

ECON 4261: Introduction to Econometrics
Fall 2009

Problem Set 1
Due: Oct 6, 2009

Exercise 1 Let A be an $(m \times n)$ matrix, with $m > n$. Show that the matrix $A'A$ is symmetric, and that if A has full rank, $A'A$ is positive definite.

Exercise 2 Characterize each of the following statements as true or false. If a statement is true, show the proof. If it is false, give a counterexample. Assume that A and B are square matrices.

(i) If A and B are non-singular, then AB is non-singular.

(ii) $(AB)' = A'B'$.

(iii) If $AB + BA = 0$, then $A^2B^3 = B^3A^2$.

(iv) If $A^2 = 0$, then $A = 0$.

(v) If A and B are symmetric, then AB is symmetric.

Exercise 3 Let X and Y be two random variables. Consider the following joint density function

$$f(x, y) = \begin{cases} A(x^2 + y^2) & \text{for } x \in [0, 1] \text{ and } y \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

(i) Find A .

(ii) Find the marginal density $f(x)$.

(iii) Find the conditional density $f(y | x)$.

- (iv) Find $Cov(X, Y)$.
- (v) Are X and Y independent?
- (vi) Find $P(\frac{1}{2} \leq x \leq 1, \frac{1}{2} \leq y \leq 1)$.

Exercise 4 Let

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 4 & 2 \\ 0 & 7 & 3 \end{bmatrix}$$

- (i) Find $A'A$. Show your work.
- (ii) Find A^{-1} . Show your work.
- (iii) Calculate the *trace* of A .

Exercise 5 Let X be a random variable with the following density function

x	$P(X = x)$
2	0.08
5	0.12
7	0.22
9	0.03
14	0.28
21	0.17
25	0.10

- (i) Calculate $E(X)$. Show your work.
- (ii) Calculate $Var(X)$. Show your work.

Exercise 6 Let X , and Y be two random variables such that $X \sim U[0, 1]$, and $Y = 3X + 2$. Calculate the values for: $(E[X])^2$, $E[X^2]$, $E[Y]$, $Var[X]$, $Var[Y]$, and $Cov[X, Y]$. Show your work.

Exercise 7 Give an example of two random variables X and Y such that $Cov(X, Y) = 0$, but X and Y are not independent.

Exercise 8 Show that if X and Y are two independent random variables, then $Var(X + Y) = Var(X) + Var(Y)$.

Exercise 9 Consider two unbiased independent estimators $\hat{\beta}$ and $\tilde{\beta}$, constructed to estimate the parameter β . Suppose you construct a third estimator β^* that is a convex combination of $\hat{\beta}$ and $\tilde{\beta}$, that is, $\beta^* = \alpha\hat{\beta} + (1 - \alpha)\tilde{\beta}$, with $\alpha \in [0, 1]$.

- (a) Is β^* an unbiased estimator?
- (b) Find the variance of β^* and express it in terms of the variance of $\hat{\beta}$ and $\tilde{\beta}$.
- (c) What value of α would you choose to calculate β^* ? Justify your answer.