denote the price of commodity j on the world market. Then country 1's prices are related to world prices by

$$p_1^1 = p_1; \quad p_2^1 = T_2 p_2. \tag{1.1}$$

Since country 2 is assumed not to retaliate, $p_j^2 = p_j$ for $j = 1, 2$.

Country 1's excess demand for its import good, as a function of the world prices and the tariff factor, is defined implicitly by

$$\hat{z}_2^1(p_1, p_2, T_2; l^1) = \hat{h}_2^1\Big(p_1, T_2 p_2, (T_2 - 1) p_2 \hat{z}_2^1(p_1, p_2, T_2; l^1)\Big). \tag{1.2}$$

Here, $(T_2 - 1) p_2 z_2^1$ consists of the tariff revenues collected by country 1's government, which we may assume to be distributed to consumers in lump-sum fashion. Since the tariff revenues represent an excess of consumption over production at domestic prices, they constitute country 1's trade deficit expressed in domestic prices. We may note, however, that the budget (balance-of-trade) equation

$$p_1 z_1^1 + T_2 p_2 z_2^1 = (T_2 - 1) p_2 z_2^1 = D^1$$

immediately implies

$$p_1 z_1^1 + p_2 z_2^1 = 0,$$

i.e., that country 1's trade is balanced when expressed in world prices.

We may complement (1.2) by defining country 2's excess demand for commodity 2 as a function of the world prices and the tariff factor by

$$\hat{z}_2^2(p_1, p_2, T_2; l^2) = \hat{h}_2^2(p_1, p_2, 0; l^2), \tag{1.3}$$

where of course $\partial \hat{z}_2^2 / \partial T_2 = 0$. The condition for world equilibrium is then

$$\hat{z}_2^1(p_1, p_2, T_2; l^1) + \hat{z}_2^2(p_1, p_2, T_2; l^2) = 0. \tag{1.4}$$

Since the trade-demand functions are homogeneous of degree 0 in the prices, we may fix $p_1 = \bar{p}_1$, choosing commodity 1 as numéraire. Fixing the endowments vectors l^1, l^2 as well, equation (1.4) implicitly defines p_2 as a function of T_2, which we shall denote $\bar{p}_2(T_2)$. Inserting it in (1.4) and then differentiating with respect to T_2, we obtain

$$\frac{d\bar{p}_2}{dT_2} = -\frac{\partial \hat{z}_2^1 / \partial T_2}{\partial \hat{z}_2^1 / \partial p_2 + \partial \hat{z}_2^2 / \partial p_2}. \tag{1.5}$$

To determine the sign of $d\bar{p}_2 / dT_2$ we need to determine the signs of both numerator and denominator.

Starting with the denominator, the time-honored procedure pioneered by Edgeworth (1908) and developed explicitly by Samuelson (1967) as the *method of comparative statics*, is to assume that the world equilibrium is *stable*. Let us replace (1.4) by a dynamic system of the Walrasian *tâtonnement* type:

$$\dot{p}_2 \equiv \frac{dp_2}{dt} = \varphi\Big(\hat{z}_2^1(\bar{p}_1, p_2, T_2; l^1) + \hat{z}_2^2(\bar{p}_1, p_2, T_2; l^2)\Big), \tag{1.6}$$

where φ is any sign-preserving function; then p_2 rises if the excess demand is positive and falls if the excess demand is negative. If equilibrium is to be *stable*, then when $p_2 < \bar{p}_2(T_2)$ we require p_2 to rise towards $\bar{p}_2(T_2)$, i.e., we want $\dot{p}_2 = dp_2/dt$ to be positive; but since (1.6) states that $\dot{p}_2 = dp_2/dt$ has the same sign as the world excess demand for commodity 2 (the argument of φ), the world excess demand for commodity 2 must be positive. Likewise, when $p_2 > \bar{p}_2(T_2)$ we require p_2 to fall towards $\bar{p}_2(T_2)$, i.e., we want $\dot{p}_2 = dp_2/dt$ to be negative; but then the world excess demand for commodity 2 must be negative. Thus, starting from the equilibrium price

$\bar{p}_2(T_2)$ where world excess demand is zero, if p_2 rises above $\bar{p}_2(T_2)$ the world excess demand must become negative, so that p_2 will fall; likewise, if p_2 falls below $\bar{p}_2(T_2)$ the world excess demand must become positive, so that p_2 will rise. What this states is that in the neighborhood of $\bar{p}_2(T_2)$, *the world excess demand for commodity 2 must be a monotone decreasing function of* p_2. Hence

$$\frac{\partial \hat{z}^1_2}{\partial p_2} + \frac{\partial \hat{z}^2_2}{\partial p_2} < 0 \tag{1.7}$$

if the equilibrium is to be stable—as we may assume, since an unstable equilibrium is unlikely ever to be observed. In section 3 below we shall show that (1.7) is equivalent to the so-called Marshall-Lerner condition.

Thus, the sign of $d\bar{p}_2/dT_2$ in (1.5) must be the same as the sign of $\partial \hat{z}^2_2/\partial T_2$. This illustrates another important principle of comparative statics (Samuelson's "correspondence principle"): to determine whether a tariff will lead to a fall in the world price of the import good, it is necessary and sufficient to determined whether, *supposing the world price of country 1's import good to be held constant*, the tariff will lead to a fall in the demand for the import good.

Let us obtain the expression for $\partial \hat{z}^1_2/\partial T_2$. Differentiating (1.2) with respect to T_2 and collecting terms we obtain

$$\frac{\partial \hat{z}^1_2}{\partial T_2} = \frac{p^1_2 \hat{s}^1_{22}/T_2}{1-(1-1/T_2)\hat{m}^1_2} \tag{1.8}$$

where

$$\hat{s}^k_{ij} = \frac{\partial \hat{h}^k_i}{\partial p^k_j} + \frac{\partial \hat{h}^k_i}{\partial D^k}\hat{h}^k_j \quad \text{and} \quad \hat{m}^k_i = p^k_i \frac{\partial \hat{h}^k_i}{\partial D^k}. \tag{1.9}$$

If both commodities are trade-normal, i.e., $\hat{m}^1_j \geq 0$ for $j = 1, 2$, it follows from $\hat{m}^1_1 + \hat{m}^1_2 = 1$ that $0 \leq \hat{m}^1_2 \leq 1$, and since $T_2 \geq 1$, it follows that $0 \leq 1 - 1/T_2 < 1$, hence $0 \leq (1 - 1/T_2)\hat{m}^1_2 < 1$; therefore $0 < 1 - (1 - 1/T_2)\hat{m}^1_2 \leq 1$. Thus the denominator of (1.8) is positive. Since the own trade-Slutsky term $\hat{s}^1_{22}$ is necessarily negative, it follows that $\partial \hat{z}^1_2/\partial T_2$ is unambiguously negative. From (1.5) it follows that $dp_2/dT_2 < 0$, i.e., the tariff will improve country 1's terms of trade.

2 Lerner's symmetry theorem and Keynes's equivalence theorem

Lerner's (1936) symmetry theorem states that exactly the same effect can be produced by an ad valorem export tax of $\tau = \tau^1_1$ (levied on the domestic price of the export good) as can be obtained by an equal percentage import tariff of $\tau = \tau_2$ (levied on the foreign price of the import good).

First, it is easily seen that in the case of an import tariff of τ_2 (and corresponding tariff factor of $T_2 = 1 + \tau_2$), the equations (1.1) yield the relation

$$\frac{p_2^1}{p_1^1} = T_2 \frac{p_2}{p_1} \tag{2.1}$$

between the domestic and external price ratios between commodities 2 and 1. Now, suppose that instead, an export tax is imposed on commodity 1, so that exporters are charged with a tax of τ_1^1 levied on the *domestic* price p_1^1. Denote the corresponding tax factor by $T_1^1 = 1 + \tau_1^1$. The relation between the domestic and world prices is then given by

$$p_1 = T_1^1 p_1^1 \quad \text{and} \quad p_2 = p_2^1. \tag{2.2}$$

The relation between the domestic and world price ratios is then

$$\frac{p_2^1}{p_1^1} = T_1^1 \frac{p_2}{p_1}. \tag{2.3}$$

Thus, if $T_1^1 = T_2 = T$, the wedges between the price ratios are the same.

Let us now confirm that the revenues from the export tax are exactly the same as those from an import tariff of the same height. In the case of an export tax, country 1's excess demand for its exportable is defined implicitly by

$$\hat{z}_1^1(p_1, p_2, T_1^1; l^1) = \hat{h}_1^1\Big(p_1/T_1^1, p_2, -(1 - 1/T_1^1)p_1 \hat{z}_1^1(p_1, p_2, T_1^1; l^1)\Big) \tag{2.4}$$

(the minus sign is needed in the deficit term since $z_1^1 < 0$; this deficit term is of course the revenue from the export tax). We need to show that this yields the same excess-demand function for commodity 2 as was defined by (1.2), when $T_1^1 = T_2$. Now, trade-demand functions satisfy the homogeneity property $\hat{h}_j(\lambda p_1, \lambda p_2, \lambda D; l) = \hat{h}_j(p_1, p_2, D; l)$; and balanced trade (in international prices) implies $p_1 z_1^1 = -p_2 z_2^1$; hence (2.4) may be written

$$\hat{z}_1^1(p_1, p_2, T_1^1; l^1) = \hat{h}_1^1\Big(p_1, T_1^1 p_2, (T_1^1 - 1)p_1 \hat{z}_2^1(p_1, p_2, T_1^1; l^1)\Big). \tag{2.5}$$

Consequently, using the fact that country 1's trade-demand function satisfies its budget eqation

$$p_1^1 \hat{h}_1^1(p_1^1, p_2^1, D^1; l^1) + p_2^1 \hat{h}_2^1(p_1^1, p_2^1, D^1; l^1) = D^1, \tag{2.6}$$

we have

$$\begin{aligned}
\hat{z}_2^1(p_1, p_2, T_1^1; l^1) &= -\frac{p_1}{p_2} \hat{z}_1^1(p_1, p_2, T_1^1; l^1) \\
&= -\frac{p_1}{p_2} \hat{h}_1^1\Big(p_1, T_1^1 p_2, (T_1^1 - 1)p_1 \hat{z}_2^1(p_1, p_2, T_1^1; l^1)\Big) \\
&= -\frac{p_1}{p_2} \cdot \frac{(T_1^1 - 1)p_2 \hat{z}_2^1(p_1, p_2, T_1^1; l^1)}{p_1} \\
&\quad + \frac{p_1}{p_2} \cdot \frac{T_1^1 p_2}{p_1} \hat{h}_2^1\Big(p_1, T_1^1 p_2, (T_1^1 - 1)p_1 \hat{z}_2^1(p_1, p_2, T_1^1; l^1)\Big),
\end{aligned} \tag{2.7}$$

hence, cancelling terms and bringing the first term on the right of the last equation of (2.7) over to the left we see that

$$T_1^1 \hat{z}_2^1(p_1, p_2, T_1^1; l^1) = T_1^1 \hat{h}_2^1\Big(p_1, T_1^1 p_2, (T_1^1 - 1)p_1 \hat{z}_2^1(p_1, p_2, T_1^1; l^1)\Big),$$

hence cancelling T_1^1 from both sides we obtain (1.2) for $T_1^1 = T_2$.

Lerner's symmetry theorem can be greatly generalized; in fact, Lerner himself (1944, p. 384n) introduced an interesting generalization. Independently, another generalization was introduced by Keynes (1931, Addendum I, p. 199, ¶34), which we shall now consider.

Lerner's 1936 theorem considered only a comparison of a pair $(T_1^1, 1)$ of tax factors with another pair $(1, T_2)$. What can be said of a *combination* of two tax factors (T_1^1, T_2)? Notice that in the Lerner symmetry theorem, the export tax is reckoned on the country-1 price of the export good as a base, while the import tariff is reckoned on the world price as a base. A more symmetric procedure would be to reckon both taxes on the world price as a base. Then, in place of (2.2) we would have

$$p_1^1 = T_1 p_1 \quad \text{and} \quad p_2^1 = p_2, \tag{2.8}$$

where $T_1^1 = (T_1)^{-1}$. The theorem would then state that the pair $(T_1, 1)$ consisting of an export-tax factor of T_1 (reckoned on the world price of commodity 1 as a base) and an import-tariff factor of 1 (zero tariff), is equivalent to the pair $(1, T_2)$ consisting of an export-tax factor of 1 (zero export tax) and an import-tariff factor of T_2 (also reckoned on the world price of commodity 2 as a base), *if and only if* $T_1 T_2 = 1$. This suggests that any pair (T_1, T_2) is equivalent to any other pair (T_1', T_2') provided $T_1 T_2' = T_1' T_2$. This in fact is true. It is easy to see that in the respective cases,

$$\frac{p_2^1}{p_1^1} = \frac{T_2}{T_1}\frac{p_2}{p_1} \quad \text{and} \quad \frac{p_2^1}{p_1^1} = \frac{T_2'}{T_1'}\frac{p_2}{p_1}$$

requiring $T_2/T_1 = T_2'/T_1'$ for the wedges to be the same. Keynes's equivalence theorem (1931) corresponds to the special case $(T_1', T_2') = (1, 1)$; in such a case, a tariff factor $T_2 > 1$ must be accompanied by an export-*subsidy* factor $T_1 = T_2 > 1$ reckoned on the world price of commodity 1 as a base, which would be equivalent to an export-subsidy factor $T_1^1 = 1/T_1 < 1$ reckoned on the domestic price of commodity 1 as a base.

3 Derivation of the "Marshall-Lerner condition"

It will be found convenient to state (1.5) in terms of elasticities, as follows:

$$\frac{T_2}{p_2}\frac{d\bar{p}_2}{dT_2} = -\frac{\dfrac{T_2}{z_2^1}\dfrac{\partial \hat{z}_2^1}{\partial T_2}}{\dfrac{p_2}{z_2^1}\dfrac{\partial \hat{z}_2^1}{\partial p_2} - \dfrac{p_2}{z_2^2}\dfrac{\partial \hat{z}_2^2}{\partial p_2}} \tag{3.1}$$

where we make use of the equilibrium condition $z_2^1 + z_2^2 = 0$. The stability condition (1.7) may then be written

$$\frac{p_2}{z_2^1}\frac{\partial \hat{z}_2^1}{\partial p_2} - \frac{p_2}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_2} < 0. \tag{3.2}$$

The Marshallian elasticities of demand for imports of the two countries are defined as

$$\eta^1 = -\frac{p_2}{z_2^1}\frac{\partial \hat{z}_2^1}{\partial p_2} \quad \text{and} \quad \eta^2 = -\frac{p_1}{z_1^2}\frac{\partial \hat{z}_1^2}{\partial p_1} \tag{3.3}$$

respectively. Thus, the first term in (3.2) is simply $-\eta^1$. Let us show that the second term is equal to $\eta^2 - 1$, so that the stability condition (1.7) is equivalent to the well-known so-called "Marshall-Lerner condition"

$$\Delta \equiv \eta^1 + \eta^2 - 1 > 0. \tag{3.4}$$

From the fact that the function $\hat{z}_2^2(p_1, p_2)$ is homogeneous of degree zero it follows by Euler's theorem that

$$\frac{\partial \hat{z}_2^2}{\partial p_1}p_1 + \frac{\partial \hat{z}_2^2}{\partial p_2}p_2 = 0,$$

hence, dividing through by z_2^2 we have

$$\frac{p_1}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_1} + \frac{p_2}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_2} = 0,$$

i.e., the cross-elasticities sum to zero; thus,

$$\frac{p_2}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_2} = -\frac{p_1}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_1}. \tag{3.5}$$

Now country 2's excess-demand functions must satisfy the balance-of-trade identity

$$p_1\hat{z}_1^2(p_1, p_2, D^1, l^1) + p_2\hat{z}_2^2(p_1, p_2, D^1, l^1) = 0. \tag{3.6}$$

Differentiating (3.6) with respect to p_1 we obtain

$$p_1\frac{\partial \hat{z}_1^2}{\partial p_1} + p_2\frac{\partial \hat{z}_1^2}{\partial p_1} + z_1^2 = 0.$$

Dividing this through by z_1^2 we obtain

$$\frac{p_1}{z_1^2}\frac{\partial \hat{z}_1^2}{\partial p_1} + \frac{p_2}{z_1^2}\frac{\partial \hat{z}_2^2}{\partial p_1} + 1 = 0. \tag{3.7}$$

Now rewriting the balance-of-trade condition (3.6) as

$$\frac{p_2}{z_1^2} = -\frac{p_1}{z_2^2},$$

and substituting this expression in the second term of (3.7), we obtain

$$\frac{p_1}{z_1^2}\frac{\partial \hat{z}_1^2}{\partial p_1} - \frac{p_1}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_1} + 1 = 0. \tag{3.8}$$

Combining (3.8) with (3.5) we obtain

$$\frac{p_1}{z_1^2}\frac{\partial \hat{z}_1^2}{\partial p_1} + \frac{p_2}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_2} + 1 = 0. \tag{3.9}$$

Thus,

$$-\frac{p_2}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_2} = 1 + \frac{p_1}{z_1^2}\frac{\partial \hat{z}_1^2}{\partial p_1} = 1 - \eta^2. \tag{3.10}$$

It follows from this and (3.3) that the inequality (3.2) is equivalent to

$$\frac{p_2}{z_2^1}\frac{\partial \hat{z}_2^1}{\partial p_2} - \frac{p_2}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_2} = -\eta^1 - \eta^2 + 1 < 0,$$

yielding (3.4).

4 The Metzler paradox—I

Defining

$$\zeta^1 = -\frac{T_2}{z_2^1}\frac{\partial \hat{z}_2^1}{\partial T_2}, \tag{4.1}$$

from (1.8) this evaluates to

$$\zeta^1 = \frac{-p_2^1 \hat{s}_{22}^1 / z_2^1}{1 - (1 - 1/T_2)\hat{m}_2^1}. \tag{4.2}$$

Using the result of the last section, we may write the above equation (3.1) as

$$\pi_2 \equiv \frac{T_2}{p_2}\frac{d\bar{p}_2}{dT_2} = \frac{-\zeta^1}{\eta^1 + \eta^2 - 1}. \tag{4.3}$$

Now consider the question of the effect of the tariff on the *domestic* price $\bar{p}_2^1(T_2) = T_2\bar{p}_2(T_2)$ of country 1's import good (commodity 2). From (1.1) we have $dp_2^1/dT_2 = \bar{p}_2(T_2) + T_2 d\bar{p}_2(T_2)/dT_2$; we may express this in elasticity form as

$$\pi_2^1 \equiv \frac{T_2}{p_2^1}\frac{d\bar{p}_2^1}{dT_2} = 1 + \frac{T_2}{p_2}\frac{d\bar{p}_2}{dT_2} = 1 + \pi_2, \tag{4.4}$$

or

$$\pi_2^1 = 1 - \frac{\zeta^1}{\eta^1 + \eta^2 - 1} = \frac{\Delta - \zeta^1}{\Delta} = \frac{\eta^1 - \zeta^1 + \eta^2 - 1}{\Delta} \tag{4.5}$$

where Δ is defined by (3.4). The "Metzler paradox" (Metzler, 1949) occurs when $\pi^1_2 < 0$, i.e., when the tariff improves the terms of trade so much that it actually *lowers* the domestic price of the import good (relative to that of the export good). Equivalently, it occurs when

$$\Delta = \eta^1 + \eta^2 - 1 < \zeta^1, \quad \text{or} \quad \eta^1 - \zeta^1 < 1 - \eta^2. \tag{4.6}$$

ζ^1 has already been evaluated by (4.2) above; let us evaluate η^1. Differentiating (1.2) with respect to p_2 we obtain

$$\frac{\partial \hat{z}^1_2}{\partial p_2} = \frac{T_2 \hat{s}^1_{22} - z^1_2 \partial \hat{h}^1_2 / \partial D^1}{1 - (1 - 1/T_2)\hat{m}^1_2}, \tag{4.7}$$

so that, from (3.3),

$$\eta^1 = \frac{\hat{m}^1_2 / T_2 - p^1_2 \hat{s}^1_{22} / z^1_2}{1 - (1 - 1/T_2)\hat{m}^1_2}. \tag{4.8}$$

From (4.8) and (4.2) we obtain

$$\eta^1 - \zeta^1 = \frac{\hat{m}^1_2 / T_2}{1 - (1 - 1/T_2)\hat{m}^1_2} = \frac{\hat{m}^1_2}{1 + \tau_2(1 - \hat{m}^1_2)} \equiv \hat{m}^{1\prime}_2. \tag{4.9}$$

Thus, from (4.4) and (4.5) we have

$$\frac{T_2}{p^1_2} \frac{d\bar{p}^1_2}{dT_2} = \frac{\hat{m}^{1\prime}_2 + \eta^2 - 1}{\eta^1 + \eta^2 - 1}. \tag{4.10}$$

The Metzler paradox therefore occurs if and only if

$$\frac{\hat{m}^1_2}{1 + \tau_2(1 - \hat{m}^1_2)} < 1 - \eta^2 \tag{4.11}$$

(cf. Metzler, 1949b). In the special case in which country 1 starts from a zero tariff this reduces to Metzler's (1949a) original simple formula

$$\hat{m}^1_2 + \eta^2 < 1. \tag{4.12}$$

5 The Metzler paradox—II

The results of the preceding section can also be obtained via another route, which also provides additional intuitive understanding.

Let us define both countries' excess-demand functions in terms of country-1 prices rather than world prices as arguments. Country 1's excess demand for its importable good is then defined implictly by

$$\tilde{z}^1_2(p^1_1, p^1_2, T_2; l^1) = \hat{h}^1_2\Big(p^1_1, p^1_2, (1 - 1/T_2)p^1_2 \tilde{z}^1_2(p^1_1, p^1_2, T_2; l^1)\Big). \tag{5.1}$$

Country 2's excess demand for its exportable is defined by

$$\tilde{z}^2_2(p^1_1, p^1_2, T_2; l^2) = \hat{h}^2_2\Big(p^1_1, p^1_2/T_2, 0; l^2\Big). \tag{5.2}$$

We note that the tariff factor T_2 enters country 1's excess-demand function only via the deficit term, while it enters country 2's excess-demand function only via the price term. The conditions for world equilibrium are exactly the same as in formula (1.4), with tildes replacing hats; however, (1.5) is replaced by

$$\frac{d\bar{p}^1_2}{dT_2} = -\frac{\partial\tilde{z}^1_2/\partial T_2 + \partial\tilde{z}^2_2/\partial T_2}{\partial\tilde{z}^1_2/\partial p^1_2 + \partial\tilde{z}^2_2/\partial p^1_2}. \tag{5.3}$$

The denominator if (5.3) is negative by stability, as before. It remains to evaluate the two expressions in the numerator.

Differentiating (5.1) with respect to T_2 and collecting terms we obtain

$$\frac{\partial\tilde{z}^1_2}{\partial T_2} = \frac{\hat{m}^1_2 z^1_2/(T_2)^2}{1-(1-1/T_2)\hat{m}^1_2} \tag{5.4}$$

so that the corresponding elasticity is

$$\frac{T_2}{z^1_2}\frac{\partial\tilde{z}^1_2}{\partial T_2} = \frac{\hat{m}^1_2/T_2}{1-(1-1/T_2)\hat{m}^1_2} = \frac{\hat{m}^1_2}{1+\tau_2(1-\hat{m}^1_2)}, \tag{5.5}$$

a formula that is in complete agreement with (4.9). Differentiating (5.2) with respect to T_2 we obtain

$$\frac{\partial\tilde{z}^2_2}{\partial T_2} = -\frac{p^2_2}{T_2}\frac{\partial\hat{h}^2_2}{\partial p^2_2} \tag{5.6}$$

so that the corresponding elasticity is

$$\frac{T_2}{z^2_2}\frac{\partial\tilde{z}^2_2}{\partial T_2} = -\frac{p^2_2}{z^2_2}\frac{\partial\hat{h}^2_2}{\partial p^2_2} = 1-\eta^2 \tag{5.7}$$

from (3.10). Summing (5.5) and (5.6) we obtain (4.11) once again.

6 The Metzler paradox—III

A third interpretation of the Metzler paradox is possible. The term $\eta^2 - 1$ in (4.10) may be decomposed using formula (3.10). We perform the Slutsky decomposition

$$\frac{\partial\hat{z}^2_2}{\partial p_2} = \frac{\partial\hat{h}^2_2}{\partial p^2_2} = \hat{s}^2_{22} - \frac{\partial\hat{h}^2_2}{\partial D^2}\hat{h}^2_2, \tag{6.1}$$

hence, from (3.10),

$$\eta^2 - 1 = \frac{p_2}{z_2^2}\frac{\partial \hat{z}_2^2}{\partial p_2} = \hat{\sigma}_{22}^2 - \hat{m}_{22}^2, \tag{6.2}$$

where $\hat{\sigma}_{22}^2 = p_2\hat{s}_{22}^2/z_2^2$ is the Hicks-Slutsky own-trade elasticity for commodity 2 in country 2. Since $\hat{s}_{22}^2 < 0$ and $z_2^2 < 0$, $\hat{\sigma}_{22}^2 > 0$. Formula (4.10) may therefore be written as

$$\frac{T_2}{p_2^1}\frac{d\bar{p}_2^1}{dT_2} = \frac{\hat{m}_2^{1\prime} - \hat{m}_2^2 + \hat{\sigma}_{22}^2}{\eta^1 + \eta^2 - 1}. \tag{6.3}$$

We see therefore that a *necessary* condition for the Metzler paradox is that $\hat{m}_2^2 > \hat{m}_2^{1\prime}$. In the case in trade is initially free, $\hat{m}_2^{1\prime}$ reduces to $\hat{m}_2^1$ and this becomes the condition that a transfer from country 2 to country 1 should improve country 1's terms of trade (which is the "orthodox presumption").

We may interpret this in the following way. Suppose that instead of country 1 imposing a tariff of T_2 on its imports (reckoned on the world or country-2 price as a base), country 2 imposes an export tax of T_2 on its exports (also reckoned on the country-2 price as a base). The only difference between this situation and the previous one is that country 2 now collects the tariff revenues instead of country 1. But we know from Lerner's symmetry theorem that an export tax imposed by country 2 is equivalent to a tariff imposed by country 2 on its imports; therefore, this export tax will improve country 2's terms of trade. If commodity 1 is chosen as numéraire, this means that the world and therefore domestic price of commodity 2 in country 1 will fall. Now suppose that country 2 transfers the revenues from its export tax back to country 1; then, in the "orthodox" case, country 1's terms of trade, having initially deteriorated, now improve; so the domestic price of its import good, having initially risen, will now fall—possibly enough to counterbalance the initial rise. A sufficient condition to *rule out* the Metzler paradox is therefore that a transfer should have the "anti-orthodox" effect of worsening the receiving country's terms of trade; then both the initial export tax of country 2 and the subsequent transfer of the tax revenues back to country 1 have the same effect, of worsening country 1's terms of trade and therefore of increasing the domestic price of country 1's import good.

All this can be treated explicitly in terms of a model that deals with the combined taxes and transfers. For details see Chipman (1990).

7 The "optimal tariff"

The indirect trade-utility function of country 1 is defined by

$$\hat{V}^1(p_1^1, p_2^1, D^1; l^1) = \hat{U}\Big(\hat{h}_1^1(p_1^1, p_2^1, D^1; l^1), \hat{h}_2^1(p_1^1, p_2^1, D^1; l^1)\Big).$$

Accordingly, we may define country 1's potential welfare as a function of the tariff factor by

$$W^1(T_2) = \hat{V}^1\Big(\bar{p}_1, T_2\bar{p}_2^1(T_2), (T_2 - 1)\bar{p}_2(T_2)\bar{z}_2^1(T_2); l^1\Big) \tag{7.1}$$

where

$$\bar{z}_2^1(T_2) = \hat{z}_2^1\Big(\bar{p}_1, \bar{p}_2(T_2), T_2; l^1\Big). \tag{7.2}$$

Differentiating (7.1) with respect to T_2 while making use of Antonelli's partial differential equation

$$\frac{\partial \hat{V}^1}{\partial p_2^1} = -\frac{\partial \hat{V}^1}{\partial D^1}\hat{h}_2^1, \tag{7.3}$$

we obtain

$$\begin{aligned} \frac{dW^1}{dT_2} &= \frac{\partial \hat{V}^1}{\partial p_2^1}\left[\bar{p}_2 + T_2\frac{d\bar{p}_2}{dT_2}\right] + \frac{\partial \hat{V}^1}{\partial D^1}\left[\bar{p}_2\bar{z}_2^1 + (T_2 - 1)\frac{d\bar{p}_2}{dT_2}\bar{z}_2^1 + (T_2 - 1)\bar{p}_2\frac{d\bar{z}_2^1}{dT_2}\right] \\ &= \frac{\partial \hat{V}^1}{\partial D^1}\left[-\bar{z}_2^1\frac{d\bar{p}_2}{dT_2} + \tau_2\bar{p}_2\frac{d\bar{z}_2^1}{dT_2}\right]. \qquad (7.4) \end{aligned}$$

From local nonsatiation of trade-preferences it follows that $\partial\hat{V}^1/\partial D^1 > 0$, hence the sign of dW^1/dT_2 is the same as that of the bracketed term in (7.4). From this we may obtain two results (Bickerdike, 1907): (1) Bickerdike's first theorem, which states that starting from a situation of free trade ($\tau_2 = 0$), a small tariff will improve a country's potential welfare, given the fact established above—and by Bickerdike—that $d\bar{p}_2/dT_2 < 0$; (2) Bickerdike's second theorem, which states that there is an optimal tariff, i.e., a τ_2 such that $dW^1/dT_2 = 0$ and $d^2W^1/d(T_2)^2 < 0$—which is true provided $d\bar{z}_2^1/dT_2 < 0$, as will be shown.

Now, multiplying the bracketed expression in (7.4) through by $T_2/\bar{p}_2\bar{z}_2^1$ and equating it to zero we obtain the equation for the optimal tariff in elasticity form:

$$\frac{T_2}{z_2^1}\frac{d\bar{z}_2^1}{dT_2}\tau_2 - \frac{T_2}{p_2}\frac{d\bar{p}_2}{dT_2} = 0. \tag{7.5}$$

From (7.2) we have

$$\frac{d\bar{z}_2^1}{dT_2} = \frac{\partial \hat{z}_2^1}{\partial p_2}\frac{d\bar{p}_2}{\partial dT_2} + \frac{\partial \hat{z}_2^1}{\partial T_2},$$

hence, in terms of elasticities,

$$\frac{T_2}{z_2^1}\frac{d\bar{z}_2^1}{dT_2} = \frac{p_2}{z_2^1}\frac{\partial \hat{z}_2^1}{\partial p_2}\cdot\frac{T_2}{p_2}\frac{d\bar{p}_2}{dT_2} + \frac{T_2}{z_2^1}\frac{\partial \hat{z}_2^1}{\partial T_2}. \tag{7.6}$$

Now we have already found from (3.1), (4.1), (3.4), and (4.4), that

$$\pi_2 = \frac{T_2}{p_2}\frac{d\bar{p}_2}{dT_2} = -\frac{\zeta^1}{\Delta}, \tag{7.7}$$

hence (7.6) may be written as

$$-\frac{T_2}{z_2^1}\frac{d\bar{z}_2^1}{dT_2} = -\eta^1\frac{\zeta^1}{\Delta} + \zeta^1 = \frac{\zeta^1}{\Delta}(\Delta - \eta^1) = \frac{\zeta^1}{\Delta}(\eta^2 - 1). \tag{7.8}$$

Thus, (7.5) may be written

$$\frac{\zeta^1}{\Delta}(\eta^2 - 1)\tau_2 + \frac{\zeta^1}{\Delta} = \frac{\zeta^1}{\Delta}[(\eta^2 - 1)\tau_2 + 1] = 0, \tag{7.9}$$

hence as long as $\zeta^1/\Delta \neq 0$,

$$\tau_2 = \frac{1}{\eta^2 - 1}, \tag{7.10}$$

which is the formula for the optimal tariff first obtained by Johnson (1950).

8 Heterogeneous preferences and terms of trade

We suppose that there are two factors of production, each with aggregable but different preferences. Assume that commodities and factors are so labelled that the production of commodity 1 uses a relatively higher ratio of factor 1 to factor 2 than the production of commodity 2, and that country 1 exports commodity 1 and imports commodity 2. We examine the consequences on country 1's terms of trade and on the welfares of the two factors of the imposition by country 1 of a tariff on its import of commodity 2 from country 2. It is assumed that the government distributes fixed fractions δ_1 and δ_2 ($\delta_i \geq 0, \delta_1 + \delta_2 = 1$) of its tariff revenues to the two factor owners in lump-sum fashion, hence factor i's income is $Y^1_i = w^1_i l^1_i + \delta_i \tau_2 p_2 z^1_2$.

The demand by factor i for commodity 2 in country 1 is given by

$$x^1_{i2} = h^1_{i2}\Big(p_1, T_2p_2, l_i \cdot \hat{w}^1_i(p_1, T_2p_2, l^1_1, l^1_2) + \delta_i(T_2 - 1)p_2[x^1_{12} + x^1_{22} - \hat{y}^1_2(p_1, T_2p_2, l^1_1, l^1_2)]\Big) \tag{8.1}$$

where $\hat{w}^1_i$ is the Stolper-Samuelson function $\partial\Pi^1/\partial l^1_i$.

Defining aggregate consumption of commodity j by $x^1_j = x^1_{1j} + x^1_{2j}$, the aggregate consumption of commodity 2 as a function of the world prices, tariff factor, distributive shares, and factor endowments, $\hat{x}^1_2(p_1, p_2, T_2; \delta, l^1)$—where δ and l^1 denote the vectors (δ_1, δ_2) and (l^1_1, l^1_2) respectively—is defined implicitly by the equation

$$\hat{x}^1_2(\cdot) = \sum_{i=1}^{2} h^1_{i2}\Big(p_1, T_2p_2, l^1_i \cdot \hat{w}^1_i(p_1, T_2p_2, l^1) + \delta_i(T_2 - 1)p_2[\hat{x}^1_2(\cdot) - \hat{y}^1_2(p_1, T_2p_2; l^1)]\Big). \tag{8.2}$$

Country 1's excess demand for commodity 2 is then defined by

$$\hat{z}^1_2(p_1, p_2, T_2; \delta, l^1) = \hat{x}^1_2(p_1, p_2, T_2; \delta, l^1) - \hat{y}^1_2(p_1, T_2p_2, l^1). \tag{8.3}$$

World equilibrium is defined by (1.4) as before. The sign of $d\bar{p}_2/dT_2$ is then, as before, determined by the sign of $\partial\hat{z}^1_2/\partial T_2$. This we now compute.

Differentiating (8.2) with respect to T_2 we obtain, upon collecting terms,

$$\begin{aligned} M\frac{\partial \hat{x}^1_2}{\partial T_2} &= p_2 \sum_{i=1}^{2} \left\{ \frac{\partial h^1_{i2}}{\partial p^1_2} + \frac{\partial h^1_{i2}}{\partial Y^1_i} \left[l^1_i \frac{\hat{w}^1_i}{\partial p^1_2} + \delta_i (x^1_2 - y^1_2) \right] \right. \\ &\quad \left. - \left(1 - \frac{1}{T_2}\right) \frac{\partial \hat{y}^1_2}{\partial p^1_2} \delta_i p^1_2 \frac{\partial h^1_{i2}}{\partial Y^1_i} \right\} \\ &= p_2 \sum_{i=1}^{2} \left\{ \frac{\partial h^1_{i2}}{\partial p^1_2} + \frac{\partial h^1_{i2}}{\partial Y^1_i} x^1_{i2} + \frac{\partial h^1_{i2}}{\partial Y^1_i} \left[l^1_i \frac{\partial \hat{w}^1_i}{\partial p^1_2} + \delta_i z^1_2 - x^1_{i2} \right] \right. \\ &\quad \left. - \left(1 - \frac{1}{T_2}\right) \frac{\partial \hat{y}^1_2}{\partial p^1_2} \delta_i p^1_2 \frac{\partial h^1_{i2}}{\partial Y^1_i} \right\} \\ &= p_2 \sum_{i=1}^{2} \left\{ s^1_{i,22} + \frac{\partial h^1_{i2}}{\partial Y^1_i} \left[l^1_i \frac{\partial \hat{w}^1_i}{\partial p^1_2} + \delta_i z^1_2 - x^1_{i2} \right] \right. \\ &\quad \left. - \left(1 - \frac{1}{T_2}\right) \delta_i m^1_{i2} t^1_{22} \right\}, \end{aligned} \tag{8.4}$$

where

$$M = 1 - \left(1 - \frac{1}{T_2}\right) \sum_{i=1}^{2} \delta_i m^1_{i2} \tag{8.5}$$

and

$$s^k_{i,jj'} = \frac{\partial h^k_{ij}}{\partial p^k_{j'}} + \frac{\partial h^k_{ij}}{\partial Y^k_i} h^k_{ij'}, \quad t^k_{jj'} = \frac{\partial \hat{y}^k_j}{\partial p^k_{j'}}, \quad \text{and} \quad m^k_{ij} = p^k_j \frac{\partial h^k_{ij}}{\partial Y^k_i}. \tag{8.6}$$

Likewise, differentiating the composed function $\tilde{y}^1_2(p_1, p_2, T_2, l^1) = \hat{y}^1_2(p_1, T_2 p_2, l^1)$ with respect to T_2 and multiplying by M we obtain

$$M\frac{\partial \tilde{y}^1_2}{\partial T_2} = \left[1 - \left(1 - \frac{1}{T_2}\right) \sum_{i=1}^{2} \delta_i m^1_{i2} \right] p_2 t^1_{22}. \tag{8.7}$$

Subtracting (8.7) from (8.4) we obtain

$$M\frac{\partial \hat{z}^1_2}{\partial T_2} = \frac{1}{T_2} \left\{ p^1_2 (s^1_{1,22} + s^1_{2,22} - t^1_{22}) + \sum_{i=1}^{2} m^1_{i2} \left[l^1_i \frac{\partial \hat{w}^1_i}{\partial p^1_2} + \delta_i z^1_2 - x^1_{i2} \right] \right\}. \tag{8.8}$$

It is interesting to note that the first term in this expression contains the sum of the two factors' own-commodity Slutsky terms, minus the own-transformation term, for commodity 2; this expression is unambiguously negative. It remains to determine the sign of the remaining term.

Let us denote

$$a_i = l^1_i \frac{\partial \hat{w}^1_i}{\partial p^1_2} + \delta_i z^1_2 - x^1_{i2}. \tag{8.9}$$

First let us show that $a_1 + a_2 = 0$; from this it will follow that if the preferences of the two factors are identical, i.e., $m^1_{12} = m^1_{22}$, then the second term on the right in

(8.8) vanishes, and we are back to the case of aggregable preferences in which the tariff necessarily improves country 1's terms of trade.

We have, using Samuelson's reciprocity theorem ($\partial\hat{w}^1_{\mathsf{i}}/\partial p^1_2 = \partial\hat{y}^1_2/\partial l^1_{\mathsf{i}}$) and the homogeneity of degree 1 of the Rybczynski function in the factor endowments (and Euler's theorem),

$$\begin{aligned} a_1 + a_2 &= \sum_{\mathsf{i}=1}^{2}\left(l_{\mathsf{i}}\frac{\partial\hat{w}^1_{\mathsf{i}}}{\partial p^1_2} + \delta_{\mathsf{i}} z^1_2 - x^1_{\mathsf{i}2}\right) \\ &= \sum_{\mathsf{i}=1}^{2} l_{\mathsf{i}}\frac{\partial\hat{y}^1_2}{\partial l^1_{\mathsf{i}}} + (\delta_1+\delta_2)z^1_2 - x^1_2 \\ &= y^1_2 + z^1_2 - x^1_2 = 0 \qquad (8.10) \end{aligned}$$

by the definition $z^1_2 = x^1_2 - y^1_2$.

Now let us show that $a_2 > 0$ so long as factor 2 does not spend all of its disposable income on the import good. We have, by the Stolper-Samuelson theorem $\partial\hat{w}^1_2/\partial p^1_2 > w^1_2/p^1_2$,

$$\begin{aligned} p^1_2 a_2 &= p^1_2\left(l^1_2\frac{\partial\hat{w}^1_2}{\partial p^1_2} + \delta_2 z^1_2 - x^1_{22}\right) \\ &> w^1_2 l^1_2 + \delta_2 p^1_2 z^1_2 - p^1_2 x^1_{22} \\ &> w^1_2 l^1_2 + \delta_2\frac{\tau_2}{1+\tau_2}p^1_2 z^1_2 - p^1_2 x^1_{22} \\ &= w^1_2 l^1_2 + \delta_2\tau_2 p_2 z^1_2 - p^1_2 x^1_{22} \\ &= Y^1_2 - p^1_2 x^1_{22} > 0. \qquad (8.11) \end{aligned}$$

From these two results it follows that $a_1 = -a_2 < 0$. Consequently,

$$\sum_{\mathsf{i}=1}^{2} m^1_{\mathsf{i}2} a_{\mathsf{i}} = a_2(m^1_{22} - m^1_{12}). \qquad (8.12)$$

We thus have a *sufficient* condition that the tariff will improve country 1's terms of trade: that $m^1_{12} \geq m^1_{22}$, i.e., factor 1 has at least as great a marginal propensity to consume the import good as factor 2. In this case, both terms in (8.8) are nonpositive, and the first negative. Likewise, we have as a *necessary* condition that the tariff will worsen country 1's terms of trade the condition $m^1_{22} > m^1_{12}$. The possibility that the tariff may worsen country 1's terms of trade may be called the *Johnson paradox*, after Johnson (1960). The intuitive explanation is simple: if labor (say) is the factor used relatively intensively in country 1's import-competing industry (industry 2), then since it gains and capital loses as a result of the tariff, and since workers will increase their consumption of the import good more than capitalists reduce theirs, there will be a net rise in demand for the import good. If this effect is strong enough to outweigh the substitution effect, the world demand for commodity 2 will increase and so will its price.

9 Heterogeneous preferences and factor welfares

The indirect utility function of the ith factor in country 1 is

$$V_i^1(p_1^1, p_2^1, Y_i^1), \tag{9.1}$$

hence we may define factor i's welfare as a function of the tariff factor by

$$W_i^1(T_2) = V_i^1\Big(\bar{p}_1^1, \bar{p}_2^1(T_2), \bar{Y}_i^1(T_2)\Big), \tag{9.2}$$

where $\bar{p}_2^1(T_2)$ is defined from (1.4) and $\bar{Y}_i^1(T_2)$ is defined by

$$\bar{Y}_i^1(T_2) = l_i^1 \hat{w}_i^1(\bar{p}_1, \bar{p}_2^1(T_2), l_1^1, l_2^1\big) + \delta_i(T_2 - 1)\bar{p}_2(T_2)\bar{z}_2^1(T_2), \tag{9.3}$$

where $\bar{p}_2^1(T_2) = T_2\bar{p}_2(T_2)$ and $\bar{z}_2^1(T_2)$ is defined by (7.2). We then have, using (7.3),

$$\frac{dW_i^1}{dT_2} = \frac{\partial V_i^1}{\partial Y_i^1}\left[-h_{i2}^1\frac{d\bar{p}_2^1}{dT_2} + \frac{d\bar{Y}_i^1}{dT_2}\right]. \tag{9.4}$$

Let us first analyze the income effect. We have

$$\begin{aligned} \frac{d\bar{Y}_i^1}{dT_2} &= l_i\frac{\partial \hat{w}_i^1}{\partial p_2^1}\frac{d\bar{p}_2^1}{dT_2} + \delta_i\bar{z}_2^1\left[\bar{p}_2 + (T_2 - 1)\frac{d\bar{p}_2}{dT_2}\right] + \delta_i(T_2 - 1)\bar{p}_2\frac{d\bar{z}_2^1}{dT_2} \\ &= \left(l_i\frac{\partial \hat{w}_i^1}{\partial p_2^1} + \delta_i\bar{z}_2^1\right)\frac{d\bar{p}_2^1}{dT_2} + \delta_i\left((T_2 - 1)\bar{p}_2\frac{d\bar{z}_2^2}{dT_2} - \bar{z}_2^1\frac{d\bar{p}_2}{dT_2}\right), \end{aligned} \tag{9.5}$$

where we use (4.4) in the last equation. Substituting this into (9.4) we obtain

$$\frac{dW_i^1}{dT_2} = \frac{\partial V_i^1}{\partial Y_i^1}\left(a_i\frac{d\bar{p}_2^1}{dT_2} + \delta_i\left[(T_2 - 1)\bar{p}_2\frac{d\bar{z}_2^1}{dT_2} - \bar{z}_2^1\frac{d\bar{p}_2}{dT_2}\right]\right), \tag{9.6}$$

where a_i is given by (8.9). We need finally to express $d\bar{z}_2^1/dT_2$ in terms of $d\bar{p}_2/dT_2$.

From (7.2) we have

$$\frac{d\bar{z}_2^1}{dT_2} = \frac{\partial z_2^1}{\partial p_2}\frac{d\bar{p}_2}{dT_2} + \frac{\partial \hat{z}_2^1}{\partial T_2}. \tag{9.7}$$

Now from (4.1) and (4.3) we have

$$\frac{\partial \hat{z}_2^1}{\partial T_2} = \Delta\frac{z_2^1}{p_2}\frac{d\bar{p}_2}{dT_2} \tag{9.8}$$

where $\Delta = \eta^1 + \eta^2 - 1$. Substituting this in (9.7) and recalling definition (3.3) we obtain

$$\frac{T_2}{z_2^1}\frac{d\bar{z}_2^1}{dT_2} = (-\eta^1 + \Delta)\frac{T_2}{p_2}\frac{d\bar{p}_2}{dT_2} = (\eta^2 - 1)\frac{T_2}{p_2}\frac{d\bar{p}_2}{dT_2}. \tag{9.9}$$

Thus, substituting

$$\frac{d\bar{z}_2^1}{dT_2} = (\eta^2 - 1)\frac{z_2^1}{p_2}\frac{d\bar{p}_2}{dT_2}$$

into (9.4) we obtain our final expression

$$\frac{dW_i^1}{dT_2} = \frac{\partial V_i^1}{\partial Y_i^1}\left(a_i\frac{d\bar{p}_2^1}{dT_2} - \delta_i z_2^1\Big[1 - \tau_2(\eta^2 - 1)\Big]\frac{d\bar{p}_2}{dT_2}\right). \tag{9.10}$$

Note that the expression $1 - \tau_2(\eta^2 - 1)$ is positive whenever $\eta^2 \leq 1$, i.e., the foreign demand for imports is inelastic; while if $\eta^2 > 1$ it is positive if and only if

$$\tau_2 < \frac{1}{\eta^2 - 1}, \tag{9.11}$$

i.e., if and only if the initial tariff rate is less than the optimal tariff. Assuming this to be the case, let us first suppose that $d\bar{p}_2/dT_2 < 0$ (the tariff improves the terms of trade, i.e., there is no Johnson paradox) and $d\bar{p}_2^1/dT_2 > 0$ (the tariff raises the domestic price of the protected good, i.e., there is no Metzler paradox). Then (9.10) is unambiguously positive for $i = 2$ (since $a_2 > 0$), i.e., factor 2 gains, regardless of how much of the tariff revenues it receives, whereas factor 1 gains only if the distribution of tariff revenues is enough to compensate for the fall in its factor rental. Assuming that factor 1 is compensated by all the tariff revenues, it gains if and only if

$$a_1\frac{d\bar{p}_2^1}{dT_2} - z_2^1\Big[1 - \tau_2(\eta^2 - 1)\Big]\frac{d\bar{p}_2}{dT_2} > 0. \tag{9.12}$$

Since $a_1 < 0$, a necessary condition for this is that $1 - \tau_2(\eta^2 - 1) > 0$.

This gives an interesting interpretation to the optimal tariff. Even though preferences in the present case cannot be aggregated, the concept of an optimal tariff makes sense in that if the foreign demand for imports is elastic and the initial tariff exceeds the optimal tariff, while factor 2 can still gain (and will gain if it has not been and is not to receive any tariff revenues), factor 1 necessarily loses; thus there is definitely an inherent conflict of interest between the two factors whenever $\tau_2 > 1/(\eta^2 - 1)$. But if $\tau_2 < 1/(\eta^2 - 1)$, there is always the possibility (though not the necessity) that both factors can gain from an increase in the tariff.

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