Economics 8101 **Exercises**

Fall Semester 2009, Session I

1. Consider the utility function $u(x_1, x_2, x_3) = x_1 + v(x_2, x_3)$ for some function $v : \Re^2_+ \to \Re$ with $D^2 v$ negative definite. Let $x_i^*(p_1, p_2, p_3, M)$ be the utility maximizing demand function, $i = 1, 2, 3$. Show that the income effect on consumption of commodities 2 and 3 is zero whenever the demand for all three commodities is interior. That is, show that $\frac{\partial x_i^*}{\partial M} = 0$ for $i = 2, 3$. Derive $\frac{\partial x_1^*}{\partial M}$.

2. Consider the demand function of two goods $d(p_1, p_2, w) = \left(\frac{2w - p_2}{p_1}, \frac{p_2 - w}{p_2}\right)$ for $p_1 > 0$, $p_2 > 0$ and $2w > p_2 > w$.

   (a) Is any of the goods a Giffen good or an inferior good?

   (b) Find the Slutsky matrix of d and verify whether it is negative semi-definite and symmetric. Is this demand function a Walrasian demand function (i.e., is it rationalizable by a utility function)?

3. Consider a reflexive, transitive, and complete preference relation on $\Re^n_+$. For every consumption bundle $x \in \Re^n_+$, let $\alpha_x \in \Re_+$ be a number (if it exists) such that x is indifferent to $\alpha_x \mathbf{e}$, where $\mathbf{e}$ denotes the unit vector in $\Re^n_+$.

   (i) Give an example of a preference relation that is continuous but not strictly increasing, and for which the function defined by $u(x) = \alpha_x$ is not be a utility representation. Justify your answer.

   (ii) Give an example of a preference relation that is strictly increasing but not continuous for which u as defined in (i) is not a utility representation. Justify your answer.

4. Consider preference relation on $\Re^n_+$, where $n > 1$, defined by

$$x \succeq x' \text{ if and only if } x \geq x',$$

   for every $x, x' \in \Re^n_+$.

   (i) Show that this preference relation does not have a utility representation.

(ii) Show that this preference relation is locally non-satiated.

(iii) Describe the demand at a price vector $p \in \Re^n_{++}$ and income $w > 0$, i.e., the set of preference maximizing consumption bundles in the budget set at (p, w).

5. There are two conditions often used to define continuity of preference relation $\succeq$ on consumption set $X = \Re^L_+$:

   (a) for every sequences $\{x^n\}$ and $\{y^n\}$ in X such that $\lim_n x^n = x,\ \lim_n y^n = y$, and $x^n \succeq y^n$, it holds $x \succeq y$.

   (b) For every $x \in X$, the preferred-to-x set $\{y \in X : y \succeq x\}$, and the lower contour set $\{y \in X : x \succeq y\}$ are closed.

   Assuming that $\succeq$ is transitive and complete, prove that conditions (a) and (b) are equivalent.

6. Give an example of a continuous and locally non-satiated utility function u on $\Re^L_+$ for which the equality of the Hicksian and the Walrasian demands proved in class does not hold for some price vector p which is positive but not strictly positive, and for $w > 0, \bar{u} > u(0)$.

7. Consider a firm that produces good 2 using good 1 as input. At prices p_1 for good 1 and p_2 for good 2 such that $p_2 > p_1$, the firm's profit is given by the following function

$$\pi(p_1, p_2) = \frac{p_2^2}{p_1} - p_1.$$

   Verify whether this profit function could result from the firm maximizing its profit on a production set. If your answer is positive, derive the firm's supply function of the two goods.

8. Suppose that utility function u of $\Re^L_+$ is continuous. Show that the expenditure minimization problem has a solution, i.e., $h(p, \bar{u}) \neq \emptyset$ for every $p >> 0$ and every $\bar{u}$ such that there exists $\bar{x} \in \Re^L_+$ with $u(\bar{x}) = \bar{u}$,