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Homework #4

Due back: Beginning of class, Friday 5pm, December 11, 2009.

Questions indicated by a "star" are required for everybody who attends the class. You can use either MatLab or Fortran to do the homework.

For each question, please discuss your answer. (Please do not merely provide some numbers and a code).

1. This question asks you to solve the baseline model in Aiyagari (1994, QJE). You are going to build on the programs you wrote for the previous homework where you solved the partial equilibrium consumption-savings problem. Aiyagari embeds that problem in general equilibrium by assuming a Cobb-Douglas production function with capital and labor as inputs. The capital is supplied by households (obtained from the consumer's savings problem). Therefore, you need to clear the capital market by finding the equilibrium interest rate. As in Aiyagari assume that the idiosyncratic income process for a typical consumer follows an AR(1) process. I want you to compare two different discretization methods to convert this AR(1) into a Markov process—Tauchen's (1986) method as described in Aiyagari (1994) as well as Rouwenhurst's method (as described in his chapter in the Cooley volume ("Frontier's of Business Cycle Research"). See the appendix to Rouwenhurst's chapter for description). Do each part below using both discretization methods and a 9-state Markov process in each case. Compare your findings for each part below.

   (a) First, take the CRRA version of E-Z preferences and set risk aversion to 2. Find the average capital stock in the stationary equilibrium of this model as well as the interest rate that clear the capital market. Report your results.

   (b) Now separate RRA from EIS. Fix EIS=0.9 and vary the risk aversion. Consider RRA=2 and 20. What happens to the capital stock and interest rate when risk aversion rises?

   (c) Now fix the RRA=2. Vary the EIS from 2 to 0.1. What happens to the capital stock and interest rate? Do you see a clear difference between the effects of the two parameters on the interest rate? Notice that with CRRA preferences you could not identify which parameter is affecting the interest rate (and capital stock) since they vary together.

2. *Krusell-Smith (1998): This question adds aggregate shocks to Aiyagari's model. Let's simplify the problem by assuming the same aggregate and idiosyncratic shock process assumed in K-S. See the paper for details. Assume log utility and no borrowing. Implement the basic K-S algorithm to solve the model. Report how long it takes to solve the model with a convergence criteria that is based on attaining $R^2 = 0.99999$ in the predictive regression: $logK' = \alpha_0 + \alpha_1 logK$.

(a) *Checking accuracy: Calculate the R^2 and regression residual variance of the predictive regression of the interest rate 25 years ahead? Report the two-standard deviation bands of this prediction of the interest rate. Also calculate the one-step ahead R^2 of the regression: $logK' - logK = \alpha_0 + \alpha_1 logK$.

(b) *Plot the "essential accuracy plot" of Den Haan as discussed in class. Are you satisfied that your solution is accurate?

(c) *Calculate the Gini coefficient for income, wealth, and consumption inequality in the stationary equilibrium. (Obviously the Gini will vary depending on aggregate state. Take the average.) How do they rank with respect to each other?

(d) *Solve the model for increasing values of the persistence of the idiosyncratic shock: 0.8, 0.9, 0.5 and 0.995. How do the dispersion measures you computed in part (a) change with persistence?

(e) *What fraction of the population are at their constraints in each parameterization in part (d)? As you make the shocks more persistent do you get more people up against the constraint? Give an economic interpretation of your finding.

(f) *Now fix persistence at 0.9 and increase the risk aversion to 5. What happens to the Gini measures? What fraction is constrained now?

Econ 8312. Computational Methods
Homework 4. Iskander Karibzhanov

Problem 1. Part (a)

In CRRA version of Aiyagari model, I solved for decision rules using policy function iteration with endogenous grid method since it is much faster than value function iteration. In parts (b) and (c) however, the PFI method no longer can be employed since value function enters Euler equation and I had to resort to VFI.

To find stationary wealth distribution I implemented CDF iteration algorithm as described in Rios Rull chapter in Marimon book. I also implemented PDF iteration algorithm from Chapter 7 of Maussner DGE modeling book but the resulting density was more jagged than with CDF method. I didn't do Monte Carlo to compute stationary distribution because I think there is no need to spend too much computing time if I can do the same thing with CDF in less than a second. I also wrote the routines for computing Lorenz curves and Gini coefficients to replicate all tables and figures in Aiyagari 94 working paper.

As in Aiyagari 94, I assumed that the idiosyncratic income process for a typical consumer follows an AR(1) process

$$y_t = \rho y_{t-1} + \sigma\sqrt{1-\rho^2}\varepsilon_t, \quad \varepsilon_t \sim \text{Normal}(0,1)$$

$$\sigma \in \{0.2, \quad 0.4\}, \qquad \rho = \{0.0, \quad 0.3, \quad 0.6, \quad 0.9\}$$

Other parameters are the same as in the paper: $\beta = 0.96$, $\alpha = 0.36$, $\delta = 0.08$. No borrowing.

Using Tauchen and Rouwenhorst methods to approximate the above AR(1) process, I obtained following results by setting relative risk aversion to 2 and using 202 grid points to compute policy functions and 1010 grid points to compute stationary distribution. As we can see both approximation methods produce almost same results.

Table 1. Net Return to Capital in %/Aggregate Capital using CDF iteration

Using Rouwenhorst method

$\sigma\backslash\rho$	0.0	0.3	0.6	0.9
0.2	4.1215 / 5.4786	4.0856 / 5.5040	4.0088 / 5.5591	3.8085 / 5.7071
0.4	3.9516 / 5.6007	3.7850 / 5.7250	3.4641 / 5.9773	2.8698 / 6.4957

Using calibrated Tauchen method

$\sigma\backslash\rho$	0.0	0.3	0.6	0.9
0.2	4.1208 / 5.4790	4.0844 / 5.5049	4.0075 / 5.5601	3.8049 / 5.7098
0.4	3.9479 / 5.6034	3.7786 / 5.7298	3.4567 / 5.9833	2.8525 / 6.5119

It turns out that some policy functions do not cross the 45 degree line. This is not a problem for endogenous grid method, but can be dangerous for value function iteration method. So I set maximum asset level to 70 because in stationary equilibrium no agent holds assets above that level.

I changed Tauchen method by calibrating the grid spread to minimize the squared percentage deviations in $\ln(1-\rho)$ and σ implied by the Markov chain. I noticed that Aiyagari did not use the Tauchen method properly. Instead of varying the spread of the grid, he fixed it to three standard deviations. If he instead used my method, the approximation would be much better as it can be seen from the table below:

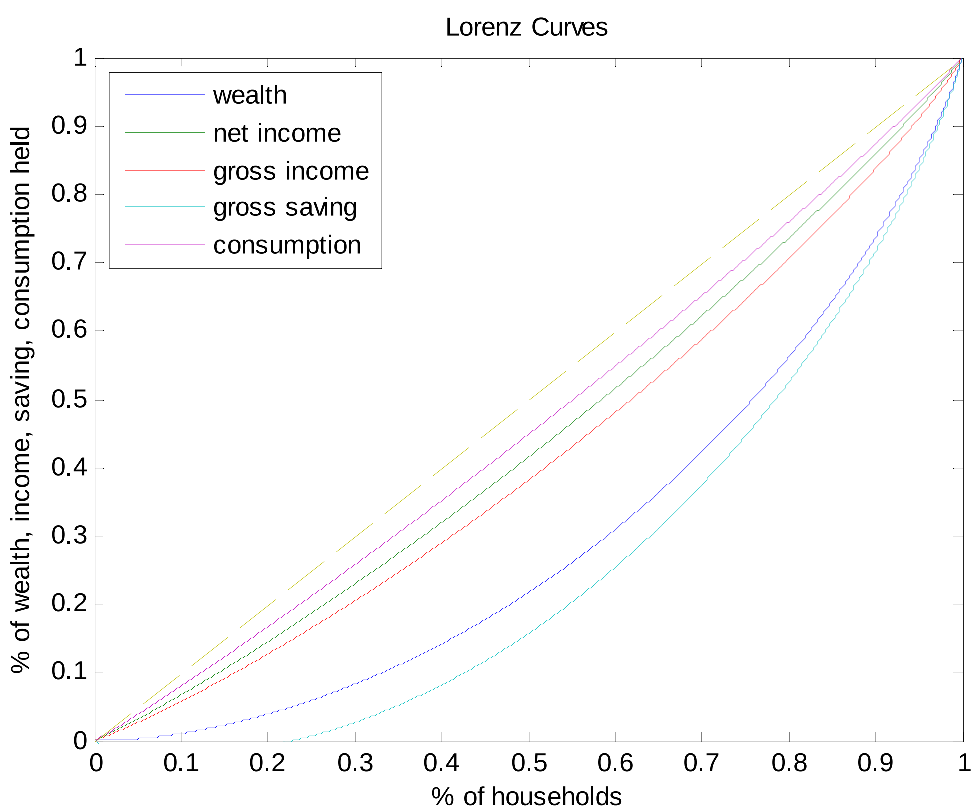
Grid spread	2.19σ	2.37σ	2.33σ	2.11σ
$\sigma \backslash \rho$	0	0.3	0.6	0.9
0.2	0.200 / 0	0.203 / 0.296	0.203 / 0.591	0.204 / 0.887
0.4	0.400 / 0	0.405 / 0.296	0.406 / 0.591	0.409 / 0.887

Comparing this table and Table 1 in Aiyagari paper, we see that even for high serial correlation $\rho = 0.9$ and coefficient of variation $\sigma = 0.4$, my method approximates serial correlation to 0.409 which is far better than 0.49 from Aiyagari paper computed using fixed grid spread.

The following results were obtained for RRA=2, and AR(1) process $\rho = 0.6$, $\sigma = 0.2$.

Interest rate = 4.0000, Average capital = 5.5655

Variable	Coefficient of variation	Gini coefficient
Wealth	0.7523	0.4034
Net income	0.2193	0.1225
Gross income	0.3108	0.1706
Gross saving	0.8871	0.4871
Consumption	0.1394	0.0764



Problem 1. Part (b,c)

To check EZ version of Aiyagari model, I firt tested it with RRA=2, EIS= 0.5 to compare with results in part (a). I obtained following similar results: interest rate = 4.0050, average capital = 5.5618

Variable	Coefficient of variation	Gini coefficient
Wealth	0.7526	0.4030
Net income	0.2157	0.1208
Gross income	0.3091	0.1696
Gross saving	0.8835	0.4852
Consumption	0.1383	0.0757

Now I separate RRA from EIS. I fixed EIS=0.9 and consider two cases RRA=2 and RRA=20. Then I fixed RRA=2 and changed EIS from 2 to 0.1. As we can see from the table below, the higher is RRA and the lower is EIS, the lower is the interest rate, the higher is the average capital. It seems like EIS has more influence on the interest rate than RRA.
Unlike RRA, increase in EIS however does not decrease the measures of inequality.

RRA	EIS	interest rate	average capital
2	0.9	4.0696	5.5154
20	0.9	3.7467	5.7541
2	2.0	4.1146	5.4834
2	0.1	3.4862	5.9593

	Coefficient of variation	Gini coefficient
	RRA=2, EIS=0.9	
Wealth	0.7506	0.4031
Net income	0.2160	0.1210
Gross income	0.3085	0.1696
Gross saving	0.8828	0.4856
Consumption	0.1392	0.0763
	RRA=20, EIS=0.9	
Wealth	0.4791	0.2682
Net income	0.1949	0.1094
Gross income	0.2277	0.1280
Gross saving	0.6600	0.3704
Consumption	0.1008	0.0567
	RRA=2, EIS=2	
Wealth	0.7358	0.3977
Net income	0.2150	0.1205
Gross income	0.3039	0.1677
Gross saving	0.8710	0.4809
Consumption	0.1379	0.0758
	RRA=2, EIS=0.1	
Wealth	0.7330	0.3914
Net income	0.2107	0.1181
Gross income	0.3027	0.1659
Gross saving	0.8593	0.4710
Consumption	0.1264	0.0692