ECON 4261: Introduction to Econometrics
Fall 2009

Some Questions

Exercise 1 *Indicate whether the following statements are true or false. Also, explain your reasoning, and show any calculations, statements and formula to support your reasoning. You do not need to provide reasoning if your answer is "true".*

1. If there is no intercept in the model, residuals will sum up to zero, i.e. $\sum_{i=1}^{n} \hat{u}_i = 0$.

2. Suppose you have the following model,

$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + u_i$$

   then the regression model suffers from perfect multicollinearity.

3. Even though the disturbance term u_i in the classical regression model is not normally distributed, OLS estimators are still unbiased.

4. Suppose the regression model specified as $y_i = \beta_2 x_i + u_{1i}$. The estimated slope coefficient $\hat{\beta}_2$ is just the inverse of the estimated slope coefficient from the regression x on y, i.e. $\hat{\alpha}_2$ obtained from $x_i = \alpha_2 y_i + u_{2i}$.

5. Suppose the regression model is specified as $Y_i = \beta_1 + u_i$. Then the intercept coefficient is estimated as $\hat{\beta}_1 = \bar{Y}$.

6. Suppose you estimate the model, $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$ and you observe that the R^2 is high but the individual coefficients are insignificant, i.e. you fail to reject the null hypothesis $H_0 : \beta_1 = 0$ and $H_0 : \beta_2 = 0$. This observation is a strong indication for multicollinearity in the model.

Exercise 2 A sample of 20 observations corresponding to the model

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

where $u_i' s$ are normally and independently distributed with zero mean and constant variance, gave the following data:

$$\sum Y_i = 21.9 \qquad \sum (Y_i - \bar{Y})^2 = 86.9 \qquad \sum (X_i - \bar{X})(Y_i - \bar{Y}) = 106.4$$
$$\sum X_i = 186.2 \qquad \sum (X_i - \bar{X})^2 = 215.4$$

Estimate β_1 and β_2 and their standard errors and construct 95% confidence intervals for β_1 and β_2.

Exercise 3 Prove that $y'y$ where $y = Ay$ with $A = I - \frac{1}{n} ii'$, is the RSS when Y is regressed on $x = i$. (i is a column vector of ones) Show also that the estimated coefficient of the regression is $\bar{Y}$.

Exercise 4 Given the following OLS estimates,

$$\begin{aligned} C_t &= constant + 0.92 Y_t + \hat{u_{1t}} \\ C_t &= constant + 0.84 C_{t-1} + \hat{u_{2t}} \\ C_{t-1} &= constant + 0.78 Y_t + \hat{u_{3t}} \\ Y_t &= constant + 0.55 C_{t-1} + \hat{u_{1t}} \end{aligned}$$

Calculate the OLS estimates of β_2 and β_3 in

$$C_t = \beta_1 + \beta_2 Y_t + \beta_3 C_{t-1} + u_t$$

Exercise 5 Prove that R^2 is the square of the correlation coefficient between Y and $\hat{Y}$ where $\hat{Y} = X\hat{\beta}$.

Exercise 6 Show that $A = I - \frac{1}{n} ii'$ is a symmetric idempotent matrix. Show also that $Ai = 0$.

Exercise 7 The following regression equation is estimated as a production function for Y:

$$lnY = 1.37 + \underset{(0.257)}{0.632} lnK + \underset{(0.219)}{0.452} lnL$$

$$R^2 = 0.98 \qquad cov(\hat{\beta_K}, \hat{\beta_K}) = 0.055$$

where standard errors are given in parentheses. Test the following hypothesis:

(i) The capital and labor elasticities of output are identical.

(ii) There are constant returns to scale.