STICKY PRICE MODELS OF THE BUSINESS CYCLE: CAN THE CONTRACT MULTIPLIER SOLVE THE PERSISTENCE PROBLEM?

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We construct a quantitative equilibrium model with firms setting prices in a staggered fashion and use it to ask whether monetary shocks can generate business cycle fluctuations. These fluctuations include persistent movements in output along with the other defining features of business cycles, like volatile investment and smooth consumption. We assume that prices are exogenously sticky for a short time. Persistent output fluctuations require endogenous price stickiness in the sense that firms choose not to change prices much when they can do so. We find that for a wide range of parameter values, the amount of endogenous stickiness is small. Thus, we find that in a standard quantitative model, staggered price-setting, alone, does not generate business cycle fluctuations.

Keywords: Staggered price-setting, endogenous price stickiness, monetary business cycles.

1. INTRODUCTION

Since the early 1970s, macroeconomists have known how to construct general equilibrium models in which monetary shocks generate contemporaneous output fluctuations. (Examples of such models include the work of Lucas (1972, 1990), Fischer (1977), Phelps and Taylor (1977), and Lucas and Woodford (1993).) The more difficult problem is to construct models in which monetary shocks generate the main feature of business cycles: persistent movements in output. In a large class of models, generating such persistence requires that prices not change much for a long time after monetary shocks. An unappealing way of generating persistence is to simply assume that prices are exogenously fixed for long periods of time. A preferable way is to construct models in which small frictions lead to long periods of endogenous price rigidity and, hence, persistent output movements.

Staggered price-setting provides a promising mechanism to generate long periods of endogenous price stickiness with the small frictions of short periods of exogenous price stickiness. Taylor (1980, p. 2) provides an eloquent exposition of this mechanism:

> Because of the staggering, some firms will have established their wage rates prior to the current negotiations, but others will establish their wage rates in future periods. Hence, when considering relative wages, firms and unions must look both forward and backward

in time to see what other workers will be paid during their own contract period. In effect, each contract is written relative to other contracts, and this causes shocks to be passed on from one contract to another—a sort of "contract multiplier."

Thus, the idea is that staggered price-setting leads to interactions among price-setters that generate longer movements in output than does a similar model with synchronized price-setting. Taylor (1980) and Blanchard (1991) investigate this mechanism for generating persistent output fluctuations, but both exogenously specify the rules for setting wages or prices.

Here we study a quantitative general equilibrium model with rational price-setting firms and staggered price-setting and ask if the model can generate a monetary business cycle. In particular, we use a variant of a standard sticky price model in which imperfectly competitive firms set nominal prices and real money balances enter the consumer's utility function.[2] To this setup we add time, uncertainty, and capital accumulation, as well as staggered price-setting. We ask whether the model with a short period of exogenous price stickiness can generate long periods of endogenous price stickiness and persistent output fluctuations. Our main finding is that it cannot: staggered price-setting, by itself, does not generate business cycles driven by monetary shocks.

We also find that the intertemporal links of capital accumulation and interest-sensitive money demand play a central role in generating the persistence problem. Specifically, we show that in versions of our model without intertemporal links, staggered price-setting leads to persistent output fluctuations after monetary shocks, but once such links are introduced, output fluctuations are no longer persistent.

Our model has a continuum of monopolistically competitive firms that produce differentiated products using capital and labor. These firms set nominal prices for a fixed number of periods and do so in a staggered fashion. In particular, each period, $1/N$ of the firms choose new prices, which are then fixed for N periods. The consumer side of the model is standard. Consumers are infinitely lived and have preferences over consumption, leisure, and real money balances. The nominal money supply follows an exogenous stochastic process.

We measure the effect of staggered price-setting on the persistence of output by the contract multiplier. This multiplier is the ratio of the half-life of output deviations after a monetary shock with staggered price-setting to the corresponding half-life with synchronized price setting. The half-life of output deviations with synchronized price-setting is roughly one-half the length of exogenous price stickiness. The half-life of output in the data is 10 quarters. Thus, if the period of exogenous price stickiness is about one quarter, then the model with

staggered price-setting must produce a contract multiplier of about 20 in order to match the data.

Our benchmark specification is the standard one in the business cycle literature. The utility function has unit elasticity of substitution between consumption and leisure and is thus consistent with balanced growth. Our specification of demand for intermediate goods has constant elasticity, as in the work of Dixit and Stiglitz (1977). Our technology has constant returns to scale. When we calibrate the model, we find that the contract multiplier is roughly 1. Thus, for our benchmark specification, the staggered price-setting mechanism does not generate persistence.

In order to gain some intuition for these results, we see what happens when we eliminate capital and impose a static money demand equation. The linearized version of this model consists of two equations, a static money demand equation and a price-setting equation, that are essentially identical to those used by Taylor (1980). The only difference between Taylor's equations and ours (besides his *wages* being relabeled as our *prices*) is that in Taylor's formulation the coefficient on the sum of future output in the price-setting equation is a free parameter, while in ours it is a function of the parameters of the underlying economy. For the model to generate a persistent output response to a monetary shock, this coefficient must be small, so that monetary shocks do not induce firms to raise their prices much. In our model, this coefficient is the product of two parameters: the sensitivity of prices to costs and the sensitivity of costs to output. With constant elasticity of demand, prices move one-for-one with costs. With unit elasticity of substitution in preferences, costs are extremely sensitive to output. Thus, in our model, the coefficient on the sum of future output is large, and monetary shocks do not have persistent effects on output.

We examine three ways of possibly increasing the persistence of output movements. One is to make consumption and leisure close to perfect substitutes in preferences, so that costs are less sensitive to output. With such preferences, the contract multiplier is 20 without intertemporal links but less than 1 with such links. Thus, while eliminating the intertemporal links of capital accumulation and interest-sensitive money demand makes developing analytical expressions easy, this procedure can be misleading. We also find that both with and without intertemporal links, prices change hardly at all after a monetary shock, but with intertemporal links, output is not persistent. This finding shows that endogenous price stickiness, by itself, does not necessarily imply persistent output fluctuations.

Another mechanism we explore, suggested by Kimball (1995), is designed to make prices less sensitive to cost changes. If intermediate goods producers face an elasticity of demand that rises as their relative prices rise, so that demand is convex, then these producers may optimally choose not to raise their prices much after a monetary shock that raises output and costs. We show that generating persistence from this feature requires extraordinarily convex demand functions for intermediate goods.

A third potentially helpful mechanism has been suggested by Rotemberg (1996). He suggests that persistence can be increased if firms face upward-sloping marginal cost curves, as they do if they use specific factors that are inelastically supplied. Adding specific factors makes costs and prices less sensitive to wage changes. Indeed, with specific factors and with extremely elastic demand, large contract multipliers can be obtained in the model without intertemporal links. But here, too, adding intertemporal links reduces the contract multiplier significantly.

Finally, we consider a model with combinations of all the variations we have explored. We find that the contract multiplier is approximately 2. Thus, to get the kind of persistence seen in the data, the exogenous price stickiness needs to last roughly $2\frac{1}{2}$ years.

2. A BENCHMARK MONETARY ECONOMY

Consider a monetary economy populated by a large number of identical, infinitely lived consumers. In each period t, the economy experiences one of finitely many events s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given.

In each period t, the commodities in this economy are labor, a consumption-capital good, money, and a continuum of intermediate goods indexed by $i \in [0, 1]$. The technology for producing final goods from intermediate goods at history s^t is

$$(1) \qquad y(s^t) = \left[\int y(i, s^t)^\theta \, di \right]^{1/\theta},$$

where $y(s^t)$ is the final good, $y(i, s^t)$ is an intermediate good of type i, and the elasticity of substitution between goods is $1/(1 - \theta)$. The technology for producing each intermediate good i is a standard constant returns to scale production function:

$$(2) \qquad y(i, s^t) = F(k(i, s^t), l(i, s^t)),$$

where $k(i, s^t)$ and $l(i, s^t)$ are the inputs of capital and labor.

Final goods producers behave competitively. In each period t, they choose inputs $y(i, s^t)$ for all $i \in [0, 1]$ and output $y(s^t)$ to maximize profits given by

$$(3) \qquad \max \bar{P}(s^t) y(s^t) - \int P(i, s^{t-1}) y(i, s^t) \, di,$$

subject to (1), where $\bar{P}(s^t)$ is the price of the final good in period t and $P(i, s^{t-1})$ is the price of intermediate good i in period t. The intermediate goods prices do not depend on s_t because period t prices in our economy are set before the realization of the period t shocks. Solving the problem in (3) gives the

input demand functions:

$$(4) \qquad y^d(i, s^t) = \left[\frac{\overline{P}(s^t)}{P(i, s^{t-1})}\right]^{1/(1-\theta)} y(s^t).$$

The zero-profit condition implies that

$$(5) \qquad \overline{P}(s^t) = \left[\int P(i, s^{t-1})^{\theta/(\theta-1)} di\right]^{(\theta-1)/\theta}.$$

Notice that in equilibrium the output price in period t depends on only s^{t-1}.

Intermediate goods producers behave as imperfect competitors. They set prices for N periods and do so in a staggered fashion. In particular, in each period t, a fraction $1/N$ of these producers choose new prices $P(i, s^{t-1})$ before the realization of the event s_t. These prices are set for N periods, so for this group of intermediate goods producers, $P(i, s^{t+r-1}) = P(i, s^{t-1})$ for $\tau = 0, \ldots, N - 1$. The intermediate goods producers are indexed so that producers indexed $i \in [0, 1/N]$ set new prices in 0, N, $2N$, and so on, while producers indexed $i \in [1/N, 2/N]$ set new prices in 1, $N + 1$, $2N + 1$, and so on, for the N cohorts of intermediate goods producers. In period t, each producer in a cohort chooses prices $P(i, s^{t-1})$ to maximize discounted profits from period t to period $t + N - 1$. That is, each solves this problem:

$$(6) \qquad \max_{P(i, s^{t-1})} \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau | s^{t-1})[P(i, s^{t-1}) - v(s^\tau)\overline{P}(s^\tau)] y^d(i, s^\tau),$$

where $Q(s^\tau | s^{t-1})$ is the price of one dollar in s^τ in units of dollars at s^{t-1}, $v(s^t)$ is the unit cost of production at s^t, and $y^d(i, s^t)$ is given in (4). The unit cost of production is given by

$$(7) \qquad v(s^t) = \min_{k, l} r(s^t)k + w(s^t)l$$

subject to $F(k, l) \geq 1$, where $r(s^t)$ is the rental rate on capital and $w(s^t)$ is the real wage rate. The solution to the problem stated in (6) is

$$(8) \qquad P(i, s^{t-1}) = \frac{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) \overline{P}(s^\tau)^{(2-\theta)/(1-\theta)} v(s^\tau) y(s^\tau)}{\theta \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau | s^{t-1}) \overline{P}(s^\tau)^{1/(1-\theta)} y(s^\tau)}.$$

Implicit in this problem are the demands for capital and labor by the intermediate goods producers. (Note that these factor demands are the unit factor demands, which solve (7), multiplied by the level of output of the intermediate good producer.) These factor demands, $k(i, s^t)$ and $l(i, s^t)$, of producer i in period t are made after the realization of the event s_t and thus depend on s^t.

Cost minimization implies that

$$(9) \qquad \frac{F_l(k(i,s^t),l(i,s^t))}{F_k(k(i,s^t),l(i,s^t))} = \frac{w(s^t)}{r(s^t)}.$$

Given constant returns to scale, this implies that capital-labor ratios are equated across the intermediate goods firms, so for all $i \in [0,1]$,

$$(10) \qquad \frac{k(i,s^t)}{l(i,s^t)} = \frac{k(0,s^t)}{l(0,s^t)}.$$

In what follows, each intermediate goods firm has the Cobb-Douglas production function, $F(k(i,s^t),l(i,s^t)) = k(i,s^t)^\alpha l(i,s^t)^{1-\alpha}$, where α is the share of capital in costs. Thus (9) can be written as

$$(11) \qquad \left(\frac{1-\alpha}{\alpha}\right)\frac{k(i,s^t)}{l(i,s^t)} = \frac{w(s^t)}{r(s^t)}.$$

Consumer preferences are given by

$$(12) \qquad \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c(s^t),l(s^t),M(s^t)/\bar{P}(s^t)),$$

where $0 < \beta < 1$ is the discount factor and where $c(s^t)$, $l(s^t)$, and $M(s^t)$ are consumption, labor, and nominal money balances, respectively. In each period $t = 0,1,\ldots$, consumers choose their period t allocations after the realization of the event s_t. The problem of consumers is to choose rules for consumption $c(s^t)$, labor $l(s^t)$, capital stocks $k(s^t)$, nominal money balances $M(s^t)$, and one-period nominal bonds $B(s^{t+1})$ to maximize (12) subject to the sequence of budget constraints

$$\bar{P}(s^t)(c(s^t)+k(s^t)) + M(s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(s^{t+1})$$

$$\leq \bar{P}(s^t)(w(s^t)l(s^t) + [r(s^t)+1-\delta]k(s^{t-1}))$$

$$+ M(s^{t-1}) + B(s^t) + \Pi(s^t) + T(s^t) \qquad\qquad (t=0,1,\ldots)$$

and borrowing constraints $B(s^{t+1}) \geq \bar{B}$ for some large negative number $\bar{B}$. Here $\Pi(s^t)$ is the nominal profits of the intermediate goods producers, $T(s^t)$ is nominal transfers, and δ is the depreciation rate of capital. The initial conditions $k(s^{-1})$, $M(s^{-1})$, and $B(s^0)$ are given. Each of the nominal bonds $B(s^{t+1})$ is a claim to one dollar in state s^{t+1} and costs $Q(s^{t+1}|s^t)$ dollars in state s^t. In terms of relating the prices in the intermediate goods producers' problem to these prices, note that $Q(s^\tau|s^t) = Q(s^{t+1}|s^t)Q(s^{t+2}|s^{t+1})\cdots Q(s^\tau|s^{\tau-1})$ for all

$\tau > t$. The first-order conditions for the consumer can be written as

$$(13) \qquad -\frac{U_l(s^t)}{U_c(s^t)} = w(s^t),$$

$$(14) \qquad \frac{U_c(s^t)}{\overline{P}(s^t)} - \frac{U_m(s^t)}{\overline{P}(s^t)} = \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\frac{U_c(s^{t+1})}{\overline{P}(s^{t+1})},$$

$$(15) \qquad U_c(s^t) = \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t)U_c(s^{t+1})[r(s^{t+1}) + 1 - \delta],$$

$$(16) \qquad Q(s^\tau|s^t) = \beta^{\tau-t}\pi(s^\tau|s^t)\frac{U_c(s^\tau)}{U_c(s^t)}\frac{\overline{P}(s^t)}{\overline{P}(s^\tau)}$$

for all $\tau > t$, where $U_c(s^t)$, $U_l(s^t)$, and $U_m(s^t)$ denote the derivatives of the utility function with respect to its arguments and $\pi(s^\tau|s^t) = \pi(s^\tau)/\pi(s^t)$ is the conditional probability of s^τ given s^t.

The nominal money supply process is given by $M(s^t) = \mu(s^t)M(s^{t-1})$, where $\mu(s^t)$ is a stochastic process. New money balances are distributed to consumers in a lump-sum fashion by having nominal transfers satisfy $T(s^t) = M(s^t) - M(s^{t-1})$.

In terms of market-clearing conditions, consider first the factor markets. Notice that the capital stock chosen by consumers in period $t - 1$ for rental in period t is $k(s^{t-1})$ while the labor supply in period t is $l(s^t)$. In turn, each intermediate goods producer i chooses factor demands after the realization of uncertainty s_t in period t, so the demands for capital and labor are $k(i, s^t)$ and $l(i, s^t)$. Factor market-clearing thus requires that

$$(17) \qquad k(s^{t-1}) = \int k(i, s^t)\, di,$$

$$(18) \qquad l(s^t) = \int l(i, s^t)\, di.$$

Bond market-clearing requires that $B(s^{t+1}) = 0$. The resource constraint for this economy is

$$(19) \qquad c(s^t) + k(s^t) = y(s^t) + (1 - \delta)k(s^{t-1}).$$

An *equilibrium* for this economy is, then, a collection of allocations for consumers $c(s^t)$, $l(s^t)$, $k(s^{t-1})$, $M(s^t)$, $B(s^{t+1})$; allocations for intermediate goods producers $k(i, s^t), l(i, s^t)$ for $i \in [0,1]$; allocations for final goods producers $y(s^t)$ and $y(i, s^t)$ for $i \in [0,1]$; together with prices $w(s^t), r(s^t), Q(s^\tau|s^t)$ for $\tau = t,\dots, t + N - 1$, $\overline{P}(s^t)$, and $P(i, s^{t-1})$ for $i \in [0,1]$ that satisfy the following conditions: (i) taking prices as given, consumer allocations solve the consumer's problem; (ii) taking all prices but his own as given, each intermediate goods producer's price solves (6); (iii) taking the prices as given, the final goods producer's allocations solve the final goods producer's problem; (iv) the factor

market conditions (17) and (18) and the resource constraint (19) hold, and the bond market clears.

3. COMPUTATION OF EQUILIBRIUM

Now we briefly describe how we compute an equilibrium in the benchmark economy. We begin by substituting out a number of variables and reducing the equilibrium to four equations: the resource constraint, a pricing equation, an Euler equation for money, and an Euler equation for capital. Once we have these four equations, we compute Markov equilibria in which allocations are functions of the state of the economy. The state variables are lagged prices, the capital stock, and money shocks. The decision variables are current prices, consumption, labor, and investment. In what follows, we will be focusing on the symmetric equilibrium in which all the intermediate goods producers of the same cohort make identical decisions. Thus, $P(i, s^t) = P(j, s^t)$, $k(i, s^t) = k(j, s^t)$, $l(i, s^t) = l(j, s^t)$, $y(i, s^t) = y(j, s^t)$ for all $i, j \in [0, 1/N]$, and so on, for the N cohorts.

We begin with the resource constraint. Equating supplies of and demands for each intermediate good using equations (2) and (4) and integrating gives

$$\overline{P}(s^t)^{1/(1-\theta)} \left(\int P(i, s^{t-1})^{1/(\theta-1)} di \right) y(s^t)$$

$$= F(k(s^{t-1})/l(s^t), 1) \left(\int l(i, s^t) di \right),$$

where we have exploited the facts that F is a constant returns to scale production function and that capital-labor ratios are equated across producers, as in (10). Simplifying using (18) gives

$$(20) \qquad y(s^t) = F(k(s^{t-1}), l(s^t)) \left(\frac{\overline{P}(s^t)^{1/(\theta-1)}}{\int P(i, s^{t-1})^{1/(\theta-1)} di} \right).$$

Substituting (20) into (19) gives the resource constraint we use in our computations.

We can now develop the pricing equation. We first express unit cost in terms of the aggregate allocations. To do so, we solve the minimization problem in (7) and use (10) and (13) to get

$$(21) \qquad v(s^t) = -\frac{1}{1-\alpha} \frac{U_l(s^t)}{U_c(s^t)} \left(\frac{l(s^t)}{k(s^{t-1})} \right)^{\alpha}.$$

Since all prices are equal, within each cohort of producers, we need only record one price per cohort and not the index identifying the producers. Thus, from now on, we drop the dependence on i, and we let $P(s^{t-1})$ denote the prices set at the beginning of period t, $P(s^{t-2})$ denote the prices set at the beginning of

period $t - 1$, and so on. Using (5), we can write the aggregate price level as

$$(22) \qquad \bar{P}(s^t) = \left[\tfrac{1}{N} P(s^{t-1})^{\theta/(\theta-1)} + \cdots + \tfrac{1}{N} P(s^{t-N})^{\theta/(\theta-1)} \right]^{(\theta-1)/\theta}.$$

Using (16), (20), (21), and (22) in (8), we obtain the pricing equation we use in our computations.

Finally, we rewrite the Euler equations for money and capital, (14) and (15), using (22) to substitute for $\bar{P}(s^t)$ and using (10), (11), and (13) to substitute for $r(s^t)$.

We are interested in a stationary equilibrium and thus restrict the stochastic processes for the growth of the money supply to be Markov. A stationary equilibrium for this economy consists of stationary decision rules that are functions of the state of the economy. Because producers choose their prices before the current money shock is realized, while other allocations are chosen after the money shock is realized, in each period we must record two states for the economy. The state for producers in period t must record the $N - 1$ intermediate goods prices in addition to the capital stock and lagged money growth rates. The state for other allocations in period t, such as consumer allocations and input allocations for producers, must record the state for producers plus the current money growth rates. In any period t, $N - 1$ prices prevail for intermediate goods, namely, those set at the beginning of period $t - 1, P(s^{t-2})$; those set at the beginning of period $t - 2, P(s^{t-3})$; and so on, through those set in period $t - (N - 1), P(s^{t-N})$. We normalize prices by dividing them by the money stock. Thus, the state for producers is $[P(s^{t-2})/M(s^{t-1}), \ldots, P(s^{t-N})/M(s^{t-1}), k(s^{t-1}), \mu(s^{t-1})]$. The state for other allocations also includes $\mu(s^t)$. The decision variables for period t are aggregate consumption in t, $c(s^t)$; aggregate labor supply in t, $l(s^t)$; and the normalized price of the cohort of intermediate goods producers that are setting their prices at the beginning of period t, $P(s^{t-1})/M(s^{t-1})$.

In some of the models we consider, investment is extremely volatile. To reduce that volatility, we introduce adjustment costs in changing the capital stock employed by each intermediate goods producer. Specifically, the law of motion for capital used in producing good i is given by

$$(23) \qquad k(i, s^t) = (1 - \delta)k(i, s^{t-1}) - \phi\left(\frac{x(i, s^t)}{k(i, s^{t-1})} \right)k(i, s^{t-1}) + x(i, s^t),$$

where ϕ is the adjustment cost function and x is investment. The problem of monopolist i who chooses prices at date t, $t + N$, $t + 2N$, and so on is to choose $P(i, s^\tau)$, $l(i, s^\tau)$, $x(i, s^\tau)$, and $k(i, s^\tau)$ for $\tau \geq t$ to maximize

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau} Q(s^\tau | s^{t-1})[P(i, s^\tau)y^d(i, s^\tau) - w(s^\tau)\bar{P}(s^\tau)l(i, s^\tau)$$

$$- \bar{P}(s^\tau)x(i, s^\tau)]$$

TABLE I

MODEL PARAMETERS

Model	Parameter Values
A. Benchmark Economy	
Preferences	$\beta = 0.97^{T/N}$, $\omega = 0.94$, $\eta = 0.39$, $\psi = 1.5$, $\sigma = 1$
Production	$\alpha = 0.33$, $\theta = 0.9$
Capital Accumulation	$\delta = 1 - 0.92^{T/N}$, b adjusted
Money Growth	$\rho = 0.57^{4T/N}$
B. Three Variations	
Near-Perfect Substitute Preferences	$\kappa = 0.0002$, $\xi = 0.00011$, $\nu = 0.0002$, ω_1, ω_2 adjusted
Convex Demand	$\varepsilon = 10$, $\chi = -289$
Specific Factors	$\alpha_1 = 0.22$, $\alpha_2 = 0.44$

Note: In our experiments, $T = 1/4$ unless otherwise noted.

subject to (4), (23), $y(i, s^t) = F(k(i, s^{t-1}), l(i, s^t))$, and the constraints $P(i, s^\tau) = P(i, s^{t-1})$ for $\tau = t, \ldots, t + N - 1$, $P(i, s^\tau) = P(i, s^{t+N-1})$ for $\tau = t + N, \ldots, t + 2N - 1$, and so on. Clearly, in computing an equilibrium for this economy, we must record the capital stock employed by the representative producer in each cohort, and we expand the state and decision variables accordingly. We also modify the resource constraint appropriately. We then linearize the resulting equations around the steady state and use standard methods to obtain linear decision rules. When $N = 1$ and 2, we check the accuracy of the linear decision rules against nonlinear decision rules obtained by the finite element method. (See the work of McGrattan (1996).)

4. CALIBRATION OF THE BENCHMARK ECONOMY

Now we describe how we chose the parameter values for the benchmark economy that are summarized in panel A of Table I. Later, we will conduct a wide range of sensitivity analyses, so we view this calibration as only setting the midpoint of the ranges for each of the parameters. These ranges are set wide enough to encompass most estimates of the various parameters in the literature.

Consider a utility function of the form

(24) $\quad U(c, l, M/\bar{P})$

$$= \frac{\left[\left(\omega c^{(\eta-1)/\eta} + (1-\omega)(M/\bar{P})^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)} (1-l)^\psi \right]^{1-\sigma}}{(1-\sigma)},$$

where ω is the share parameter, η is interest elasticity, ψ is the weight on leisure, and σ is risk aversion. Utility functions of this form are attractive because they are consistent with balanced growth. (See the work of Cooley and

Prescott (1995).) But these functions have the unattractive feature that the share of time devoted to the market is linked to the labor supply elasticity, which is given by $(1 - l)/l$. In Section 5, we consider preferences that do not have this feature.

We chose parameter values for the utility function (24) as follows. For ω and η, we drew on the money demand literature. Our model can be used to price a variety of assets, including a nominal bond that costs one dollar at s^t and pays a gross interest rate of $R(s^t)$ dollars in all states s^{t+1}. The first-order condition for this asset can be written as $U_m(s^t) = U_c(s^t)(R(s^t) - 1)/R(s^t)$. Using our specification of utility, we can rewrite the first-order condition as

$$(25) \qquad \log\frac{M(s^t)}{P(s^t)} = -\eta\log\frac{\omega}{1-\omega} + \log c(s^t) - \eta\log\left(\frac{R(s^t)-1}{R(s^t)}\right).$$

We regressed the log of consumption velocity on the interest rate variable in (25) using several data series from Citibase for 1960:1–1995:4: M1; the GDP deflator; the consumption of services, nondurables, and durables; and the three-month U.S. Treasury bill rate. The implied value of ω from our regression is 0.94. Our estimate of the interest elasticity is $\eta = 0.39$, with a standard error of 0.033. This elasticity is similar to that estimated by Mankiw and Summers (1986) and Lucas (1988) and smaller than that of Stock and Watson (1993).

Based on the work of Basu and Fernald (1994, 1995), Basu and Kimball (1997), and Basu (1996), we chose $\theta = 0.9$, which implies a markup of 11 percent and an elasticity of demand of 10.

We set the parameters β, δ, ψ, and α using balanced growth path relationships in our model and various statistics in the data. (Again, the parameter values are given in Table I.) With these parameters, our model predicts an annualized capital-output ratio of 2.65, an investment-output ratio of 0.23, and a share of time allocated to the market of $1/3$. The capital-output ratio and the investment-output ratio are the same as those calculated by Christiano (1988), and the time allocated to the market is similar to that of Cooley and Prescott (1995). In using the national income accounts to determine the capital share in our model, we assumed that monopoly profits are allocated proportionately to capital and labor. With this assumption, the capital share of income in our model is $1/3$, which is similar to that estimated by Christiano (1988).

We chose an adjustment cost function of the form $\phi(x/k) = b[(x/k) - \delta]^2/2$, where $b \geq 0$. We set the adjustment cost parameter b so that the Hodrick-Prescott-filtered data from the model produce a standard deviation of investment relative to that of output similar to the corresponding statistic for the U.S. economy (3.25). We set $b = 0$ when the relative volatility of investment in the economy with no adjustment costs is too small.

The stochastic process for the growth rate of the money stock is given by $\log \mu_t = \rho \log \mu_{t-1} + \epsilon_t$, where ϵ is an independent and identically normally distributed mean-zero shock. The serial correlation parameter for money growth rates, ρ, was calculated using Citibase's quarterly data on M1 from 1959:3 through 1995:2. We obtained $\rho = 0.57$.

In our experiments, we will vary the amount of staggering, N, as well as the length of time of exogenous price stickiness, which we denote by T. When we vary these, we also adjust the discount factor β, the depreciation rate δ, and the serial correlation parameter ρ so as to keep the capital-output and investment-output ratios and the quarterly serial correlation unchanged. Specifically, we set $\beta = 0.97^{T/N}$, $\delta = 1 - 0.92^{T/N}$, and $\rho = 0.57^{4T/N}$, where T is measured in years. For most of the experiments, we set T equal to one-quarter of one year and $N = 13$, so that prices are fixed for one quarter at a time, a cohort of producers chooses prices every week, and consumers make their decisions weekly.

Our notion of a contract multiplier is designed to measure how much staggered price-setting increases the persistence of output relative to synchronized price-setting. We measure the persistence of output by its *half-life*, defined as the length of time after a shock before the deviation in output shrinks to half of its impact value. With synchronized price-setting, on average the half-life of output is one-half the length of exogenous stickiness. The reason is that monetary shocks occur randomly between price adjustments, and the associated movements in output last only until the next price adjustment. Thus, we define the *contract multiplier* in a model with staggered price-setting as the ratio of the half-life of output to one-half the length of exogenous stickiness. Hence, a multiplier of, say, 5 means that staggered price-setting generates output movements that last 5 times longer than they would with synchronized price-setting. An attractive feature of the contract multiplier is that in our quantitative models, this multiplier is approximately invariant to the length of exogenous price stickiness.

We measured the half-life of output in the data by fitting an ARMA process to the logarithm of real GDP, after removing a quadratic trend. The fitted equation is

$$\log y_t = \underset{(0.066)}{1.30} \log y_{t-1} - \underset{(0.066)}{0.38} \log y_{t-2} + \epsilon_t,$$

where the numbers in parentheses are standard errors. The Ljung-Box Q test indicates little evidence for serial correlation of the residuals ($Q = 13.8$ with significance level 0.46). From this equation, we calculated the half-life of output to be 10 quarters, or $2\frac{1}{2}$ years. This measure is admittedly imperfect, but provides a useful benchmark.

For the bulk of our analysis, we consider economies with a period of exogenous price stickiness of 1 quarter. The contract multiplier needed to match this measure of persistence is 20. With one month of exogenous price stickiness, the needed contract multiplier is 60; with one year, 5.

5. RESULTS FOR THE BENCHMARK ECONOMY

The main question we address here is whether staggered price-setting can lead to persistent movements in output. We begin by considering a stripped-down

version of the benchmark model that we can solve analytically for the equilibrium. In this version, we abstract from capital and impose a static money demand equation. We show that in order to get persistence, the equilibrium wage rate must change little when consumption changes. We show that for the class of preferences in our benchmark model, the equilibrium wage changes so much that we cannot get persistence.

Consider a version of the benchmark model without capital ($\alpha = 0$) and with two cohorts of monopolists staggering their price-setting ($N = 2$). Suppose that the utility function is given by equation (24), as in the benchmark economy. We log-linearize the equilibrium conditions around a deterministic steady state with a constant money supply. Let $\hat{x}_{it}$, $\hat{x}_t$, $\hat{p}_t$, $\hat{w}_t$, $\hat{c}_t$, $\hat{y}_t$, and $\hat{m}_t$ represent the logarithmic deviations from a steady-state for $P(i, s^{t-1})$, $P(s^{t-1})$, $\bar{P}(s^t)$, $w(s^t)$, $c(s^t)$, $y(s^t)$, and $M(s^t)$, respectively.

Now, simply impose a static money demand equation in which consumption equals real balances, so $M(s^t)/\bar{P}(s^t) = c(s^t) = y(s^t)$ and hence

$$(26) \qquad \hat{m}_t - \hat{p}_t = \hat{y}_t.$$

Next, consider the pricing equation (8). Substituting for Q from equation (16), setting $\beta = 1$, and linearizing the resulting equation around the deterministic steady state gives

$$(27) \qquad \hat{x}_{it} = \tfrac{1}{2}E_{t-1}\hat{p}_t + \tfrac{1}{2}E_{t-1}\hat{p}_{t+1} + \tfrac{1}{2}E_{t-1}(\hat{w}_t + \hat{w}_{t+1}),$$

where E_{t-1} denotes the expectation as of period $t - 1$. The price level $\hat{p}_t$ is a weighted average of the individual prices:

$$(28) \qquad \hat{p}_t = \frac{\hat{x}_t}{2} + \frac{\hat{x}_{t-1}}{2}.$$

Substituting (28) into (27) and imposing equilibrium gives

$$(29) \qquad \hat{x}_t = \tfrac{1}{2}\hat{x}_{t-1} + \tfrac{1}{2}E_{t-1}\hat{x}_{t+1} + E_{t-1}(\hat{w}_t + \hat{w}_{t+1}).$$

Next, note that with $\alpha = 0$ and $N = 2$, using (22), (20) can be written as

$$y(s^t) = l(s^t)\left[\tfrac{1}{2}P(s^{t-1})^{\theta/(\theta-1)} + \tfrac{1}{2}P(s^{t-2})^{\theta/(\theta-1)}\right]^{1/\theta}$$

$$\times \left[\tfrac{1}{2}P(s^{t-1})^{1/(\theta-1)} + \tfrac{1}{2}P(s^{t-2})^{1/(\theta-1)}\right]^{-1}.$$

Log-linearizing this equation around a deterministic steady state gives

$$(30) \qquad \hat{y}_t = \hat{l}_t.$$

Log-linearizing the labor supply equation (13) around the deterministic steady state gives

$$(31) \qquad \hat{w}_t = \frac{\partial \log\left(-\frac{U_l}{U_c}\Big|_{ss}\right)}{\partial \log c} \hat{c}_t + \frac{\partial \log\left(-\frac{U_l}{U_c}\Big|_{ss}\right)}{\partial \log M/\overline{P}} (\hat{m}_t - \hat{p}_t)$$

$$+ \frac{\partial \log\left(-\frac{U_l}{U_c}\Big|_{ss}\right)}{\partial \log l} \hat{l}_t,$$

where the subscript ss denotes that the derivatives are evaluated at the steady state allocations. With our functional forms (31) becomes

$$(32) \qquad \hat{w}_t = \left[\omega\left(\frac{\eta-1}{\eta}\right) + \frac{1}{\eta}\right]\hat{c}_t + \left[(1-\omega)\left(\frac{\eta-1}{\eta}\right)\right](\hat{m}_t - \hat{p}_t) + \frac{\theta\omega}{\psi}\hat{l}_t,$$

where we have used the facts that the steady state allocations satisfy $c_{ss} = l_{ss} = (M/\overline{P})_{ss}$ and

$$-\frac{U_l}{U_c}\Big|_{ss} = \frac{\psi}{\omega}\frac{c_{ss}}{(1-c_{ss})} = w_{ss} = \theta.$$

Substituting (26), (30), and $\hat{c}_t = \hat{y}_t$ in (32) gives the following log-linearized labor supply equation:

$$(33) \qquad \hat{w}_t = \gamma \hat{y}_t,$$

where

$$(34) \qquad \gamma = 1 + \frac{\theta\omega}{\psi}.$$

Here γ is the elasticity of the equilibrium real wage rate with respect to consumption. Substituting (33) into (29) gives

$$(35) \qquad \hat{x}_t = \tfrac{1}{2}\hat{x}_{t-1} + \tfrac{1}{2}E_{t-1}\hat{x}_{t+1} + \gamma E_{t-1}(\hat{y}_t + \hat{y}_{t+1}).$$

The system of equations (26), (28), and (35) is the same as that in Taylor (1980). The only difference between our price-setting equation and Taylor's is that our value of γ depends on the underlying preferences and technology while Taylor's γ is a structural parameter.

The system of equations (26), (28), and (35) can be solved to determine how many shocks are divided into movements in prices and movements in output. Equation (26) implies that large movements in output require small movements in the price level. But equation (35) implies that large movements in output have small effects on the price level only if γ is small.

To see how γ influences the division of money shocks into price and output movements, we solve the model for $\hat{x}_t$, $\hat{p}_t$, and $\hat{y}_t$. Substituting for $\hat{y}_t$ in (35) using (26) and (28), we obtain

$$E_{t-1}\hat{x}_{t+1} - 2\frac{1+\gamma}{1-\gamma}\hat{x}_t + \hat{x}_{t-1} = -\frac{2\gamma}{1-\gamma}E_{t-1}(\hat{m}_t + \hat{m}_{t+1}).$$

We can use standard methods to write $\hat{x}_t$ as

$$(36) \qquad \hat{x}_t = a\hat{x}_{t-1} + \frac{2a\gamma}{1-\gamma}E_{t-1}\sum_{i=0}^{\infty} a^i(\hat{m}_{t+i} + \hat{m}_{t+1+i}),$$

where a is the root with absolute value less than 1 which solves $a^2 - \lambda a + 1 = 0$, where $\lambda = 2(1 + \gamma)/(1 - \gamma)$. This root is equal to

$$(37) \qquad a = \frac{1 - \sqrt{\gamma}}{1 + \sqrt{\gamma}}.$$

Now suppose that $\hat{m}_t$ is a random walk. After simplifying (36), we can write

$$(38) \qquad \hat{x}_t = a\hat{x}_{t-1} + (1-a)\hat{m}_{t-1}.$$

Using (28) to substitute for $\hat{x}_t$ in (38), we obtain

$$(39) \qquad \hat{p}_t = a\hat{p}_{t-1} + (1-a)(\hat{m}_{t-1} + \hat{m}_{t-2})/2.$$

Finally, using (26) to substitute for $\hat{p}_t$ in (39), we obtain

$$(40) \qquad \hat{y}_t = a\hat{y}_{t-1} + (\hat{m}_t - \hat{m}_{t-1}) + (1-a)(\hat{m}_{t-1} - \hat{m}_{t-2})/2.$$

As should be clear from (40), the persistence properties of output with respect to money shocks depend critically on the value of a and, therefore, on the value of γ. Recall from (34) that $\gamma > 1$. Hence it follows from (37) that $a < 0$, so output is not persistent.

When γ is treated as a free parameter, Taylor's model described by (26), (28), and (35) can produce the needed contract multiplier. When $N = 2$, one-half of the length of exogenous price stickiness is 1. Thus, in order to get a multiplier of 20, we need a half-life of 20. If $a > 0$, it is easy to show that the half-life of output is given by $t = 1 - \log(1 + a)/\log a$. For a multiplier of 20, we need $a = 0.965$ and hence $\gamma = 0.00031$. In our model, γ is not a free parameter and is necessarily greater than 1.

In Table II we report values of the contract multiplier for a variety of economies that use our calibrated parameters, except where noted.[3] We investi-

[3] To calculate the contract multiplier for our economies in which output is measured at discrete intervals, we need a measure of the half-life of output for such economies. We calculate the half-life by finding the first date at which output is lower than half its impact value and then linearly interpolate output between this date and the previous date.

gate how the intertemporal links of interest-sensitive money demand and capital accumulation affect the contract multiplier separately and jointly. The first column of the table refers to economies without capital and with interest-insensitive money demand, while the next three columns refer to economies with only interest-sensitive money demand, with only capital, and with both interest-sensitive money demand and capital, respectively. We report multipliers for economies with 2 cohorts and i.i.d. money growth since these are comparable to results for the stripped-down version of our model. We also report multipliers for economies with 13 cohorts and serially correlated money growth and think of these multipliers as more relevant for comparing with data.

In Table II we see that for all versions of our benchmark model, the contract multiplier is roughly 1. With this contract multiplier, in order to generate the half-life of output seen in the data, the exogenous length of price stickiness would have to be 5 years.

Taylor (1980) and Blanchard (1991) conjecture that, if the length of time that prices are fixed is held constant, then increasing the amount of staggering increases the persistence of output fluctuations. We investigate this conjecture by considering versions of our benchmark economy with $N = 2$, 13, and 26. The resulting multipliers are 1.06, 0.99, and 0.87. These results show that increasing the amount of staggering does not increase the persistence of output responses.

We also examine whether the contract multiplier varies with the length of exogenous price stickiness, and we find that it does not. For example, for our

TABLE II

THE CONTRACT MULTIPLIER WITH VARIOUS INTERTEMPORAL LINKS

Economy[a]	Types of Intertemporal Links			
	None	Only Money	Only Capital	Both
Benchmark				
2 cohorts, iid money growth	0.90	0.90	1.02	1.05
13 cohorts, serially correlated money growth	1.01	0.87	1.11	0.99
Near-Perfect Substitute Preferences[b]				
2 cohorts, iid money growth	21.97	20.00	1.07	0.50
13 cohorts, serially correlated money growth	139.10	22.72	8.31	0.13
Convex Demand				
2 cohorts, iid money growth	2.17	2.17	1.57	1.62
13 cohorts, serially correlated money growth	3.79	2.44	1.76	1.55
Specific Factors				
2 cohorts, iid money growth	1.43	1.43	1.30	1.30
13 cohorts, serially correlated money growth	1.82	1.54	1.59	1.33
Combined Model				
2 cohorts, iid money growth	1.96	1.96	1.75	1.75
13 cohorts, serially correlated money growth	3.26	2.19	2.39	1.81

[a] See Table I for parameter values. No capital link implies that the capital share is 0. No money link implies that the interest elasticity of money is 0. If money growth is serially correlated, then $\rho = 0.57^{1/13}$; otherwise, it is 0.
[b] If there is no money link, we assume that $M/\bar{P} = c$.

benchmark economy, changing the length of price stickiness from one quarter to one year does not change the contract multiplier.

6. ROBUSTNESS

In our benchmark economy, the contract multiplier is extremely small because prices are extremely sensitive to movements in marginal costs. These results suggest that to generate a large contract multiplier, we must find some mechanism that makes prices insensitive to movements in marginal costs. We consider three such mechanisms in what follows. Then, we examine how the contract multiplier changes in the various models as we vary parameters, and we explore how combining the various mechanisms changes the multiplier.

6.1. Near-Perfect Substitute Preferences

First, we consider a specification of preferences in which consumption and leisure are near-perfect substitutes. With these preferences, γ is small, so that prices are insensitive to movements in marginal costs. Intuition suggests that these preferences should generate a large contract multiplier. We find that without the intertemporal links of capital accumulation and interest-sensitive money demand, they do. Once we add intertemporal links, however, the contract multiplier is small.

To see how the substitutability of consumption and leisure is related to γ, consider the following measure. Let η_e denote the *equilibrium elasticity of substitution* between leisure and consumption in a steady state with $c_{ss} = l_{ss} = (M/\bar{P})_{ss}$ defined by

$$\eta_e = - \frac{\partial \log\left(\left.\frac{1-l}{c}\right|_{ss}\right)}{\partial \log\left(\left.-\frac{U_l}{U_c}\right|_{ss}\right)}.$$

This elasticity measures the percentage change in the equilibrium ratio of leisure to consumption induced by a percentage change in the equilibrium wage rate. In equilibrium, the wage rate equals the marginal rate of substitution between leisure and consumption, consumption equals labor input, and, with static money demand, $c = M/\bar{P}$. It is straightforward to show that $\eta_e = 1/[\gamma(1 - c)]$. Clearly, for γ to be small, η_e must be large.

This argument leads us to consider utility functions of the form:

$$(41) \qquad U(c,l,M/\bar{P}) = \left[\frac{c^{1-\kappa}}{1-\kappa} - \omega_1\frac{l^{1+\xi}}{1+\xi} + \omega_2\frac{(M/\bar{P})^{1-\nu}}{1-\nu}\right]^{1-\sigma} \bigg/ (1-\sigma).$$

For such utility functions, if κ and ξ are small, so that the utility function is close to linear in both consumption and leisure, then η_e is large. Log-linearizing

the labor supply equation (13) gives $\gamma = \kappa + \xi$, where $1/\xi$ is the labor supply elasticity evaluated at the steady state. With these preferences, we can get a large contract multiplier as long as κ and ξ are small enough. For example, a κ of 0.0002 and a labor supply elasticity of 9,091 yield $\gamma = 0.00031$ and a contract multiplier of 20. These calculations suggest that preferences with high substitutability between consumption and leisure offer one route to persistence. (Blanchard and Fischer (1989) and Blanchard (1990) point to the importance of high labor supply elasticities in generating persistence. Our calculations suggest that near quasi-linearity in consumption is also crucial.)

To set the parameter ν, we use the money demand equation for this economy:

$$(42) \qquad \log\frac{M}{P} = \text{constant} + \frac{\kappa}{\nu}\log c - \frac{1}{\nu}\log\frac{R-1}{R},$$

where R is the gross nominal interest rate. This money demand equation makes clear that once κ is given, we cannot vary the interest elasticity of money demand without also varying the elasticity of consumption. Therefore, when we eliminate the intertemporal link of interest-sensitive money demand, we replace the money demand equation with a static money demand equation of the form $M/P = c$. When we allow intertemporal links, we set $\kappa = \nu$, so that the consumption elasticity of money demand is 1 (as balanced growth dictates).

The results for this model show that intertemporal links can have a dramatic effect on the contract multiplier. (See Table II.) With $N = 2$, no persistence in money growth, and no intertemporal links, the multiplier is 21.97. The same economy with intertemporal links has a multiplier of only 0.50. The results also show that the key intertemporal link that affects the multiplier is capital accumulation. For the economy with $N = 13$ and persistent money growth, the reduction in the multiplier is even more dramatic. Eliminating intertemporal links makes developing analytical solutions easy; that step is, therefore, usually thought of as a useful shortcut. The results here show that this shortcut can be misleading.

For this economy, the impulse responses following a one-time money shock are as follows. Output and consumption rise by a large amount in the impact period and by a much smaller amount thereafter. The contract multiplier is small because money shocks lead to an investment boom in the impact period, which raises the demand for labor and, therefore, wage rates. Since utility is nearly linear in both consumption and labor, the rise in wage rates leads to a large increase in both consumption and labor in the impact period relative to future periods. This makes both the half-life of output and the multiplier small.

Adding adjustment costs would increase the contract multiplier, but would also raise problems. With $N = 13$, persistent money growth, and both intertemporal links, the volatility of investment relative to output is 0.42, which is substantially less than the corresponding statistic in the data. Adding adjustment costs to investment will decrease this relative volatility even further, making this a poorer business cycle model along this important dimension. Experiments with

a large range of adjustment costs do not change the multiplier much: they reduce the size of the output response in the impact period but also in subsequent periods.

The results for this model also emphasize the role of interest-elastic money demand. In models with a static money demand equation of the form $\hat{m} - \hat{p} = \hat{y}$, the contract multiplier clearly is large if and only if the price level responds slowly over time. In our near-perfect substitutes model, money demand is interest elastic. It turns out that although the price level responds slowly, output is not affected very much, and the contract multiplier is tiny. The reason is that interest rates fall enough so that households willingly demand higher real balances, even though output has not risen much.

One problem with extreme substitutability between consumption and leisure is that these preferences have troublesome implications for the behavior of labor supply with even modest growth in wage rates and consumption. Suppose, for example, that wage rates and consumption grow 2 percent per year. Then, with our parameter values, labor input rises 57 percent each day.

6.2. Convex Demand

Now we explore an avenue suggested by Kimball (1995), designed to make prices less sensitive to marginal costs, which should tend to increase the size of the multiplier. To motivate Kimball's idea, note that the log-linearized pricing equation (29) implies that a 1 percent increase in marginal costs (which are simply wage rates in our model without capital) results in a 1 percent increase in prices set by the current cohort of intermediate goods producers. The intuition for this feature is as follows. In static models, a monopolist's markup of price over marginal cost is a simple function of the elasticity of demand. With constant elasticity of demand, the markup is constant, so that when costs rise 1 percent, so do prices.

Kimball (1995) suggests an alternative formulation of the final goods production technology that results in demand functions with the following feature: as prices rise, the elasticity of demand increases, so that the demand curve is convex. Demand functions of this kind can make the markup decline as prices rise, thus implying that a 1 percent increase in marginal costs results in a less than 1 percent increase in prices. Thus, after a monetary shock, each cohort of firms does not raise prices much, so that output deviates from its steady state for a long time.

We begin by analyzing this alternative formulation in a version of our model without capital and with $N = 2$. This alternative formulation closely follows Kimball's (1995) work. Let the technology for producing final goods be implicitly defined by $\int g(y_i/y)\,di = 1$, where y_i is the use of the intermediate good i and y is the output of the final good. Profit maximization by final goods producers, together with zero profits, implies that $P_i/\overline{P} = g'(y_i/y)/\int g'(y_i/y)(y_i/y)\,di$. This condition can be inverted to yield the demand function confronting each intermediate goods producer: $y_i^d/y = d((P_i/\overline{P})\int g'(y_i/y)(y_i/y)\,di)$, where $d \equiv$

$(g')^{-1}$. Substituting this demand function into the maximization problem (6), we can solve for the optimal price for intermediate goods producer i. Log-linearizing the resulting optimal pricing equation yields

$$(43) \qquad \hat{x}_{it} = \tfrac{1}{2}E_{t-1}\hat{p}_t + \tfrac{1}{2}E_{t-1}\hat{p}_{t+1} + \tfrac{\varphi}{2}E_{t-1}(\hat{w}_t + \hat{w}_{t+1}),$$

where again prices are written in log-deviation form. The parameter φ is given by

$$(44) \qquad \varphi = \frac{1 - (1/\varepsilon)}{2 - (\chi/\varepsilon)},$$

where ε is the elasticity of demand evaluated at the steady state (that is, $\varepsilon = -d'(g'(1))g'(1))$ and χ is the parameter governing the curvature of the demand function (that is, $\chi = -d''(g'(1))g'(1)/d'(g'(1)))$. Substituting $\hat{w}_t = [1 + \theta\omega/\psi]\hat{y}_t$ and (28) into (43), we obtain

$$(45) \qquad \hat{x}_t = \tfrac{1}{2}\hat{x}_{t-1} + \tfrac{1}{2}E_{t-1}\hat{x}_{t+1} + \gamma E_{t-1}(\hat{y}_t + \hat{y}_{t+1}),$$

where $\gamma = \varphi[1 + \theta\omega/\psi]$. Note that (45) has the same form as (35) but a different value for γ. The analysis above demonstrates that lower values of γ lead to more persistent output movements. The parameter φ is determined by the values of ε and χ. From equation (44), we can see that if the demand function is sufficiently convex, so that χ is sufficiently negative, then φ, and hence γ, can be made arbitrarily small and output can be made arbitrarily persistent. Note also that increases in ε, with χ held fixed, reduce φ. We show later that this relation is important.

The key question now is, what values of χ imply substantial persistence of output? Kimball (1995) chooses a parameterization in which a 1 percent increase in market share, y_i/y, leads to a decline in the elasticity of demand from 11 to 8. In our model, the elasticity of demand in the steady state is 10, so we choose a parameterization in which a 1 percent increase in market share, y_i/y, leads to a decline in the elasticity of demand from 10 to 7. We use a Taylor series expansion of the elasticity of demand around the steady state to calculate the value of the curvature parameter associated with Kimball's parameterization. The elasticity of demand is given by $\varepsilon(y_i/y) = -d'(g'(y_i/y))g'((y_i/y)/(y_i/y))$. Since $g'' = 1/d'$, we can see that the Taylor series expansion of $\varepsilon(y_i/y)$ around $y_i/y = 1$ is given by $\varepsilon(y_i/y) \cong \varepsilon(1) - [1 + \varepsilon(1) - \chi](y_i/y - 1)$. Using Kimball's parameterization, we obtain a value of $\chi = -289$. This value implies that $\varphi = 0.029$, and with our other parameters, the resulting value of γ is 0.045. This value implies a contract multiplier of 2.2.

Unfortunately, however, this parameterization implies that the demand function must be extraordinarily convex. To appreciate how convex this demand function must be, consider a second-order Taylor series expansion of the

demand function at the steady state, given by

$$(46) \qquad d\left(\frac{P_i}{\overline{P}}g'(1)\right) \cong 1 - \varepsilon(1)\left(\frac{P_i}{\overline{P}} - 1\right) + \frac{\varepsilon(1)\chi}{2}\left(\frac{P_i}{\overline{P}} - 1\right)^2.$$

Using the parameterization that gives a multiplier of 2.2, we see that this demand function implies that a 2 percent increase in relative prices results in a 78 percent reduction in demand, and a 2.3 percent increase in relative prices results in zero demand. A demand function with this extreme level of convexity is clearly inconsistent with both casual empiricism and a wide variety of demand studies for a wide range of products.

The contract multiplier is smaller in the presence of intertemporal links. (See Table II.) When $N = 13$ and money growth is persistent, the multiplier is 3.79 without intertemporal links and only 1.55 with such links. The intuition for these results is similar to that in the near-perfect substitutes model.

6.3. Specific Factors

Now we assume that each intermediate good is produced using a specific factor in addition to labor and capital. Each of these specific factors is inelastically supplied. The motivation for including these specificities, is, of course, to see if they lead to greater persistence in output. As our discussion of the benchmark model makes clear, a key factor determining persistence in models without intertemporal links is the sensitivity of the monopolist's price to changes in economy-wide output. With specific factors, this sensitivity is affected by two forces—the *output effect* and the *wage effect*. Increases in economy-wide output induce increases in the monopolist's output and raise the monopolists costs because of decreasing returns to scale. This increase in costs induces the monopolist to raise the price. Alone, the output effect makes prices more sensitive to changes in output. Increases in wage rates induce a smaller increase in the monopolist's price with decreasing returns to scale. Alone, the wage effect makes prices less sensitive to changes in output. The wage effect is stronger the more elastic is demand. When demand is sufficiently elastic, the wage effect dominates, the monopolist's price is relatively insensitive to aggregate output, and monetary shocks have more persistent effects.

We begin by analyzing a version of this model without intertemporal links. The production function for producing intermediate goods is given by $y(i, s^t) = l(i, s^t)^{1-\alpha}$. One interpretation of this production function is that it is a constant returns to scale production function in labor and a specific factor, such as land. The price-setting equation for the monopolist is given by the analog of (27):

$$(47) \qquad \hat{x}_{it} = \frac{1}{2}E_{t-1}\hat{p}_t + \frac{1}{2}E_{t-1}\hat{p}_{t+1} + \frac{1}{2}\frac{E_{t-1}(\hat{w}_t + \hat{w}_{t+1})}{1 + A\varepsilon}$$

$$+ \frac{A}{2}\frac{E_{t-1}(\hat{y}_t + \hat{y}_{t+1})}{1 + A\varepsilon},$$

where $A = \alpha/(1 - \alpha)$ and $\varepsilon = 1/(1 - \theta)$, which is the elasticity of demand. Comparing (47) to (27), we see that with decreasing returns to scale, two changes have occurred. Relative to (27), (47) has an extra term. This extra term, the last one in (47), is the output effect discussed above and reflects the increase in marginal costs associated with an increase in output. The response of the price to the wage rate is reduced by $1/(1 + A\varepsilon)$ and is the wage effect discussed above. With constant returns to scale ($A = 0$), this price-setting equation does not depend on the elasticity of demand or directly on aggregate output. With decreasing returns to scale, the larger is the demand elasticity, the less sensitive is the best response to wage rates and economy-wide output. Substituting $\hat{w}_t = [1 + \theta\omega/\psi]\hat{y}_t$ and (28) into (47) and imposing equilibrium gives an equation of the form in (35) with

$$(48) \qquad \gamma = \frac{1 + \theta\omega/\psi + A}{1 + A\varepsilon}.$$

For the parameters given in Table I—namely, $\theta = 0.9$, $\varepsilon = 10$, $\psi = 1.5$, and $\omega = 0.94$—and with $A = 3/2$, we get $\gamma = 0.19$. This value of γ yields a contract multiplier of only 1.35. If, however, the elasticity of demand is large enough, then γ can be made arbitrarily close to 0 and the contract multiplier can be made arbitrarily large. For example, we can generate a contract multiplier of 20 with an elasticity of demand of $\varepsilon = 6500$ ($\theta = 0.9998$).

While high enough demand elasticities can lead to substantial persistence, they also lead to extremely counterfactual implications regarding the distribution of output across firms in our economy. To see this, note that relative outputs of monopolists i and j are given by $y(i, s^t)/y(j, s^t) = [P(j, s^t)/P(i, s^t)]^\varepsilon$. For example, with $\varepsilon = 6500$, a 1 percent difference in relative prices implies a difference in relative outputs equal to 1.2×10^{30} percent (1.01^{6500}).

Introducing intertemporal links into this model also reduces the size of the contract multiplier. Assume that the technology for producing each intermediate good i is given by $F(k(i, s^t), l(i, s^t)) = k(i, s^t)^{\alpha_1} l(i, s^t)^{\alpha_2}$, where $\alpha_1 + \alpha_2 < 1$. The results in Table II show how the multiplier changes once we include capital and interest-sensitive money demand. For the specific factors economy, we set $\alpha_2 = 4/9$, and when capital is included, we set $\alpha_1 = 2/9$. We also assume that adjustment costs to capital are set so that the relative volatility of investment to output is similar to that in the data. Without capital or interest-sensitive money demand links, the multiplier is 1.82 with $N = 13$ and $\rho = 0.57^{1/13}$. Introducing capital and interest-sensitive money demand reduces this multiplier to 1.33, which is not much higher than the multiplier's value in our benchmark economy.

6.4. Sensitivity Analysis

Finally, we examine how the contract multiplier changes in the various models as we vary parameters, both one at a time and simultaneously. We also consider how combining the various models we have explored changes the multiplier.

Here we focus on key parameters. (In Chari, Kehoe, and McGrattan (1998), we report on how the multiplier varies with all of the parameters.) We find that the value of the multiplier is roughly 2 and that combining the various models does not raise the multiplier much.

We begin with the benchmark economy. In Figure 1 we show how the multiplier varies as we vary one parameter at a time for that economy, holding fixed the relative volatility of investment to output. The other parameters for this economy, with the exception of the adjustment cost parameter, are set at their calibrated values given in Table I. In Figure 1, panels A and B illustrate the roles of the intertemporal links. Panel A shows that the multiplier is roughly constant in the interest elasticity of money demand and that the multiplier rises only when the interest elasticity is less than 0.05. Panel B shows that the multiplier does not change much as the capital share varies.

Panel C in Figure 1 shows that making the utility function more concave than log (increasing the risk aversion parameter σ above 1) reduces the multiplier. The contract multiplier varies with the risk aversion parameter because in all our experiments we chose the adjustment cost parameter to produce the volatility of investment relative to output in the data. As the utility function gets more concave, the volatility of investment falls without adjustment costs. To keep the volatility of investment constant, we must reduce adjustment costs as we raise risk aversion. But lowering adjustment costs reduces the contract multiplier. When the risk aversion parameter is above 2.2, adjustment costs are not needed and the contract multiplier is small.

Panel D in Figure 1 illustrates that the contract multiplier increases as the demand elasticity ε $(= 1/(1 - \theta))$ increases. The intuition for this finding is similar to that in the specific factors economy. Since capital does not move much, it acts like a factor in fixed supply.

We also experiment by varying some key parameters simultaneously within the ranges reported in Table III. We set the discount factor β, the depreciation rate δ, and the share parameter in utility ω at their calibrated values. (See Table I.) Our ranges for the interest elasticity η, the capital share α, risk aversion σ, and the weight on leisure ψ are chosen to include most of the estimates provided in the literature. We choose ρ so that money growth ranges from independent and identically distributed to very persistent. We choose the elasticity of demand ε to range from fairly inelastic to an upper bound dictated by computational feasibility. Within these parameter ranges, the largest value of the multiplier we find is 3.05. This occurs with an interest elasticity of 0.2, a capital share of 0.22, a weight of 9 on leisure in utility, risk aversion of 1 (that is, log utility), a demand elasticity of 300, and a serial correlation of 0.96 on money growth.

In panel A of Figure 2, we show how the multiplier varies as we vary the demand curvature parameter χ for the convex demand economy, holding fixed the relative volatility of investment. The other parameters for this economy, with the exception of the adjustment cost parameter, are set at their calibrated values. (See Table I.) The inset to the figure shows that the contract multiplier is
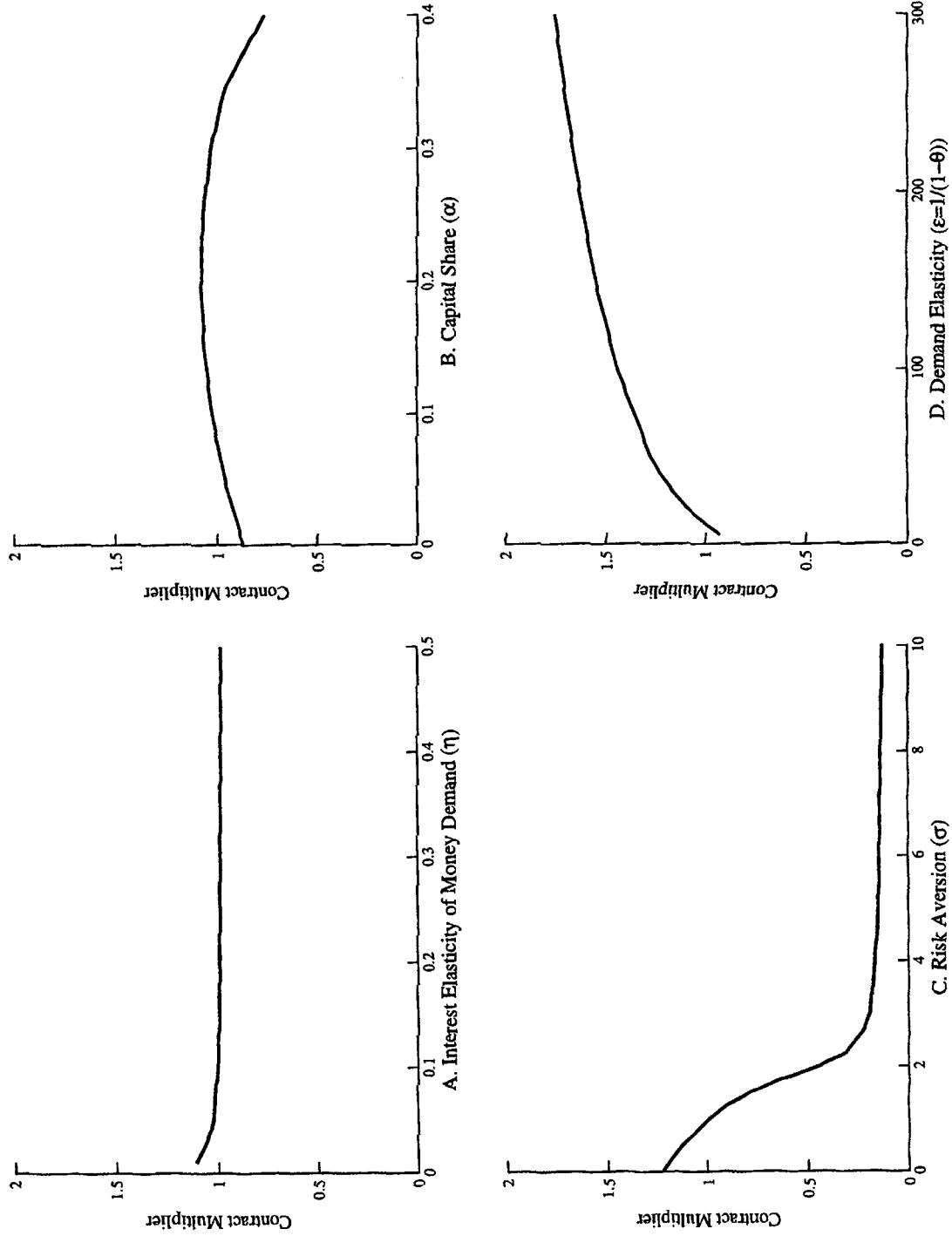
FIGURE 1.—Sensitivity analysis for the benchmark economy.

TABLE III

PARAMETER RANGES AND THE RESULTS OF SENSITIVITY ANALYSIS

Economy	Parameters	Values
Benchmark	Interest Elasticity (η)	[0.2, 0.5]
	Capital Share (α)	[0.22, 0.44]
	Weight on Leisure in Utility (ψ)	[0.11, 9]
	Risk Aversion (σ)	[1, 10]
	Demand Elasticity (ε)	[5, 300]
	Serial Correlation of Money Growth (ρ)	[0, 0.99]
	Maximum Multiplier	3.05
	($\eta = 0.2$, $\alpha = 0.22$, $\psi = 9$,	
	$\sigma = 1$, $\varepsilon = 300$, $\rho = 0.96$)	
Convex Demand	Demand Curvature (χ)	[−289, 11]
	Maximum Multiplier	3.20
	($\eta = 0.2$, $\alpha = 0.22$, $\psi = 9$,	
	$\sigma = 1$, $\varepsilon = 5$, $\rho = 0.96$, $\chi = -289$)	
Specific Factors	Capital Share (α_1)	[0.22, 0.44]
	Labor Share (α_2)	[0.33, 0.78]
	Maximum Multiplier	4.17
	($\eta = 0.2$, $\psi = 9$, $\sigma = 1$, $\varepsilon = 300$,	
	$\rho = 0.96$, $\alpha_1 = 0.22$, $\alpha_2 = 0.33$)	

1.8 with a demand curvature of −578—which is twice as large as that used by Kimball (1995). But even if we set the curvature parameter 100 times higher at $\chi = -57{,}800$, we do not get a contract multiplier above 5. Furthermore, with this curvature parameter, equation (46) implies that a 0.5 percent increase in relative prices results in zero demand. Even with such an extremely convex demand function, when we raise σ from 1 to 2.2, the contract multiplier falls from a value above 4 to a value close to 0. Again, this fall in the contract multiplier is due to the decrease in the adjustment cost parameter when σ is raised. The empirical asset pricing literature suggests a risk aversion parameter significantly above 1, so that even this level of convexity of demand may not be enough to generate the requisite persistence. When we vary the parameters simultaneously within the ranges reported in Table III, we find that the largest value of the multiplier is 3.20.

In panel B of Figure 2, we show how the multiplier varies as we vary the extent of decreasing returns to scale in the specific factors economy. We do so by varying the share of total income going to the fixed factor $(1 - \alpha_1 - \alpha_2)$. When varying this share, we keep α_2/α_1 equal to 2 and vary α_1 and α_2. When we do this, we see that the contract multiplier does not change much. When we vary the parameters simultaneously in the range reported in Table III, we find that the largest value of the multiplier is 4.17.

Finally, we consider a model in which we incorporate near-perfect substitutes, convex demand, and specific factors in order to look for significant interaction effects. In Table II we report the results for this *combined* model at our
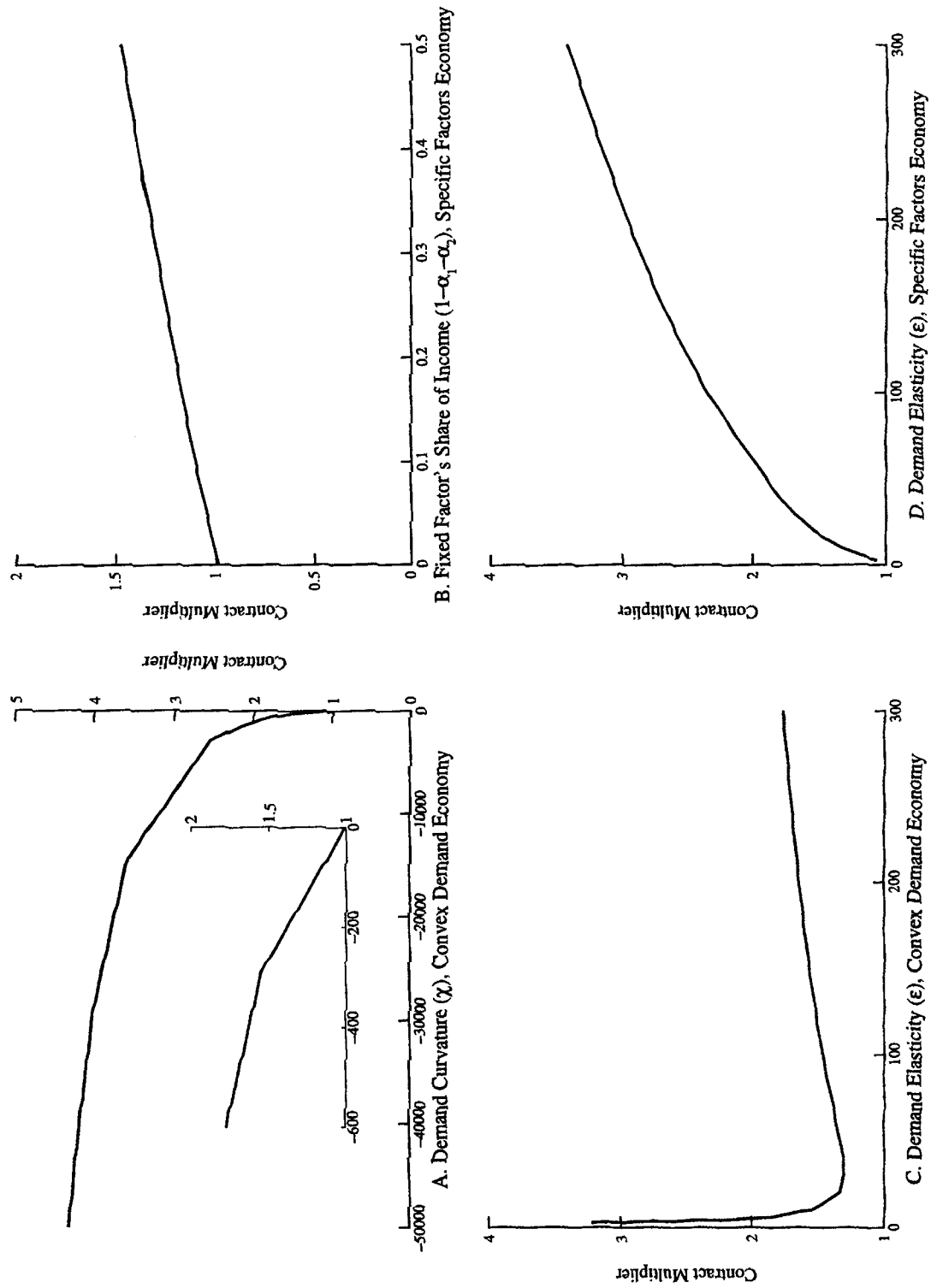
FIGURE 2.—Sensitivity analysis for two economies.

calibrated values. The last column of Table II shows that combining these features raises the multiplier modestly to 1.81 from 1.55 (convex demand only) and 1.33 (specific factors only). When we vary the parameters simultaneously in the ranges reported in Table III, the maximum value of the contract multiplier is 4.17 and occurs with the same extreme parameter settings as the maximum of the specific factors economy.

One way to understand this is to note that any model with near-perfect substitutes gives a tiny contract multiplier. Thus, the only possible interesting interactions are between the convex demand and specific factors specifications. Panels C and D of Figure 2 show that the convex demand specification gives a large multiplier when the demand elasticity is small, while the specific factors specification gives a large multiplier when the demand elasticity is large. So when we combine the two features, we can either choose to make the elasticity small and get essentially the same multiplier as with convex demand alone or choose to make the elasticity large and get essentially the same multiplier as with specific factors alone. In no sense do these three variations tend to reinforce each other, and in no sense is the multiplier for the model with all three variations the sum of the multipliers for each one separately.

7. CONCLUSION

The central challenge to a theory of monetary business cycles is to find a solution to the persistence problem. Monetary economists have long searched for a mechanism that has a multiplier effect in the sense that small frictions lead to long periods of endogenous price rigidity and, hence, persistent output movements. Here we find that the staggered price-setting mechanism is not the long-sought solution.

One interpretation of our findings is that if 5 years of exogenous price stickiness is plausible, then conventional models used in the business cycle literature can generate persistent output fluctuations. With the amount of convexity in demand posited by Kimball (1995), or with extremely elastic demand and sizable departures from constant returns to scale, generating persistence requires $2\frac{1}{2}$ years of exogenous price stickiness. If one quarter of exogenous price stickiness is plausible, then generating persistence requires extraordinarily convex demand or extremely elastic demand.

Altogether, these findings suggest that mechanisms to solve the persistence problem must be found elsewhere.

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