Economics 8101 **Exercises 2**

Fall Semester 2009, Session I

1. There are two states, $s = 1, 2$, with respective probabilities $\pi_1 = 1/4$ and $\pi_2 = 3/4$. Consider two state-contingent consumption plans: $z = (9, 1)$, and $y = (6, 2)$. Note that $E(z) = E(y)$.

   (a) Does y dominate z in the sense of the First-Order Stochastic Dominance? Justify your answer.

   (b) Is z more risky than y (that is, does y dominate z in the sense of the Second-Order Stochastic Dominance)? Justify your answer.

2. Consider an agent with expected utility $E[v(c)]$, where the von Neumann-Morgenstern utility function $v: \Re \to \Re$ is strictly increasing and continuous. Prove that the agent is risk averse if and only if $\rho(w, \tilde{y}) \geq \rho(w, \tilde{z})$ for every $w \in \Re$ and every random variables $\tilde{y}$ and $\tilde{z}$ such that $\tilde{y}$ is more risky than $\tilde{z}$ and $E(\tilde{y}) = E(\tilde{z}) = 0$.

3. Let $\tilde{y}$ and $\tilde{z}$ be arbitrary random variables on some finite state space, with $E(\tilde{z}) = 0$.

   (i) Give an example of $\tilde{y}$ and $\tilde{z}$ such that $\tilde{y} + \tilde{z}$ is **not** more risky than $\tilde{y}$.

   (ii) Show that if $\tilde{y}$ and $\tilde{z}$ are independently distributed, then $\tilde{y} + \tilde{z}$ is more risky than y.

4. Let v_1 and v_2 be two twice-differentiable von Neumann-Morgenstern utility functions. Show that v_1 is more risk averse than v_2 (i.e., $A_1(x) \geq A_2(x)\ \forall x$) if and only if, for every deterministic $w \in \mathcal{R}$ and every random variable $\tilde{z}$ (that may havbe non-zero expectation),

$$E[v_2(w + \tilde{z})] \leq v_2(w) \quad \text{implies} \quad E[v_1(w + \tilde{z})] \leq v_1(w).$$

   That is, if agent 2 rejects gamble $\tilde{z}$, then so does agent 1.

5. Consider two real-valued random variables Y and Z on a probability space. You may think about Y and Z as two contingent claims on a state space. You may assume

that the state space is finite. Suppose that Y can take only one of two possible values y_1, y_2 with respective probabilities $\pi_1 > 0$ and $\pi_2 > 0$ such that $\pi_1 + \pi_2 = 1$. Suppose further that the expectations of Z conditional on $\{Y = y_1\}$ and $\{Y = y_2\}$ are zero, that is $E[Z|Y = y_1] = 0$ and $E[Z|Y = y_2] = 0$.

(i) Prove that $Y + Z$ is more risky than Y.

6. Consider expected utility $\pi v(c_1) + (1 - \pi)v(c_2)$ defined on $(c_1, c_2) \in \Re^2$ with $0 < \pi < 1$ and twice differentiable v. Show that the slope of the indifference curve at $c_1 = c_2$ equals $-\frac{\pi}{1-\pi}$ for each $c_1 > 0$. Further, show that the second order derivative (curvature) of the indifference curve at $c_1 = c_2$ is proportional to $-\frac{v''(c_1)}{v'(c_1)}$.

7. Give and example of two von Neumann-Morgenstern utility functions v_1 and v_2 such that neither v_1 is more risk averse than v_2 nor v_2 is more risk averse than v_1.

8. Consider an agent whose preferences over risky consumption plans have an expected utility representation with strictly increasing and continuous utility function $v\colon \Re \to \Re$. Prove that the agent is risk averse if and only if $Ev(\tilde{z}) \geq Ev(\tilde{y})$ for every $\tilde{y}$ and $\tilde{z}$ such that $E(\tilde{y}) = E(\tilde{z})$ and $\tilde{y}$ is more risky than $\tilde{z}$.

9. Let $\tilde{y}$ and $\tilde{z}$ be two normally distributed random variables with the same expected value and with variances σ_y^2 and σ_z^2. Show that $\tilde{y}$ is more risky than $\tilde{z}$ if and only if $\sigma_y^2 \geq \sigma_z^2$.