ECON 4261: Introduction to Econometrics
Fall 2009

Problem Set 2

Due: Oct 22, 2009

Exercise 1 Suppose you model a variable Y depending on a non-stochastic variable X, according to the relation $Y_i = \beta X_i + u_i$. The disturbance terms are iid with mean zero and variance σ^2. Considering estimating β by the slope of a line between the origin and one out of the n plotted observations (X_i, Y_i).

(a) Is this estimator unbiased?

(b) Find the variance of this estimator.

(c) Which of the observations would you choose to calculate the estimate? Justify your answer.

Exercise 2 You are given 30 pairs of observations (X_i, Y_i) which are to be represented by the following model $Y_i = \beta_0 + \beta_1 X_i + u_i$, and the errors are iid with mean zero and variance σ^2. Suppose you know that $R^2 = 0.25$, $\sum_{i=1}^{n}(X_i - \overline{X})^2 = 49$ and $\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 49$. Calculate the OLS estimate for β_1 (assume is positive).

Exercise 3 (*Regression without any regressor*) Suppose you are given the model $Y_i = \beta_1 + u_i$. Use OLS to find the estimator of β_1. What is its variance and RSS? What is the coefficient of determination? (R^2)

Exercise 4 Suppose you estimate the consumption function

$$Y_i = \alpha_1 + \alpha_2 X_i + u_{1i}$$

and the savings function

$$Z_i = \beta_1 + \beta_2 X_i + u_{2i}$$

where Y is consumption, X is income, Z is saving and $X = Y + Z$, that is income is equal to consumption plus savings.

(a) What is the relationship between $\hat{\alpha}_2$ and $\hat{\beta}_2$? ($\hat{\alpha}_2$ and $\hat{\beta}_2$ are the OLS estimators for α_2 and β_2) Show your calculations.

(b) Will the residual sum of squares, RSS, be the same for the two models? Explain.

Exercise 5 Consider the following models:

$$Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + u_{1i}$$

$$(Y_i - X_{2i}) = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_{2i}$$

(a) Will the OLS estimates of α_1 and β_1 be the same?

(b) Will OLS estimates of α_3 and β_3 be the same?

(c) What is the relationship between α_2 and β_2?

(d) Can you compare the R^2 of the two models? Why or why not?

Exercise 6 Suppose you have the following model $Y_i = \beta_1 + \beta_2 X_i + u_i$ where u_i satisfies the assumptions of the classical linear regression model. Suppose you have fixed values for X's: $X_1 = 1$, $X_2 = 2$, $X_3 = 3$, $X_4 = 4$, $X_5 = 5$ and $X_6 = 6$. An econometrician with no calculator and an aversion to arithmetic proposes to estimate the slope of the linear relation between Y and X by:

$$\hat{\beta}_2 = \frac{1}{8}(Y_6 + Y_5 - Y_2 - Y_1)$$

Show that this estimator is unbiased, i.e. $E(\hat{\beta}_2) = \beta_2$

Exercise 7 Let $\hat{\beta}_{YX}$ and $\hat{\beta}_{XY}$ represent the slopes in the regression of Y on X and X on Y, respectively. Show that

$$\hat{\beta}_{YX}\hat{\beta}_{XY} = r^2$$

where r is the coefficient of correlation between X and Y.

Exercise 8 In class we showed that the OLS estimator of β_1 could be written as

$$\widehat{\beta}_1 = \frac{\sum_{i=1}^n X_i Y_i - n\overline{XY}}{\sum_{i=1}^n X_i^2 - n\overline{X}^2} \text{ or } \widehat{\beta}_1 = \frac{\sum_{i=1}^n (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^n (X_i - \overline{X})^2}$$

Prove that $\widehat{\beta}_1$ can also be written as

$$\widehat{\beta}_1 = \frac{\sum_{i=1}^n (X_i - \overline{X}) Y_i}{\sum_{i=1}^n (X_i - \overline{X})^2} \text{ or } \widehat{\beta}_1 = \frac{\sum_{i=1}^n X_i (Y_i - \overline{Y})}{\sum_{i=1}^n (X_i - \overline{X})^2}.$$