

ECON 4261: Introduction to Econometrics
Fall 2009

Problem Set 3
Due: Nov 17, 2009

Exercise 1 Given the following OLS estimates,

$$\begin{aligned}C_t &= \text{constant} + 0.92Y_t + u_{1t} \\C_t &= \text{constant} + 0.84C_{t-1} + u_{2t} \\C_{t-1} &= \text{constant} + 0.78Y_t + u_{3t} \\Y_t &= \text{constant} + 0.55C_{t-1} + u_{4t}\end{aligned}$$

Calculate the OLS estimates of β_2 and β_3 in

$$C_t = \beta_1 + \beta_2 Y_t + \beta_3 C_{t-1} + u_t$$

Exercise 2 Prove that $y'y$ where $y = Ay$ with $A = I - \frac{1}{n}ii'$, is the RSS when Y is regressed on $x = i$. (i is a column vector of ones) Show also that the estimated coefficient of the regression is \bar{Y} .

Exercise 3 The following regression equation is estimated as a production function for Y :

$$\begin{aligned} \ln Y &= 1.37 + 0.632 \ln K + 0.452 \ln L \\ &\quad (0.257) \quad (0.219) \\ R^2 &= 0.98 \quad \text{cov}(\hat{\beta}_K, \hat{\beta}_L) = 0.055 \end{aligned}$$

where standard errors are given in parentheses. Test the following hypothesis:

- (i) The capital and labor elasticities of output are identical.
- (ii) There are constant returns to scale.

Exercise 4 Show that the F-statistic for testing the joint significance ($H_0 : \beta_2 = \beta_3 = \dots = \beta_k = 0$) can be written as the following:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

(Hint: How does the restricted model look like? What is the RSS_r , ESS_r , and TSS_r ?)

Exercise 5 Show that $A = I - \frac{1}{n}ii'$ is a symmetric idempotent matrix. Show also that $Ai = 0$. Show that $M = (I - x(x'x)^{-1}x')$ is a symmetric idempotent matrix and $\hat{u} = MY$ where $Y = X\beta + u$. Show also that $\hat{u} = Mu$.

Exercise 6 Consider the following model :

$$Y_i = \beta_0 + \beta_1 D_i + u_i$$

where $D_i = 0$ for the first 20 observations and $D_i = 1$ for the remaining 30 observations. You are also told that $var(u_i) = 300$.

(i) Suppose Y denotes annual starting salary and the dummy variable D denotes 1 if the applicant is a graduate of UMN and 0 if the applicant is a graduate of UWisconsin. In words, how would you interpret β_0 and β_1 ?

(ii) What are the mean values annual starting salary of the two groups (UMN graduates vs. UWisc graduates)?

(iii) How would you compute the variance of $(\hat{\beta}_0 + \hat{\beta}_1)$? Note: You're given that the $cov(\hat{\beta}_0, \hat{\beta}_1) = -15.0$

Exercise 7 Consider the following model:

$$Y = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u$$

The number of observations is $n = 50$. Suppose we want to test:

$$H_0 = \beta_2 + 2\beta_3 = 1$$

Write the restricted model and the procedure to test the given null hypothesis. Clearly indicate the distribution and the degrees of freedom of the test statistic.