1. (20 points) Consider security markets with $I+1$ agents. All agents' preferences have expected utility representation with strictly increasing utilities and with common probabilities of states. One agent is risk neutral while the remaining I agents are strictly risk averse. The consumption set of each risk-averse agents is the positive orthant $\mathcal{R}^S_+$ (there is no consumption at date 0). The consumption set of the risk-neutral agent consists of all consumption plans $c \in \mathcal{R}^S$ that satisfy $E(c) \geq 0$. Show that consumption plans of all risk-averse agents are risk-free at any Pareto optimal consumption allocation. Do not assume that utility functions of the strictly risk averse agents are differentiable.

2. (20 points) Consider security markets with two dates and two states at date 1. There is no consumption at date 0. There are two agents: a risk-neutral agent 1, and a strictly risk-averse agent 2 with von Neumann-Morgenstern utility function $v^2(y) = \ln(y)$. Endowments at date 1 are $\omega^1 = (3, 2)$, and $\omega^2 = (5, 2)$. Probabilities of states are $\frac{1}{3}$ for state 1 and $\frac{2}{3}$ for state 2, the same for both agents. There are two securities with payoffs $x_1 = (1, 1)$ and $x_2 = (2, 1)$.
   (i) Consider consumption allocation $c^1 = (5, 1)$, $c^2 = (3, 3)$. Show that this allocation in an equilibrium allocation in security markets. Find equilibrium security prices and portfolio allocation.
   (ii) Suppose now that agents have multiple-prior expected utilities instead of expected utilities. Their vNM utility functions remain unchanged. The set of priors is $\{(\pi_1, \pi_2) : 0.3 \leq \pi_1 \leq 0.4,\ \pi_1 + \pi_2 = 1\}$ for both agents. Are the allocation and prices of (i) an equilibrium for multiple-prior utilities?