1. (10 points) Consider production set given by

$$Y = \left\{(y_1, y_2)\colon y_1 \leq 0, \quad y_2 \leq \max\{\ln(\frac{-y_1+1}{3}), 0)\right\}.$$

Find the profit function π^* and the supply function (or correspondence) s^* associated with Y. Clearly specify the sets of price vectors for which π^* and s^* are well defined. Find the range of price vectors for which π^* is differentiable.

2. (10 points) Consider production function of two inputs given by

$$f(x_1, x_2) = x_1 + \ln(x_2 + 1),$$

for $x_1 \geq 0$ and $x_2 \geq 0$.

(a) Verify whether this production function exhibits decreasing returns to scale, increasing returns to scale, or neither one.

(b) What is the range of prices $p > 0$, $w_1 > 0$, and $w_2 > 0$ for which there exist profit-maximizing input quantities? Find these quantities as functions of prices.

3. (20 points) Consider two production sets $Y_1, Y_2 \subset \Re^L$. Assume that both sets are closed and convex and such that $0 \in Y_1$ and $0 \in Y_2$. Let π_1^* and π_2^* denote the profit functions associated with Y_1 and Y_2.

(i) Prove that

$$Y_1 \supset Y_2 \quad \text{if and only if } \pi_1^*(p) \geq \pi_2^*(p) \text{ for every } p \in \Re^L.$$

[$\pi_1^*(p)$ and $\pi_2^*(p)$ can take value $+\infty$ in this inequality.]

(ii) Does the equivalence from (i) hold when sets Y_1, Y_2 are not necessarily convex? Justify your answer.

Economics 8101 **Problem Set 2**
Fall Semester 2009 **Due: Thursday, September 24**
Jan Werner

1. (10 points) Consider the following supply function of a producer

$$\psi(p_1, p_2) = \left(-\frac{2p_2}{p_1}, \frac{p_2}{p_1}\right) \quad \text{for} \quad p_1 > 0, p_2 > 0.$$

   The producer produces good 2 using good 1 as input.

   Can this supply function result from profit maximization on a production set? Justify your answer.

2. (15 points) Consider the following lexicographic preferences on the consumption set $\Re^2_+$: the value of $x_1 + x_2$ has the first priority; the value of x_2 has the second priority.

   (i) Prove that this lexicographic preference has no utility representation on $\Re^2_+$.

   (b) Does this lexicographic preference have a utility representation on the set N^2 of (ordered) pairs of natural numbers? Justify your answer.

   (c) Derive demand functions $x_i^*(p_1, p_2, w)$, $i = 1, 2$ for $p_1 > 0$, $p_2 > 0$ and $w > 0$ for this lexicographic preference on $\Re^2_+$.

3. (15 points) Consider a preference relation on $\Re^L_+$ defined by

$$x \succeq x' \quad \text{if and only if} \quad qx \geq qx' \;\; \forall q \in Q,$$

   where Q is a closed and convex subset of the unit simplex Δ in $\Re^L$.

(i) Show that $\succeq$ is complete if and only if Q consists of a single vector in Δ.

(ii) Show that $\succeq$ is continuous and strictly increasing (strongly monotone).

1. (15 points) Consider the demand function of two goods $d(p_1, p_2, M) = \left(\frac{p_2}{p_1}, \frac{M - p_2}{p_2}\right)$ for $p_1 > 0$, $p_2 > 0$ and $M > p_2$.

   (a) Is any of the goods a Giffen good or an inferior good?

   (b) Find the Slutsky matrix of d and verify whether it is negative semi-definite and symmetric. Is this demand function a Walrasian demand function (i.e., is it rationalizable by a utility function)?

2. (10 points) Show that the equality of the Hicksian and the Walrasian demands proved in class under assumptions of continuity and local non-satiation of utility function may fail to hold if one of these assumptions is violated. That is, give examples of utility function u such that

   (a) u is continuous but not locally non-satiated,

   (b) u is locally non-satiated but not-continuous,

   and the equality of the demands does not hold for some $p >> 0,\ w > 0, \bar{u} > u(0)$.

3. (15 points) Consider the following function of prices and income:

$$v(p_1, p_2, w) = \frac{(w + p_1)^2}{4p_1p_2},$$

   where w is income and p_i is price of good $i = 1, 2.$. Consider only strictly positive prices and income.

   (i) Assuming that function v is an indirect utility function of a consumer who maximizes some continuous and l.n.s utility function subject to the budget constraint, derive this consumer's expenditure function and Hicksian demand functions for goods 1 and 2. Specify domains on which these functions are positive.

   (ii) Verify whether the expenditure function of part (i) is concave and homogeneous of degree one.

Ad 1. The Slutsky matrix $S = [s_{ij}]$ of d is

$$s_{11} = -\frac{p_2}{p_2^2}, \quad s_{12} = \frac{1}{p_1}, \quad s_{21} = \frac{1}{p_1}, \quad s_{22} = \frac{1}{p_2}.$$

It obtains from deriving partial derivatives $\frac{\partial d}{\partial p_i}$ and $\frac{\partial d}{\partial w}$ and arranging them the right way to get Slutsky terms s_{ij}.

Matrix S is negative-semi definite and symmetric.

Demand function d satisfies all assumptions of Theorem 12.1 (except for not being defined for all p and M.)

Ad 2. (a) ... ask Joe for this one.

(b) this part is easy - just take a utility function that give a thick indifference "curve."

Ad 3. The expenditure function is

$$e(p, \bar{u}) = 2\sqrt{p_1 p_2 \bar{u}} - p_1.$$

It is positive as long as $4p_2\bar{u} \geq 1$. Hicksian demand obtain from $De = h$.

(ii) e is concave and hom. 0 in prices.

1. (20 points) Consider firm 1 which competes with firm 2 in the market for output, as in the Cournot duopoly model. The firm chooses output quantity $z_1 \geq 0$ taking the quantity $z_2 \geq 0$ of firm 2 as given. The profit of firm 1 is

$$P(z_1 + z_2)z_1 - C(z_1)$$

where P is the inverse demand function in the output market, that is, P assigns price to the quantity demanded at this price in the market. C is the cost function of the firm. Assume that P is strictly decreasing and C is strictly increasing. Let $z_1^*(z_2)$ denote the solution, assumed unique, to the profit-maximization problem.

(i) Show that if P is a concave function of total demand, then z_1^* is monotone non-increasing in z_2.

A full-credit solution to this question should not have an additional assumption that functions P and C be differentiable. If you choose to impose differentiability, then the maximum score on this question will be 15 points.

2. (10 points) Consider a firm with production function $f : \Re^n_+ \to \Re_+$. The firm maximizes its profit at prices $w \in \Re^n_{++}$ for inputs and $q \in \Re_{++}$ for output. In addition, the firm is taxed at rate $t \geq 0$ of its total cost. The firm's profit maximization problem is

$$\max_{x \geq 0} qf(x) - wx(1+t).$$

Show that if production function f is supermodular, then firms input demand x^* is monotone nonincreasing in tax rate t. Production function f is strictly increasing but need not be differentiable.

3. (10 points) Derive a weak axiom of cost minimization in a way analogous to the other weak axioms discussed in class and so that production choices of a cost-minimizing firm with strictly increasing production function satisfy this axiom. Then derive a "Delta property" which is a counterpart of negative semi-definiteness of the matrix $D_w x^*$ of factor demand x^* established in Corollary 9.2 for cost minimization.

1. (10 points) Consider an agent whose preferences have an expected utility representation with strictly increasing vNM utility function. The agent is strictly risk averse and has a risk-free initial wealth $w > 0$. Consider a gamble $(g, -l)$ where g has probability π and $-l$ has probability $1 - \pi$ with $g > 0, l > 0$ and $0 < \pi < 1$.

   Show that if the agent rejects gamble $(g, -l)$, then she also rejects the gamble $(tg, -tl)$ for every $t \geq 1$.

2. (15 points) Consider the following preference relation on $\mathcal{R}_+^S$ for $S \geq 2$:

   $c \succeq c'$ if and only if $\min_{\pi \in \mathcal{P}} E_\pi(c) \geq \min_{\pi \in \mathcal{P}} E_\pi(c')$ for some closed and convex set of probability vectors $\mathcal{P} \subset \Delta$.

   Show that $\succeq$ has state-separable utility representation if and only if $\mathcal{P}$ consists of a single probability vector.

3. (15 points) Suppose that there are two states, $s = 1, 2$, with equal probabilities $\frac{1}{2}$. An agent has an expected utility function $E[v(c)] = \frac{1}{2}v(c_1) + \frac{1}{2}v(c_2)$ with $v(c) = \ln(c)$, for $c > 0$.

   (i) Suppose that the agent has deterministic wealth $w > 0$ and faces risk $z = (z_1, z_2)$ such that $E(z) = 0$. Calculate risk compensation for $w = 12.5$ and $z = (-7.5, 7.5)$, that is, for risk of losing 7.5 in state 1 and winning 7.5 in state 2.

   (ii) Suppose that there are two assets: a risk-free asset with return in two states given by $\bar{r} = (1, 1)$, and a risky asset with return in two states given by $r = (2, 0.5)$. Find the optimal investment in the risky asset for the agent's initial wealth w, as a function of w. Is the optimal investment an increasing function of wealth w?