ECON 4261: Introduction to Econometrics
Fall 2009

Formulas for the Midterm

Regression with one explanatory variable

- Model: $Y_i = \beta_1 + \beta_2 X_i + u_i$
- Assumptions
  - $E(u_i) = 0$, for all i
  - $Var(u_i) = \sigma^2$, for all i
  - $cov(u_i, u_j) = 0$ for all i, j
- Normal equations: $\sum_{i=1}^{n} \widehat{u}_i = 0$ and $\sum_{i=1}^{n} X_i \widehat{u}_i = 0$
- $\widehat{\beta}_2 = \frac{\sum_{i=1}^{n} X_i Y_i - n\overline{XY}}{\sum_{i=1}^{n} X_i^2 - n\overline{X}^2} = \frac{\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})}{\sum_{i=1}^{n}(X_i-\overline{X})^2} = \frac{\sum_{i=1}^{n}(X_i-\overline{X})Y_i}{\sum_{i=1}^{n}(X_i-\overline{X})^2} = \frac{\sum_{i=1}^{n} X_i(Y_i-\overline{Y})}{\sum_{i=1}^{n}(X_i-\overline{X})^2}$
- $\widehat{\beta}_1 = \overline{Y} - \widehat{\beta}_2 \overline{X}$ $\quad var(\widehat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{n}(X_i-\overline{X})^2}$ $\quad \widehat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n} \widehat{u}_i^2$
- $TSS = \sum_{i=1}^{n}(Y_i-\overline{Y})^2$ $\quad ESS = \sum_{i=1}^{n}(\widehat{Y}_i-\overline{\widehat{Y}})^2$ $\quad RSS = \sum_{i=1}^{n}(Y_i-\widehat{Y}_i)^2 = \sum_{i=1}^{n}\widehat{u}_i^2$
- $R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = \frac{\widehat{\beta}_2^2 \sum_{i=1}^{n}(X_i-\overline{X})^2}{\sum_{i=1}^{n}(Y_i-\overline{Y})^2} = \frac{\left[\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})\right]^2}{\sum_{i=1}^{n}(X_i-\overline{X})^2 \sum_{i=1}^{n}(Y_i-\overline{Y})^2}$
- If $u_i \sim iidN(0, \sigma^2) \Rightarrow \frac{\widehat{\beta}_i - \beta_i}{std(\widehat{\beta}_i)} \sim t_{(n-2)}$ $\quad i = 1, 2$

Regression with more than one variable

- Model: $Y_i = \beta_1 + \beta_2 X_{2i} + ... + \beta_k X_{ki} + u_i$ $\quad \rightarrow Y = X\beta + u$
- Normal equations: $X'X\widehat{\beta} = X'Y$ $\quad X'\widehat{u} = 0$
- $\widehat{\beta} = (X'X)^{-1}X'Y$
- $var(\widehat{\beta}) = \sigma^2(X'X)^{-1}$