

Complements versus Substitutes and Trends in Fertility Choice in Dynastic Models*

Larry E. Jones[†] and Alice Schoonbroodt[‡]

First version: March 2007

This version: March 2008

Abstract

The Barro-Becker dynastic model is a simple intuitive model of fertility choice. In its original formulation, however, it has not been very successful at reproducing the changes in fertility choice in response to decreased mortality and increased income growth that demographers have emphasized in explaining the demographic transition. In this paper we show that this is due to an implicit assumption that number and utility of children are complements, which is a byproduct of the high intertemporal elasticity of substitution (IES) typically assumed in the fertility literature. We show that, not only is this assumption not necessary, but both the qualitative and quantitative properties of the model in terms of fertility choice change dramatically when substitutability and high curvature are assumed. We find that with an IES less than one, model predictions of changes in fertility amount to about two-thirds of those observed in U.S. data since 1800. There are two major sources to these predicted changes: the increase in the growth rate of productivity which accounts for about 90 percent of the predicted fall in fertility before 1880; and changes in mortality which account for 90 percent of the predicted change from 1880 to 1950.

Keywords: Fertility choice, substitutes, demographic changes.

JEL Classification Numbers: J11, J13, E13, O11

*The authors thank the National Science Foundation for financial support. We thank Martin Gervais, John Knowles, Jeremy Lise, and Michèle Tertilt as well as seminar participants at SITE 2007, Vienna Macro Workshop 2007, NYU, CEMFI, EUI, and the University of Minnesota for their comments. We also thank Michael Bar and Oksana Leukhina for their valuable comments and for making their data available to us and Anderson Schneider for valuable research assistance.

[†]University of Minnesota and Federal Reserve Bank of Minneapolis

[‡]University of Southampton

1 Introduction

The seminal papers by Barro and Becker (Becker and Barro (1988) and Barro and Becker (1989)) have generated increased interest in the study of the macroeconomics of fertility choice. The model is intuitive and analytically tractable. In its original formulation, however, it has not been very successful in generating the desired effects of changes in mortality and productivity on fertility choices and population growth rates as emphasized by demographers (see Doepke (2005), Bar and Leukhina (2007)). In this paper we show that this is mainly due to the high intertemporal/intergenerational elasticity of substitution (IES) typically assumed in the fertility literature.

There are two reasons why the choice of the IES matters for qualitative as well as quantitative results. The first is that an implicit complementarity/substitutability assumption is embedded in these preference formulations. High IES, as in the original Barro-Becker formulation, implicitly introduces the assumption that family size and utility of children are complements in utility. To the contrary, low IES implies that they are substitutes in utility. This has important implications for the *sign* of first order effects. For instance, if number and utility of children are complements, the marginal utility of an extra child increases when productivity growth (and hence, utility of children) increases. All else equal, this implies an increase in fertility and the population growth rate. With low elasticity and substitutability, the opposite is true.

The second reason is related to children being partly an investment good in these models and is most important for the *size* of the fertility response to changes in mortality and productivity. Here, much of what we find comes from basic intuitions about the desirability of changes in the growth rate of consumption for different values of IES—again, when IES is low, the value of increasing the growth rate of consumption is also low. In a dynastic model the principal effect of permanently increasing fertility, that is, the population growth rate, is to permanently increase growth rates in dynasty aggregates. Hence, because of the low IES any change in economic environment that facilitates this growth is met with a higher *fraction* of output going to current consumption and less to the investment good—fertility. This causes the stock of population to *grow* more slowly than with high values for the IES.¹

In this paper we study a simple version of the original Barro-Becker model to determine how important these effects are both qualitatively and quantitatively. First, we revisit the properties of Barro-Becker preferences related to the intertemporal elasticity of substitution. Here, we show that one implicit restriction, namely positive utility, has led to the assumption of complementarity between number and utility of children and consequently high IES in the fertility literature. This restriction can be relaxed to also allow negative utility and substitutability between number and utility of children for low values of IES.

To build intuition, we derive simple comparative statics at the household level

¹Other papers with children as an investment good include Ehrlich and Lui (1991), Boldrin and Jones (2002), Boldrin, De Nardi, and Jones (2005).

to highlight the effect of the complementarity versus substitutability assumption for changes that exogenously increase children’s utility. Throughout, we abstract from bequests and assume (exogenously growing) labor income only. We then introduce longevity through random survival of adults and child mortality to derive the dynasty’s utility and law of motion for population. In the resulting dynasty utility, it becomes apparent how the choice of elasticity above or below unity leads to population size and aggregate consumption being complements or substitutes, respectively.

In solving the dynastic planner’s problem, the simplicity of the model allows us to derive analytical comparative statics across balanced growth paths (BGP) for high versus low IES. We analyze the fertility response to permanent changes in productivity growth rates, longevity and youth mortality and distinguish between three measures of fertility, namely population growth rates, surviving children and total births. Our findings are summarized here:

1. The effect of an increase in the growth rate of productivity is to increase population growth and fertility if the IES is greater than one (complementarity) and decrease both population growth rates and surviving fertility if the IES is less than one (substitutability).²
2. A reduction in youth mortality decreases the cost of producing a surviving child and hence, increases surviving fertility and population growth rates for any value of IES. The size of this increase is increasing in the IES—in a quantitatively significant way. We also find that this tends to give a large decrease in births for low IES, and a small decrease in births for high IES.
3. Increasing expected work life conditional on reaching adulthood increases population growth rates for all values of IES, but typically decreases surviving fertility if the IES is low enough. However, the increase in population growth rates is increasing in the IES, while the decrease in surviving births, if any, is decreasing in the IES.

Since population growth rates slightly decreased over the last 200 years, while the number of births decreased substantially, these comparative statics results are interesting, both in a qualitative and in a quantitative sense. Qualitatively, commonly used values for the IES in the growth and real business cycle (RBC) literatures (below one) also produce the observed direction of changes in fertility choice in response to changes in productivity growth due to substitutability. Quantitatively, with low IES changes in mortality generate only small increases in population growth rates which may be overturned by the effects of productivity growth. This resolves a puzzle often encountered in the fertility literature, where due to low values of IES, population

²Our model, like many based on dynastic altruism, has a Balanced Growth Path (BGP) property. Changes in the *level* of productivity has no permanent effect on fertility choice. Rather it is the acceleration in income growth that causes permanent changes. This distinguishes our model from many other papers in the field, see the discussion below.

growth rates tend to increase dramatically in response to these changes—unless more elaborate effects are taken into account (see below).

Finally, we study quantitative predictions of the model in relation to U.S. history. First, we calibrate child costs in the model to match the recent fertility experience in the U.S. given its economic circumstances. That is, we match population growth of 0.65 percent per year given the observed productivity growth rate, infant and child mortality rates and longevity. We then simulate the U.S. experience since 1800, taking the timing of events and all three changes in economic environment into account. We find that with an IES of one third, the model predicts that the Crude Birth Rate would fall from 36.3 to 17.2 births per 1000 population. This corresponds to about two-thirds of the observed change in U.S. data. In terms of population growth rates, the model predicts a fall from 1.4 to 0.65 percent per year which captures about one half of observed changes in the U.S.. Interestingly, about 90 percent of model predicted changes in fertility before 1880 are accounted for by changes in productivity growth rates, while changes in mortality account for about 90 percent of the predicted fertility decrease thereafter. This finding speaks to the debate about what the root cause of the fertility decline was—was it reductions in youth mortality rates or was it accelerating economic development? What we find is: it was a combination of both with the latter being the most important factor early in the transition and mortality being the most important factor later on.

These findings also help clarify the results of several authors who have recently studied quantitative versions of the Barro-Becker model to examine its ability to track the basic trends over the last 200 years in fertility choices. These include Mateos-Planas (2002), Doepke (2005), and Bar and Leukhina (2007). They study the quantitative response of fertility to changes in the environment including infant, youth, and adult mortality, as well as changes in productivity growth rates in a variety of different, but related economic environments. Surprisingly, they reach very different conclusions. For example:

1. In Mateos-Planas (2002) changes in mortality have only small effects on population growth and hence, he concludes that changes in productivity growth and/or changes in the cost of child-rearing are more likely to be the sources of the changes in fertility seen in the data.
2. Doepke (2005) also finds that reductions in child mortality appear not to be responsible for the observed changes in fertility. In fact, not only is the predicted change in births small, but the simple version of the model also has the counterfactual implication that population growth rates and surviving fertility increase substantially.
3. Bar and Leukhina (2007) study both, the effects of changes in young-age mortality and productivity growth, and conclude that the observed decreases in mortality are responsible for a large fraction of the reduction in fertility, but that changes in rates of productivity growth have very little effect.

Doepke assumes that the IES is greater than one; and Bar and Leukhina use a value equal to one; Mateos-Planas assumes that the IES is less than one but focuses only on population growth rates rather than births. Given the discussion above, we see that the differences in their assumptions regarding the IES, and hence, implicitly, complementarity versus substitutability between number and utility of children, is key to reconciling these seemingly contradictory results.

Other papers have also studied trends in fertility but have focused on different channels and setups including increased *levels* of income/productivity and changes in education laws/subsidies. Examples include Becker, Murphy, and Tamura (1990), Tamura (1994, 1996), where the crucial feature for the fertility decrease is an increase in the rate of return to human capital across multiple equilibria in a model with increasing returns; Galor and Weil (2000) and Greenwood and Seshadri (2002), where the change in fertility is generated by increased *levels* of income with non-homotheticities in utility and non-convexities in the education choice/technology; Doepke (2004), where the change in fertility is also driven by increases in the *level* of productivity, here in skill intensive technologies, and the interaction between this and changes in child labor laws and education subsidies is studied (see also Doepke and Zilibotti (2005)); Fernandez-Villaverde (2001), where the key feature is falling capital prices when capital and skilled labor are complements; Kalemli-Ozcan (2002, 2003), where the reduction in risk concerning family size that comes about from a reduction in mortality decreases fertility (see also Doepke (2005) who adds sequential fertility choice to this mechanism); and de la Croix and Licandro (2007) where a decrease in fertility is generated through an increase in the productivity for health production; etc.. We view these channels as complementary to ours and discuss some of these mechanisms in more detail later in the paper. In particular, our finding that the model predictions are so much more in line with the data when family size and descendant utility are substitutes does not mean either that this is the correct specification of preferences or that these other channels are unimportant. Rather, it shows that this feature of the model—which we know very little about—is key and worthy of further research. Alongside other extensions, we discuss potential ways of getting more direct evidence that may support (or refute) the assumption of substitutability in the last section.

The remainder of the paper is organized as follows. Section 2 revisits preference configurations and gives intuitive results. In Section 3 we lay out the basic model of fertility choice that we focus on. In Section 4 we solve the dynastic planner’s problem and derive analytical comparative statics for population growth, surviving children and all births across BGPs. Section 5 presents model predictions for the U.S. experience since 1800. In Section 6 we discuss various issues and extensions. Section 7 concludes.

2 Barro-Becker altruism revisited

In this section, we discuss Barro-Becker type preferences and show how one implicit restriction, namely positive utility, has led to the assumption of complementarity between number and utility of children. We also show why this assumption is crucial in determining the fertility response to permanent changes in productivity growth, youth mortality and longevity.

2.1 The household problem: complements versus substitutes

The standard presentation of the Barro-Becker model usually begins with a description of the preferences of a period- t adult. It is assumed that parents care about three separate objects:

- i) their own consumption in the period, c_t ;
- ii) the number of children they have, n_t ;
- iii) the average utility of their children, U_{t+1} .

This is usually specialized further. It is assumed that utility of the typical time- t household is of the form:

$$U_t = u(c_t) + \beta g(n_t) \sum_{i=1}^{n_t} \frac{1}{n_t} U_{it+1}$$

where U_{it+1} is the utility of the i^{th} child of the parent. Assuming equal treatment $U_{it+1} = U_{i't+1} = U_{t+1}$ for all i, i' , this simplifies to:

$$U_t = u(c_t) + \beta g(n_t) U_{t+1}.$$

Intuitively, the following are desirable properties of utility:

- 1.) Parents like the consumption good:
utility is increasing and concave in own consumption;
- 2.) Parents are altruistic (with respect to born children):
holding n_t fixed and increasing U_{t+1} increases (strictly) the utility of the parent;
- 3.) Parents like having children:
holding U_{t+1} fixed and increasing n_t increases (strictly) the utility of the parent.
- 4.) The increase described in 3.) is subject to diminishing returns.

The first property is satisfied as long as u is increasing and concave. The second has implications for what g can be. Since $\partial(u(c) + \beta g(n)U)/\partial U = \beta g(n)$, it follows that (2.) implies that $g(n) > 0$ for all n . The third requirement is less straightforward. Although this requirement makes intuitive sense, some issues arise because of the special restrictions implicit on functional forms. For example, suppose that $U_{t+1} > 0$. Then (3.) implies that $g(n)$ must be increasing in n . On the other hand, if $U_{t+1} < 0$, (3.) implies that $g(n)$ should be decreasing in n . It follows that if it is possible

for U_{t+1} to be either positive OR negative, it is impossible to satisfy all of these requirements simultaneously. In sum, (1.)-(3.) are mutually inconsistent without some sort of restrictions on the possible values for U_{t+1} . Similar issues arise with respect to (4.). If U_{t+1} is restricted to be positive, (4.) requires g to be concave while if U_{t+1} is restricted to be negative, (4.) requires that g is convex.

This is not to say that these conditions cannot be satisfied. We must simply assume that either $U_{t+1} > 0$ always or $U_{t+1} < 0$ always and then make the appropriate assumptions on g . Without an assumption like this, the natural monotonicity properties of utility cannot be guaranteed. Thus, we are left with two options:

- I. Assume $g(n)$ is non-negative, strictly increasing and concave and $U > 0$.
- II. Assume $g(n)$ is non-negative, strictly decreasing and convex and $U < 0$.³

As it turns out, the choice between these two alternatives has important implications for the properties of the model. This can be illustrated in a simple example. First, consider how the solution of the problem of a time zero parent changes when the growth rate in wages is changed. In the simplest case, they face a problem of the form:

$$\begin{aligned} \max_{\{c_0, n_0\}} \quad & u(c_0) + \beta g(n_0)U_1 \\ \text{s.t.} \quad & c_0 + \theta_0 n_0 \leq w_0, \end{aligned}$$

where θ_0 is the cost of raising a child to survive to adulthood and w_0 is the wage rate in time 0. Increased wage growth only enters this problem through the indirect effect of changing U_1 . That is, if wages grow faster, future generations will have larger choice sets and hence, U_1 will be larger. The first order condition for this problem is:

$$LHS(n_0) \equiv \theta_0 u'(w_0 - \theta_0 n_0) = \beta U_1 g'(n_0) = RHS(n_0). \quad (1)$$

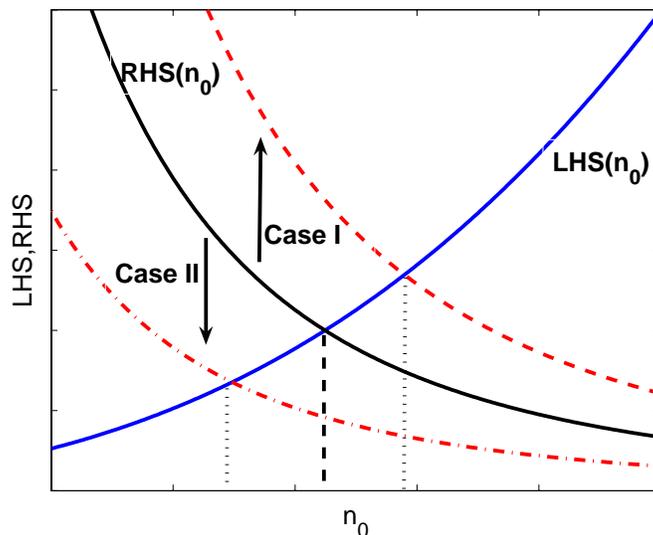
The left hand side of this equation is the marginal cost in terms of period 0 consumption of having an extra child and is increasing in n_0 , while the right hand side is the marginal benefit and is decreasing.

A change in U_1 has different effects depending on which case we are in. In particular, whether an increase in U_1 increases or decreases the right hand side depends on whether U_1 is positive or negative—see Figure 1. That is:

$$\frac{\partial RHS(n_0, U_1)}{\partial U_1} = \beta g'(n_0) = \frac{\partial^2 U_0}{\partial n_0 \partial U_1}. \quad (2)$$

³One issue that arises here is whether or not, with negative utility of born children, parents are necessarily making potential children worse off by having them. This will not be true if the utility of unborn potential children is a large enough negative number. Further, the same representation for the choice problem of the individual parent will hold (approximately) if the fraction of potential children that can feasibly be born is small. See Section 6 for more discussion on this point.

Figure 1: Comparative Statics with respect to U_1



When U_1 is positive, g is increasing and hence, the cross partial in (2) is positive—children and the utility of children are complements in the utility of the parent.⁴ In this case, it follows that a change in wage growth shifts the right hand side up causing n_0 to increase. Fertility is increasing in the rate of growth of wages. When U_1 is negative, g is decreasing and hence, this is negative—children and the utility of children are substitutes in utility. In this case, it follows that the right hand side shifts down and n_0 falls. Fertility is decreasing in the rate of growth of wages. Thus, whether an increase in wage growth (i.e., an acceleration of industrialization) increases or decreases fertility is completely determined by this assumption. Also clear is that this effect is quite general—it is not restricted to changes in the rate of growth of wages.

- I. If $U_1 > 0$, increasing U_1 increases the marginal utility of children, MU_n . Because of this, anything that increases U_1 will lead to a greater desire for children (unless something else changes to offset that).
- II. If $U_1 < 0$, increasing U_1 (smaller number in absolute value) lowers MU_n , making larger family sizes LESS desirable. Again, this is holding everything else equal.

From this discussion, we can also get a sense about how changes in the survival rates of children depend on this choice. It is common in the literature (e.g., Barro

⁴Note that in this simple version U_1 is endogenous to the child or period-1 adult, but exogenous to the period-0 parent. Hence, "complements" and "substitutes" are a slight abuse of language. In Section 6.2, we introduce bequests which gives parents some control over children's utility alleviating the abuse of language.

and Becker (1989), Doepke (2005), Bar and Leukhina (2007)) to model decreases in youth mortality as a reduction in the cost of producing a surviving child—a reduction in θ_0 . Notice that this will also typically increase U_1 if the reduction in mortality is permanent (i.e. future costs of producing surviving children also decreases). A decrease in θ_0 causes the left hand side of the equation to shift down. Hence, when children and their utility are complements ($U_1 > 0$), it follows that fertility will increase. In the opposite case ($U_1 < 0$), there are off-setting effects and the sign of the change cannot be predicted without more detailed analysis. This discussion is complicated by the fact that U_1 is not exogenous. Below we derive a planner’s problem with explicit mortality formulations to generalize these intuitions. Finally, note that in this simple case, the parent has no direct method of affecting U_1 , and hence, although one part of a quality/quantity trade-off is present—the direct preference part—other aspects of it are missing (e.g., increasing U_1 through leaving a larger bequest or spending more on the education of the child). This is a weakness in some ways, but allows us to focus our attention on the importance of the role of preferences much more transparently. In Section 6, we address extensions in which the parent has some control over children’s utility and show that the main qualitative results derived so far go through.

We conclude that the assumption one makes about whether children and their utility are substitutes or complements—and with it the implicit assumptions about both the sign of U and the monotonicity of g —has important qualitative implications about the properties of such models. Almost all work based on the Barro-Becker model to date has focused on the first case and this has had important implications about quantitative results using these extensions.

To the best of our knowledge, there is no good evidence for assuming that number and well-being of children are complements. Instead, we find indirect evidence to the contrary since with low elasticity and substitutability, the model is able to generate trends in fertility similar to those observed over the past 200 years in response to observed changes in mortality, longevity and productivity growth rates. These are also the drivers most emphasized by demographers.⁵

3 A simple dynastic model of fertility choice

In this section we lay out the basic model we will analyze for the remainder of the paper and link choices for the IES in dynastic planner’s problems to the discussion in Section 2. The model we study is a version of the Barro-Becker model with two changes. The first is to include adult survival. The second is to restrict attention to wage income (for simplicity).

⁵We suggest ways of getting more direct evidence in Section 6.

3.1 Youth mortality and longevity

To address the effect on fertility choice of changes in mortality rates, we introduce child mortality and adult longevity using certainty equivalence.⁶

We follow the original Barro-Becker work and assume that parents care only about surviving children, $n_{s,t} = \pi_s n_{b,t}$, where π_s is survival rate of a birth to adulthood and $n_{b,t}$ is the number of births. The total cost to the household of all births is then given by $\theta_{b,t} n_{b,t}$, where $\theta_{b,t}$ is the cost, in terms of goods, of the birth of each child.⁷ Thus, even children that do not survive to adulthood can be costly.

We assume that the survival probability for adults is age independent and given by $\pi < 1$ (see Blanchard (1985)). This formulation allows us to perform comparative statics with respect to longevity and calibrate to reasonable life lengths without adding additional state variables.⁸

To simplify preference aggregation below, and keep the number of state variables in the model to a minimum, it is useful to impose some structure to the preferences of parents over their own future utility (if they survive) and that of their surviving children. We assume that parents continuation utility is a function only of expected family size and future per capita utility:

$$U_t = u(c_t) + \beta g(\pi + \pi_s n_{b,t}) U_{t+1}.$$

For example, when $\pi = 0$ these preferences revert to those used in the original Barro-Becker model. At the other extreme, $\pi = 1$, people live forever and care about themselves exactly as much as they care about each surviving child.

Finally, the household's budget constraint is:

$$c_t + \theta_{b,t} n_{b,t} \leq w_t.$$

Notice that all the intuitive results relating to complementarity and substitutability derived in the previous subsection go through with these changes. An additional effect is also present. This is that increased longevity (i.e. π) decreases the marginal utility of having an extra child, regardless of the sign of U_{t+1} .

3.2 Dynasty utility and intertemporal elasticity

To fully specify the model we need an explicit mechanism for the determination of the utility of subsequent generations. Note that by construction, any time- t preferences have a natural time consistency property: there is no inherent conflict in preferences between the agents in period t and period $t + 1$. Therefore, the sequence of decisions

⁶See Ben-Porath (1976), Sah (1991), Wolpin (1997), Eckstein, Mira, and Wolpin (1999), Kalemli-Ozcan (2002, 2003), and Doepke (2005) for models where family level uncertainty is included.

⁷Although we write the costs of children here in terms of goods, it is straightforward to reinterpret it as requiring the time of the parent. In this case, $\theta_{b,t}$ is directly proportional to the wage.

⁸Barro and Sala-i-Martin (1995), Chapter 9, introduce longevity in a similar manner in a continuous time version of the Barro-Becker model.

made by the individual time t agents are exactly the same as what would be decided for them by the time 0 agent. To study this version of the problem, we can sequentially substitute $U_1, U_2, \dots, U_{t+1}, \dots$ into the utility function of the time 0 agent, the dynasty head, to get:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[\prod_{k=0}^{t-1} g(\pi + \pi_s n_{b,k}) \right] u(c_t).$$

Because of the term $\prod_{k=1}^{t-1} g(\pi + \pi_s n_{b,k})$, this utility function is typically not concave as written. However, as discussed in Alvarez (1999), under certain conditions, this can be rewritten as a concave problem in dynasty aggregate variables. Assume that $g(x) = x^\eta$, $\eta < 1$, and let $N_0 = 1$ and $N_t = \prod_{k=0}^{t-1} (\pi + \pi_s n_{b,k})$, the expected total number of adults (parent and descendants) alive during period t evaluated in $t - 1$. Assuming certainty equivalence/law of large numbers, we get the following law of motion for population:

$$N_{t+1} = \pi N_t + N_{s,t} = \prod_{k=0}^t (\pi + \pi_s n_{b,k})$$

where $N_{s,t} = n_{s,t} N_t$ is the total number of surviving children born in period t . Then $\prod_{k=1}^{t-1} g(\pi + \pi_s n_{b,k}) = g(\prod_{k=1}^{t-1} (\pi + \pi_s n_{b,k})) = g(N_t)$, and so preferences for the dynasty head can be rewritten as:

$$U_0 = \sum_{t=0}^{\infty} \beta^t g(N_t) u \left[\frac{C_t}{N_t} \right]$$

where $C_t = N_t c_t$ is total consumption in period t . Note that this assumes that consumption is the same for all adults in a period. U_t for $t > 0$ is defined similarly.

Following the discussion above, since $g(N) = N^\eta$ is always positive, there are two possible ways to satisfy conditions (1.)-(4.) above:

- I. Assume that $u(c) \geq 0$ for all $c \geq 0$, that u is strictly increasing and strictly concave and that $0 < \eta < 1$;
- II. Assume that $u(c) \leq 0$ for all $c \geq 0$, that u is strictly increasing and strictly concave and that $\eta < 0$.

Either of these are consistent with the entire set of intuitive requirements laid out in the original Barro-Becker papers. Typically we want more, however. For existence and uniqueness of a solution to the planner's problem below, the extra desirable properties are that U_0 as written here is increasing and concave in (C, N) . We therefore specialize further and assume that $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $\sigma > 0$. Let $V(C_t, N_t) = \frac{N_t^{\eta+\sigma-1} C_t^{1-\sigma}}{1-\sigma}$ denote the period- t flow utility in aggregates. Given this functional form, there are two sets of parameter restrictions that satisfy the natural monotonicity and concavity restrictions of V :

AI. The first completes the standard assumption in the fertility literature:

$$0 < 1 - \sigma \leq \eta < 1$$

In this case, $U_t > 0$ for all $(C, N) \in R_+^2$.

AII. The second allows for intertemporal elasticities of substitution in line with the standard growth and business cycle literature:

$$0 > 1 - \sigma \geq \eta$$

In this case, $U_t < 0$ for all $(C, N) \in R_+^2$.⁹

The conditions in assumption AI can be read as follows: if V is positive (first inequality) then for $V_N > 0$, we need $\eta + \sigma - 1 \geq 0$ (second inequality). To ensure overall concavity, we need $V_{CC} < 0$, $V_{NN} < 0$, and $V_{CC}V_{NN} - V_{CN}V_{NC} > 0$. This requires $\eta < 1$ (third inequality). In case AI, $V_{CN} = V_{NC} > 0$ and hence C and N are complements in utility. The conditions in configuration AII can be read as follows: if V is negative (first inequality) then for $V_N > 0$, we need $\eta + \sigma - 1 \leq 0$ (second inequality). This immediately ensures overall concavity. In case AII, $V_{CN} = V_{NC} < 0$ and hence C and N are substitutes in utility. In the case where, $\eta = 1 - \sigma$ (allowed under both configurations), utility becomes a function of aggregate consumption only. Hence, conditions for monotonicity and concavity of V involve V_C and V_{CC} only.¹⁰

3.3 The planner's problem

Using the dynasty's preferences derived above, we follow the approach from Alvarez (1999) in which a time zero dynastic head chooses the time paths of aggregate, dynasty level variables. The problem solved is:

$$\begin{aligned} & \max_{\{C_t, N_{b,t}, N_{s,t}, N_t\}} \sum_{t=0}^{\infty} \beta^t \frac{N_t^{\eta+\sigma-1} C_t^{1-\sigma}}{1-\sigma} \\ \text{s.t:} & \\ & C_t + \theta_{b,t} N_{b,t} \leq w_t N_t, \text{ and} \\ & N_{t+1} \leq \pi N_t + N_{s,t}, \\ & N_{s,t} \leq \pi_s N_{b,t}, \\ & N_0 \text{ given.} \end{aligned}$$

This problem can be reformulated by eliminating $N_{b,t}$ and defining $\theta_{s,t} \equiv \frac{\theta_{b,t}}{\pi_s}$, the cost of producing a surviving child. Thus, the cost of raising a child to working age

⁹The low elasticity case, $\eta < 0$, has also been considered in Alvarez (1999), Proposition 2, and mentioned in Barro and Sala-i-Martin (1995), Chapter 9, footnote 18. Mateos-Planas (2002) used $\sigma = 3$ and $\eta = -2.78$ in his quantitative analysis. Although this formulation is not common in the fertility literature, a similar formulation for utility, where the arguments are consumption and leisure, is quite common in the growth literature. There are numerous examples of this, see Ales and Maziero (2007) for a recent one. A few remarks and issues with this utility formulation are addressed in Section 6.

¹⁰Note that the intertemporal elasticity of substitution in consumption is only partially expressed in σ . The actual elasticity also involves N_t . In the case where $\eta = 1 - \sigma$, the analogy is exact. We will nevertheless stick to this abuse of language for the remainder of the paper.

depends on the survival probability—an increase in π_s decreases $\theta_{s,t}$. Further one can eliminate $N_{s,t}$ and C_t and solve for N_{t+1} , the period- $t+1$ stock of population given N_t .

Under either of the sets of parameter restrictions given above (AI and AII), this (time zero) maximization problem has a concave objective function and a convex constraint set. Thus, the problem has a unique solution, concave value functions, etc. (see Alvarez and Stokey (1998)).

4 Equilibrium populations and comparative statics

In this section, we describe the solution to the model and present analytical comparative statics results across Balanced Growth Paths (BGPs). We discuss how fertility changes when productivity growth rates and survival rates are changed exogenously. We find that both the sign and size of these effects depend critically on which parameter configuration holds.

Note that this, like many models based on dynastic altruism, is a balanced growth model. Thus, increases in *income (wages)* do not, by themselves, trigger a change in fertility (at least, as long as costs change proportionally). Rather, it is a change in *growth rates* that is required. This feature brings focus to the hypothesis in demography that the Fertility Transition is in part caused by 'industrialization'. That is, in this particular model at least, it was the *acceleration* of productivity growth that mattered, not the change in *levels* per se.

4.1 Equilibrium populations

The first order condition for the stock of population in period- $t+1$, N_{t+1} , is given by:

$$\begin{aligned} & \theta_{s,t} N_t^{\eta+\sigma-1} C_t^{-\sigma} \\ &= \underbrace{\beta [w_{t+1} + \theta_{s,t+1} \pi] N_{t+1}^{\eta+\sigma-1} C_{t+1}^{-\sigma}}_A + \underbrace{\beta \frac{(\eta + \sigma - 1)}{(1 - \sigma)} N_{t+1}^{\eta+\sigma-2} C_{t+1}^{1-\sigma}}_B. \end{aligned} \quad (3)$$

The intuition for this is as follows. On the left is the marginal cost in terms of changed current utility of increasing N_{t+1} (i.e., of producing an extra child). This cost is just the direct utility cost of reduced consumption today (rescaled by the fact that it only takes $\theta_{s,t}$ units of C to make one extra unit of N). On the right hand side are the two pieces of the marginal benefits next period from increasing N_{t+1} . These are: (A.) the value of the extra output the dynasty will have next period; (B.) the marginal value of utility from having extra children.

To gain some more insight, consider the special case in which $\eta = 1 - \sigma$. In this case, N is exactly like a capital good since the two utility effects of increasing N

exactly cancel out. These two effects are: first, the direct benefit of having extra children in the utility function, $\frac{g(N)}{1-\sigma} = \frac{N^\eta}{1-\sigma}$; second, the direct cost of having children by diluting per capita consumption in the utility function, $\frac{[\frac{C}{N}]^{1-\sigma}}{1-\sigma}$. As we can see in the first order condition (3), two simplifications result. The first is that $(\eta + \sigma - 1) = 0$ and so term (B) disappears entirely, and second that $N_t^{\eta+\sigma-1} = N_{t+1}^{\eta+\sigma-1} = 1$, i.e., the marginal value of increased total consumption by the dynasty in periods t and $t + 1$ no longer depend on the size of the dynasty in the period. Hence, equation (3) simplifies to:

$$\left[\frac{C_{t+1}}{C_t} \right]^\sigma = \beta \left[\frac{w_{t+1}}{\theta_{s,t}} + \frac{\theta_{s,t+1}}{\theta_{s,t}} \pi \right]. \quad (4)$$

This is the standard Euler Equation from an Ak model in terms of *aggregate consumption* and the stock of people, N , the investment good with $(1 - \pi)$ corresponding to depreciation, and time varying costs and benefits of flow investments (fertility), i.e., w_t and $\theta_{s,t}$.

Equation (3) or (4) together with the feasibility constraint and the initial condition N_0 completely describe the equilibrium path.

Next, we make an analytically and quantitatively useful distinction between several fertility measures. The first fertility measure we consider is the population growth rate between period t and period $t + 1$ given by:

$$\gamma_{N,t} = \frac{N_{t+1}}{N_t}.$$

Adjusting for the length of the period, this easily maps into annual population growth rates (net of immigration) observed in the data.

Further, we discuss two other measures of fertility related to common measures used in demography. These are: the Crude Birth Rate (CBR) and the Cohort Total Fertility Rate (CTFR). CBR is defined as the number of births in a period per person. Typically it is a yearly measure. Since our model period will be 20 years below, we adapt this to include those children born in the early part of a period and that have survived to the end of the period. Since it will be useful below to distinguish between overall births (CBR) and those births that actually survive to adulthood, we introduce one further concept, the Surviving Crude Birth Rate (CBR_s):

$$\begin{aligned} CBR_t &= \frac{N_{b,t}}{N_t + N_{s,t}} \\ CBR_{s,t} &= \frac{N_{s,t}}{N_t + N_{s,t}}. \end{aligned}$$

Finally, the Cohort Total Fertility Rate (CTFR) is the number of children an adult woman in a particular birth cohort had over her lifetime. Since the model only makes predictions about how many children are born, not to whom they are born, further assumptions are required to develop a model analog. We therefore assume that only

one-half (i.e., the females) of the surviving children from among those born in the previous period can have children in the current period. When a period is long, e.g., 20 years, which we will assume below, this is a natural assumption to make. Under this assumption, there is a simple expression for the analog of CTFR in the model:¹¹

$$CTFR_t = 2 \frac{N_{bt}}{N_{st-1}}.$$

4.2 Balanced growth

One advantage of the version of the model with labor income only is that it delivers simple analytic comparative statics results across Balanced Growth Paths (BGPs).¹²

Assume that wages and costs of children grow at rate γ , i.e. $w_t = \gamma^t w$ and $\theta_{s,t} = \gamma^t \theta_s$. In this case, it can be shown directly that, both $\gamma_{N,t} = \frac{N_{t+1}}{N_t}$ and $\frac{C_t}{\gamma^t N_t}$ are independent of t (see Jones and Schoonbroodt (2007)). Because of this, it follows that $\gamma_{C,t} = \frac{C_{t+1}}{C_t}$ (the growth rate in aggregate consumption) and $\gamma_{c,t} = \frac{C_{t+1}/N_{t+1}}{C_t/N_t}$ (the growth rate in per capita consumption) are also independent of t and that $\gamma_c = \frac{\gamma_C}{\gamma_N} = \gamma$. Using this, the feasibility constraint and $\theta_s \equiv \theta_b/\pi_s$ in equation (3) after dividing both sides by $\gamma^{t+1} N_{t+1}^{\eta+\sigma-1} C_{t+1}^{-\sigma}$ and rearranging, we get

$$\frac{1}{\beta} \gamma_N^{1-\eta} \gamma^{\sigma-1} + \frac{(\eta + \sigma - 1)}{(1 - \sigma)} \gamma_N = \frac{\eta}{(1 - \sigma)} \left[\frac{\pi_s w}{\theta_b} + \pi \right]. \quad (5)$$

Further, equation (5) simplifies considerably when $\eta = 1 - \sigma$. In fact,

$$\gamma_N^\sigma = \beta \gamma^{1-\sigma} \left[\frac{\pi_s w}{\theta_b} + \pi \right]. \quad (6)$$

We use this version of the Euler equation in our quantitative analysis below.

Given parameters, equation (5) or (6) can be solved for the BGP population growth rate, γ_N . The other fertility measures are then given by:

$$\begin{aligned} CBR_s &= \frac{\gamma_N - \pi}{1 + \gamma_N - \pi} \\ CBR &= \frac{\gamma_N - \pi}{\pi_s [1 + \gamma_N - \pi]} = \frac{CBR_s}{\pi_s} \\ CTFR &= \frac{2\gamma_N}{\pi_s}. \end{aligned}$$

¹¹A more common measure in demography is the Total Fertility Rate (TFR). This is a point in time measure constructed by adding age specific fertility rates of woman of different cohorts alive fecund at a point in time. CTFR and TFR coincide when age specific fertility rates are constant over time. In the model, TFR and CTFR are the same since one model period also corresponds to the entire fertile period (e.g., from age 20 to age 40). In the data we will compare this object to CTFR whenever data is available.

¹²Some, but not all of these results carry over to generalized versions of the model with capital. See Section 6.2 for a more detailed discussion.

Since these are all increasing functions of the population growth rate, γ_N , the comparative statics analysis below simplifies greatly.

4.3 Comparative statics

In this section, we derive analytic comparative statics across BGPs for all the fertility measures given above. We focus on three distinct changes leading to the quantitative experiments we explore in the next section, namely changing γ , π_s and π . These correspond to three commonly discussed driving forces of the demographic transition over the period from 1800 to 1990: 1) the increased growth rate of labor productivity that came with industrialization (γ); 2) the substantial reduction in infant and youth mortality rates (π_s); and 3) the significant increase in (adult) life expectancy (π).

Note that the only endogenous variable in equation (5) is the population growth rate, γ_N , which only enters on the left-hand side. Moreover, the productivity growth rate, γ , only enters on the left-hand side while survival rates, π_s and π , only enter on the right-hand side. That is, holding $(\sigma, \beta, \eta, w, \theta_b)$ fixed, this equation is of the form: $LHS(\gamma_N; \gamma) = D(\pi_s, \pi)$, where $D(\pi_s, \pi) = \frac{\eta}{(1-\sigma)} \left[\frac{\pi_s w}{\theta_b} + \pi \right]$ (see Figure 2). $LHS(\gamma_N; \gamma)$ is increasing in γ_N , for all values of γ , since in both parameter configurations, AI and AII, we have that $\eta \in [-\infty, 1)$ and $\frac{(\eta+\sigma-1)}{(1-\sigma)} > 0$. Similarly, $D(\pi_s, \pi) > 0$ since $\frac{\eta}{(1-\sigma)} > 0$.

It is easy to see that in response to an increase in γ , $LHS(\gamma_N; \gamma)$ decreases (shifts right) if $\sigma < 1$ and increases (shifts left) if $\sigma > 1$. Hence, γ_N is increasing in γ if $\sigma < 1$ (AI) and decreasing in γ if $\sigma > 1$ (AII). That is, the *sign* of first order effect of productivity growth on population growth changes across the parameter configurations AI and AII. Since all other fertility measures are increasing functions of the population growth rate, one can see that whether CBR , CBR_s and $CTFR$ increase or decrease in response to an increase in γ analogously depends on the parameter configuration.

D , on the other hand, is increasing in both survival rates, π_s and π , for either parameter configuration, and hence, so is γ_N . That is, a decrease in either youth or adult mortality always leads to an increase in the population growth rate. The effect of mortality on the other fertility measures differs, however.

An increase in survival to adulthood, π_s , will also increase the crude birth rate in terms of surviving children, CBR_s , for AI and AII. The effects of youth mortality on total births, $CBR = \frac{CBR_s}{\pi_s}$, however, depends on the size of the response in CBR_s to an increase in π_s relative to the increase in π_s itself. A similar relationship holds between the size of the change in γ_N and the sign of the response of $CTFR$. Similarly, even though an increase in π always causes γ_N and $CTFR$ to increase, whether or not it increases CBR_s and CBR depends on the size of $\frac{\partial \gamma_N}{\partial \pi}$.

While the *sign* of the effects of π_s and π on γ_N is the same under AI and AII, the parameter configuration matters for the *size* of these effects. To see this, note that the left-hand side of equation (5) is increasing in γ_N . Further, it is concave if

Figure 2: Comparative statics of γ_N

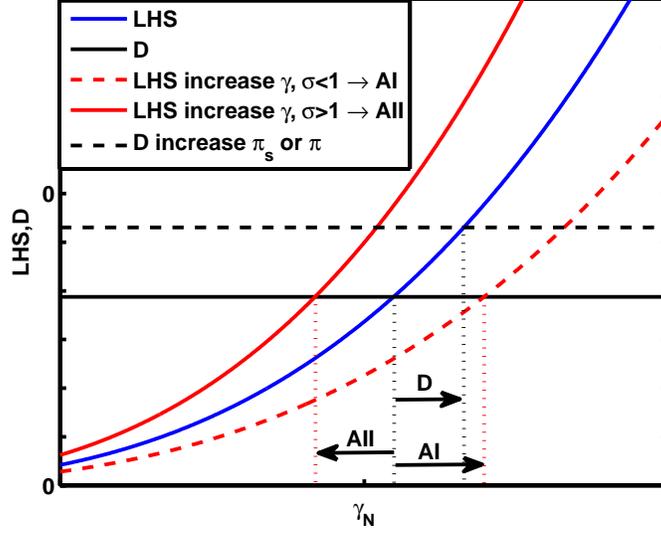
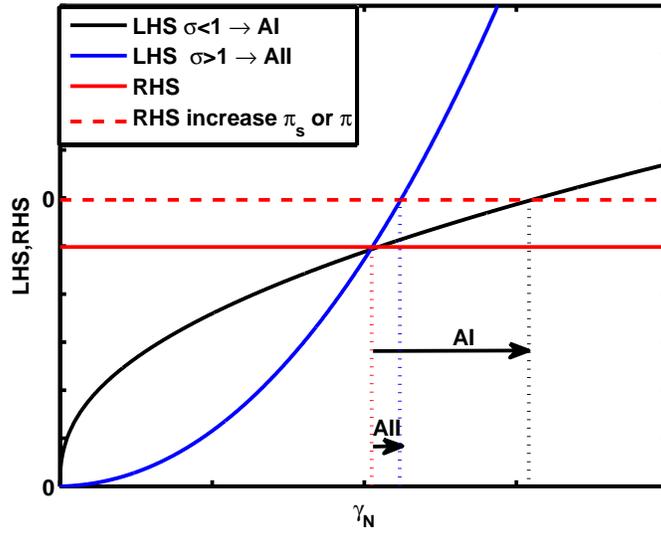


Figure 3: Increase in γ_N is smaller for AII than for AI



$1 > \eta \geq 1 - \sigma > 0$ (i.e. under AI), and convex in γ_N if $\eta \leq 1 - \sigma < 0$ (i.e. under AII) (see Figure 3). This implies that in the range where $\gamma_N \geq 1$ the slope of *LHS* is larger under AII than it is under AI. Because of this, the response of γ_N to a given change in $D(\pi_s, \pi)$ is smaller under AII than AI.

More precisely, the empirically relevant ranges for parameters are when productivity growth is positive but small, $\gamma \approx 1$, and the same holds for γ_N . It is therefore of special interest to know the properties of the model when $\gamma = 1$ and $\gamma_N \geq 1$ (i.e., $\beta D \geq 1$). Suppose for simplicity that $\eta = 1 - \sigma$. In this case, holding all parameters but σ fixed, it follows that a given size change in π_s and π has larger effects on γ_N in the low curvature case, AI, than in the high curvature case, AII. This is also true for CBR_s in response to a change in π_s . Because of this, it follows that when $\beta D = 1$, we have $\frac{\partial CBR}{\partial \pi_s}|_{\sigma < 1} > \frac{\partial CBR}{\partial \pi_s}|_{\sigma > 1}$. For example, if CBR falls from a given change in π_s when $\sigma < 1$, it falls more when $\sigma > 1$, etc. It can also be shown that, in the special case where $\pi = 0$, $\frac{\partial CBR}{\partial \pi_s} < 0$ under both AI and AII.

In sum then, when γ and γ_N are both near one and π isn't too large, an increase in π_s will cause CBR to fall under both AI and AII, and this fall will be larger when $\sigma > 1$. Similar results hold for *CTFR*.

Finally, we know that under AI, $\frac{\partial \gamma_N}{\partial \pi}$ is larger than under AII. Hence, as above, if parameters are such that CBR_s and CBR fall in response to an increase in adult survival, π , when AI holds, they fall more when AII holds. When $\gamma = 1$, $\beta D = 1$, $\eta = 1 - \sigma$, and $\sigma > 1$, it can be shown that an increase in π causes CBR_s and CBR fall.

We summarize the results most relevant for the quantitative analysis in a proposition:

Proposition 1. *The following comparative statics results across hold BGPs .¹³*

1. *If AI holds, then, $\frac{\partial \gamma_N}{\partial \gamma}$, and $\frac{\partial CBR}{\partial \gamma}$ are positive, while if AII holds, then $\frac{\partial \gamma_N}{\partial \gamma}$, and $\frac{\partial CBR}{\partial \gamma}$ are negative.*

- (a) *In both cases AI and AII, $\frac{\partial \gamma_N}{\partial \pi_s} > 0$. However, when $\gamma = 1$, $\eta = 1 - \sigma$ and $\beta D \geq 1$ (i.e., $\gamma_N \geq 1$), then, $\frac{\partial \gamma_N}{\partial \pi_s}|_{\sigma < 1} > \frac{\partial \gamma_N}{\partial \pi_s}|_{\sigma > 1}$.*
- (b) *If $\gamma = 1$, $\eta = 1 - \sigma$ and $\beta D = 1$ then, $\frac{\partial CBR}{\partial \pi_s}|_{\sigma < 1} > \frac{\partial CBR}{\partial \pi_s}|_{\sigma > 1}$. If in addition, $\pi = 0$ then, $\frac{\partial CBR}{\partial \pi_s} < 0$ for AI and AII.*

2. *In both cases AI and AII, $\frac{\partial \gamma_N}{\partial \pi} > 0$. However, when $\gamma = 1$, $\eta = 1 - \sigma$ and $\beta D \geq 1$ (i.e., $\gamma_N \geq 1$), then, $\frac{\partial \gamma_N}{\partial \pi}|_{\sigma < 1} > \frac{\partial \gamma_N}{\partial \pi}|_{\sigma > 1}$ and $\frac{\partial CBR}{\partial \pi}|_{\sigma > 1} < 0$.*

¹³When $\eta = 1 - \sigma$ and $\sigma \rightarrow 1$, the utility function we use converges to $\sum \beta^t \log(C_t)$. All of the results from this Proposition go through in this case. In particular there is no change in γ_N when γ is changed. See Bar and Leukhina (2007) for an explicit derivation of Barro-Becker preferences in this case.

It also follows that $\frac{\partial \gamma_N}{\partial \beta} > 0$ and $\frac{\partial \gamma_N}{\partial w} > 0$ if $\theta_b = a$ (goods cost) and $\frac{\partial \gamma_N}{\partial w} = 0$ if $\theta_b = bw$ (time cost). We do not emphasize these because they play no role in the quantitative discussion we focus on below.¹⁴

These comparative statics have important implications for studying trends in fertility using this type of model. From the beginning of the 19th century to the end of the 20th century, Crude Birth Rates (CBR) fell substantially, while population growth rates (net of immigration) decreased only slightly. Since the rate of growth of productivity has increased over the period describing these demographic changes, it follows from the proposition that we would expect population growth rates to fall as a result as long as high curvature and substitutability between number and utility of children are assumed (AII). Offsetting this effect is the fact that both youth and adult survival rates rose over this period. Thus, under AII, increases in population growth in response to changes in mortality declines (through π_s or π) will mitigate the negative effect from increases in productivity growth, but these effects are small. As we shall see, overall we see a substantial decrease in γ_N in this case. Under AI, however, population growth rates increase in response to all, increased productivity growth, decreased youth mortality and longevity. Hence, the model has no shot at reproducing observed changes in population growth rates.

With respect to CBR, the effect of productivity growth is the same as for population growth rates, while the effect of decreases in mortality could, in principle, be either an increase or a decrease. As we shall see, the model predicts, from this source alone, a small decrease in CBR under AI and hence, as in the proposition, a larger decrease under AII. Because of this, the combined effect will be to increase CBR when AI holds, and decrease CBR when AII holds.

5 The U.S. experience 1800-1990¹⁵

Our next objective is to extend the insights gained above to get quantitative estimates of the model predictions in terms of population growth rate and CBR in response to permanent (unanticipated) changes in productivity growth (γ), longevity (π), and youth mortality (π_s) as observed in the data.

First, we discuss how the model can be calibrated to match (current) levels of fertility and/or population growth rates in the U.S.. Most of the parameters needed can be taken directly from the quantitative growth and RBC literatures. The exceptions to this are θ , the cost of raising a child, and η the utility parameter governing curvature over dynasty size. For simplicity we set $\eta = 1 - \sigma$ throughout.¹⁶ One

¹⁴If all child costs are goods costs, increases in labor income taxes are equivalent to reductions in w . Thus, it follows that increasing the labor income tax rates will decrease both population growth rates and fertility on the BGP, see also Manuelli and Seshadri (2007). At the other extreme, if all costs are in terms of time ($\theta = bw$), fertility and population growth rates are independent of labor income taxes.

¹⁵See the Appendix for the case of the U.K.

¹⁶Note that given our two admissible parameter configurations, η would have to be adjusted

puzzle in the quantitative growth literature with endogenous fertility has been that calibrated costs of children were unrealistically high. We find that when $\sigma > 1$ (AII), the needed calibrated costs to match any given fertility level are substantially lower.

Next, we simulate the predicted effects on fertility from changing all three of the above driving forces with the appropriate timing of events in the U.S. over the last 200 years. We find that how well the model does at reproducing the historical facts depends critically on the size of the IES. When the IES is high, as is commonly assumed in the fertility literature, the model (counterfactually) predicts large increases in both CBR and γ_N . On the other hand when the IES is low, the model captures two thirds of the decrease in CBR observed in the data and about half of the decrease in population growth rates—and with similar timing.

Finally, we decompose the contribution of each factor separately for the low IES case and find that the increase in the growth rate of productivity accounts for about 90 percent of the predicted fall in fertility before 1880, and changes in mortality account for 90 percent of the predicted change from 1880 to 1950.

Overall, these results are consistent with the qualitative comparative statics results given in Proposition 1. This must be interpreted with caution however since the results in the Proposition hold $\frac{\theta}{w}$ fixed, while here we are recalibrating this so as to match final levels of fertility.

5.1 Calibration: fertility levels and costs of children

In this subsection we use the Euler equation in (6) to calibrate costs of children given all other parameters and targeting 0.65 annual population growth. This target roughly corresponds to choosing a cost of children that matches the U.S. fertility experience (i.e., about 15 births per 1000 population) over the 1970 to 1990 period.

From the results of the previous section, we have:

$$\gamma_N = \left[\beta \left[\frac{w}{\theta_s} + \pi \right] \right]^{1/\sigma} [\gamma]^{1/\sigma-1}.$$

Now, suppose there are T years in a period. Then we can rewrite the equation above in annual terms as (the subscript ann denotes annual values):

$$\gamma_{N,ann}^T = \left[\beta_{ann}^T \left[\frac{w}{\theta_s} + \pi_{ann}^T \right] \right]^{1/\sigma} [\gamma_{ann}^T]^{1/\sigma-1}.$$

Solving for $\frac{\theta_s}{w}$ gives

$$\frac{\theta_s}{w} = \frac{1}{\left[\frac{\gamma_{N,ann}^\sigma}{\beta_{ann} \gamma_{ann}^{(1-\sigma)}} \right]^T - \pi_{ann}^T}.$$

where the relevant π_{ann}^T is the probability of an adult surviving (in the workforce) for T years. We choose $T = 20$ and hence $\pi_{ann} = 0.977$ to match an expected adult

whenever we compare results for values of $\sigma > 1$ and $\sigma < 1$. Hence, the assumption that $\eta = 1 - \sigma$ is fairly innocuous. In this case, the two utility effects of increasing dynasty size cancel out and children are a pure investment good—independently of σ .

(working) life, $\frac{T}{1-\pi}$, of 53 years. Moreover, we assume an annual discount factor of $\beta_{ann} = 0.96$ and compute an annual productivity growth rate of $\gamma_{ann} = 1.018$ from wage data. Note that over the period 1970 to 1990 survival to adulthood is close to certain, i.e., $\pi_s \approx 1$ and hence $\theta_s \approx \theta_b$. We introduce additional layers of complexity in the next subsection so that we can use detailed data on infant, child and youth mortality rates in the time series experiments below. Throughout, we consider values of $\sigma \in \{0.5, 1, 3\}$.

We report our results in Table 1. The units of $\frac{\theta_s}{w}$ are the fraction of one periods per capita output that it costs to raise one child. To get a sense of how costly children must be in the model in order to match realistic growth rates of population, it is more convenient to express it in terms of the number of *years* of output that are required to raise a child, i.e., $T \times \frac{\theta_s}{w}$. Our choice of $T = 20$ also corresponds to assuming that it takes 20 years for a newborn to become a productive worker. We also report the maximal number of children a two parent household can (feasibly) have over one (fertile) period (Max *CTFR*).

Table 1: U.S. Costs of children in 1990, Time Series Experiment

σ	$\frac{\theta_s}{w}$	$T \times \frac{\theta_s}{w}$	Max <i>CTFR</i>
0.5	0.72	14.32	2.80
1.0	0.51	10.25	3.90
3.0	0.16	3.24	12.34

Clearly, the values of the costs of raising children—in the range of 3 to 14 years of one person’s output—are very sensitive to the choice of the intertemporal elasticity of substitution in consumption (*IES*). The intuition for this is as follows. When σ is high (i.e., there is a strong desire to smooth consumption), high growth in aggregate consumption is not valuable (since $\eta = 1 - \sigma$ and the two utility effects of dynasty size in equation 3 cancel out this is the only effect that matters). Aggregate consumption grows at rate $(\gamma\gamma_N)$. Hence, everything else equal, the population growth rate is decreasing in the desire to smooth consumption (σ). Vice versa, to get a *given* population growth rate (i.e., fertility level) to be the optimal choice, one needs higher costs of children when σ is lower. With $\sigma = 0.5$, a household can maximally have 2.8 children during one 20 year period, while with $\sigma = 3.0$ as many as 12.34 are feasible. This shows that with low *IES*, costs of children lie in a much more reasonable range.

In the historical experiments we perform below we will examine how these costs change over time due to changes in mortalities. Since age specific mortality rates

changed unevenly since 1800, we follow Doepke (2005) and assume that there are three relevant subperiods of childhood: infancy (up to age 1), childhood (ages 1 to 5) and youth (from age 5 to adulthood at age 20). Let π_i , π_{ic} and π_{cy} and θ_i , θ_c , and θ_y denote the three conditional survival rates and costs for these sub-periods. Then, the cost of producing one surviving child is:

$$\theta_s = \frac{\theta_i}{\pi_i \pi_{ic} \pi_{cy}} + \frac{\theta_c}{\pi_{ic} \pi_{cy}} + \frac{\theta_y}{\pi_{cy}}. \quad (7)$$

As in Doepke (2005) we assume that $\theta_y = 0$ and costs are the same for every year age 0 to 5, i.e. $\frac{\theta_c}{\theta_i} = 4$. Given these assumptions, the calibrated values of θ_s/w given σ , and historical data on mortality rates, we can back out θ_i/w and θ_c/w from equation (7) for each value of σ . Over time, we assume that the base costs ($\theta_i, \theta_c, \theta_y$) remained constant, while survival rates ($\pi_i, \pi_{ic}, \pi_{cy}$) have increased—decreasing the effective cost of surviving children, θ_s .¹⁷

5.2 U.S. Fertility decline: model versus data

Next, we examine the model predictions for fertility and population growth from changing γ , θ_s (through changes in π_i , π_{ic} , and π_{cy}) and π in line with the experience of the U.S. over the period from 1800 to 1990. These predictions are then to be compared with their data counterparts. All data series and sources can be found in Table A.1 in the Appendix.

In the data, CBR was roughly constant at 45 births per 1000 population from 1800 to 1860, then decreased to 19 in 1930.¹⁸ From Haines (1994b), we get that from 1930, the bottom of the pre-WWII baby bust, CBR increased to 23 in the 1950s and 1960s, the peak of the post-WWII baby boom, and finally fell to 15.8 in 1990.¹⁹ CTFR and TFR show similar patterns. Finally, also from Haines (1994b), population growth rates (net of immigration) decreased from 2.6 percent population growth per year to 0.65 percent per year, again with a down then upward swing from 1930 to 1960.

We use data on productivity growth, survival probabilities to adulthood and expected lifetimes in the U.S. from 1800 to 1990. This data implies that productivity growth, γ_{ann} , increases from zero to 1.8 percent per year, adult survival, π_{ann} , increases from 0.964 to 0.977 and survival to adulthood, $\pi_{s,ann}$, increases from 0.632 to 0.989.²⁰ Note that, in the model, agents assume that current values of these

¹⁷See Mateos-Planas (2002) for an estimate of how these base costs must have changed in several European countries to fully match their fertility experience.

¹⁸Note that earlier estimates of CBR in the 19th century were higher, at about 55 births per 1000 population in 1800 to 45 in 1850 (e.g. Haines (1994b)).

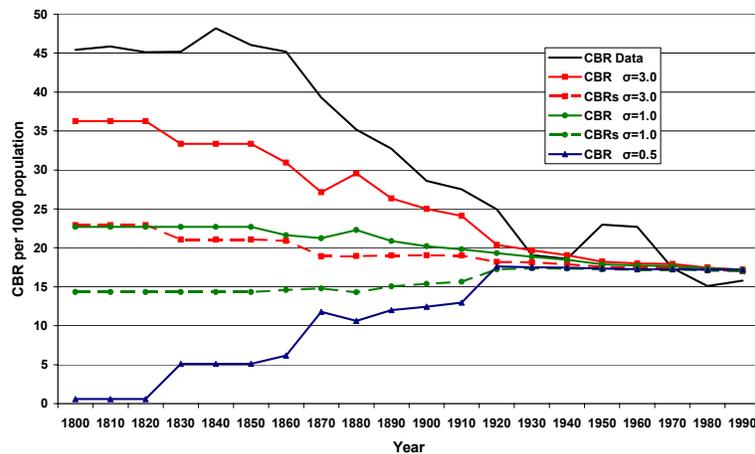
¹⁹For an application of a stochastic version of the present model to address these fertility fluctuations, see Jones and Schoonbroodt (2007).

²⁰The range for survival probabilities conditional on reaching age 20, π , are deduced from measures of expectation of life (EL) at age 20 (see Table A.1, column g). One issue related to this is that expected time in retirement has increased dramatically over the past 150 years (see Lee (2001)). We performed the same experiment using expected working life at age 20 (EWL) (i.e. the difference

parameters will prevail forever.²¹

The results of this experiment are shown in Figure 4 and Table 2. As can be seen, for high IES ($\sigma = 0.5$) the model predicts a substantial increase in $CTFR$ and CBR . The model also predicts a large increase in population growth rates in this case. These predictions are in part due to the increase in survival to adulthood and in part due to the increased growth rate in productivity, both of which always imply an increase in population growth rates for $\sigma < 1$. The first of these findings is consistent with Doepke (2005) and is the reason for his conclusion that the basic Barro-Becker model doesn't fit the facts (even though a more sophisticated model with sequential fertility choice performs slightly better).²²

Figure 4: The U.S. experience from 1800 to 1990, CBR



For $\sigma = 1$, the model predicts a sizable fall in $CTFR$ and CBR . However, since the probability of surviving to adulthood is increasing over the period, the number of surviving children (CBR_s) may increase even if the number of births (CBR) falls. Indeed, the case shown with $\sigma = 1$ has CBR falling but CBR_s increasing. This property of CBR_s is directly reflected in the predicted time path of population growth rates (see Table 2). This is partly due to the fact that changes in productivity growth do not affect CBR_s or population growth rates when $\sigma = 1$. These results are in

between expected lifetime and expected years of retirement) and obtained very similar results (details are available in the Appendix).

²¹The assumption that agents believe that productivity growth, mortality and longevity will be constant forever is an extreme one but greatly simplifies the analysis. An alternative extreme would be to assume that agents perfectly foresee the exact future path of parameter changes. The results of this exercise for $\sigma = 3$ can be found in the Appendix. The predictions are virtually identical.

²²This version of the model overstates the poor performance when $\sigma = 0.5$ to some degree. When physical capital is also included, there is a small decrease in CBR over the period from about 20 to 17. See the discussion in Section 6 and the Appendix.

keeping with Bar and Leukhina (2007) who find that changes in productivity have only a small effect on fertility while changes in mortality have a relatively large effect.

Finally, for low IES ($\sigma = 3$), the overall changes give rise to a predicted fall in $CTFR$ from 4.2 in 1800 to 3.8 in 1850 (the 1826 birth cohort) and 2.3 in 1980 (the 1958 birth cohort) and a predicted fall in CBR from 36.3 in 1800 to 17.2 in 1990. In this case, CBR_s is decreasing over the period (see Figure 4). This is mainly due to increased productivity growth. In terms of population growth rates, the model predicts a fall from 1.4 to 0.6 percent per year. This follows from the effect of changing productivity growth which overturns the effect of decreased mortality. Hence, changes in CBR predicted by the model capture about two-thirds of changes observed in the data, while model predictions of changes in $CTFR$ and population growth rates, γ_N , capture about one half of observed changes.²³

Table 2: Time Series Experiment: Data versus Model, for several values of σ

		1800	1880	1990
$\gamma_{N,ann}$	Data	1.027	1.016	1.006
	$\sigma = 0.5$	0.965	0.979	1.006
	$\sigma = 1.0$	0.994	.995	1.006
	$\sigma = 3.0$	1.014	1.005	1.006
CBR_{ann}	Data	45.4	35.2	15.8
	$\sigma = 0.5$	0.59	10.6	17.2
	$\sigma = 1.0$	22.7	22.3	17.2
	$\sigma = 3.0$	36.3	29.6	17.2
$CTFR$	Data	7.04	4.90	1.97
	$\sigma = 0.5$	1.54	2.04	2.30
	$\sigma = 1.0$	2.79	2.79	2.30
	$\sigma = 3.0$	4.20	3.45	2.30

²³We have also experimented with even higher values for σ (not shown here). Although the implied levels for the CBR are even higher in the earlier periods, this change is not large, and even levels of σ close to 1,000 do not generate the entire change seen in the data.

5.3 Productivity versus mortality: decomposition

Next, we decompose the sources of these changes (for $\sigma = 3$) into the components separately. Since the effects of changing π are quantitatively quite small we don't emphasize those changes. That is, we ask what the model would predict for *CTFR*, *CBR* and γ_N if two of the three forcing variables had stayed at their 1800 levels, while the other changed as per the experiment above. The results from these calculations are shown in Table 3. We find that, taken one at a time, both changes productivity growth (γ) and changes in survival probability to adulthood (π_s) have sizable impacts on both *CTFR* and *CBR*. Changes in expected lifetime (π) generate a hardly noticeable increase in *CTFR* and account for a relatively small decrease in *CBR*. As can be seen, the effect is largest for changes in π_s which by itself shows a decrease in *CTFR* from 4.2 to 2.9 and in *CBR* from 36.3 to 24.7. The effect of changing γ alone is smaller but still significant, reducing *CTFR* from 4.2 to 3.3 and *CBR* from 36.3 to 28.6. The effect of a change in π is substantially smaller, causing a reduction in *CBR* from 36.3 to 33.0.

On the other hand, the changes in both π_s and π cause population growth, γ_N , to increase. It is the increase in productivity growth, γ , that alleviates this so that, in sum, the effect of the three changes taken together on γ_N is negative, i.e., the increase in γ_N from increases in π_{cy} and π is more than offset by the decrease in γ_N resulting from the increase in γ . These findings are consistent with Mateos-Planas (2002) who focuses on population growth rates (rate of natural increase) and therefore understates the importance of mortality to understand the facts relating to births of the demographic transition.

Table 3: Decomposition: Productivity (γ) vs. Mortality (π_s), $\sigma = 3.0$

		1800	1880	1990
$\gamma_{N,ann}$	Productivity (γ)	1.014	1.005	1.002
	Mortality (π_s)	1.014	1.014	1.018
<i>CBR</i> _{ann}	Productivity (γ)	36.3	30.3	28.6
	Mortality (π_s)	36.3	35.9	24.7
<i>CTFR</i>	Productivity (γ)	4.20	3.49	3.30
	Mortality (π_s)	4.20	4.15	2.90

The results of the decomposition exercise are also interesting because of their implication about the timing of fertility decline. Previous authors have criticized the hypothesis that the fertility decline was a byproduct of a reduction of infant mortality rates because of questions about the relative timing of these two changes (see for example, Van de Walle (1986), Doepke (2005), Fogel (1990)). The model predictions here also show that the decrease in youth mortality has very little effect before 1880. Then, from 1880 to 1950, changes in youth mortality (from survival to age 20 of $\pi_s = 0.64$ in 1880 to $\pi_s = 0.96$ in 1950) account for 93 percent of the total model predicted change in CBR. However, changes in productivity growth rates (from 0 in 1800 to 1.4 percent per year in 1880) account for about 90 percent of the total model predicted change in CBR for the early period.²⁴

In sum then, all three effects are quantitatively important in understanding the model predictions about the history of fertility and population growth over the last 200 years. An important requirement for the success of the model is low values for the IES in consumption.

6 Discussion

In this section, we discuss various issues related to and extensions of the model laid out so far. First, we address issues related to the assumptions on utility. We then briefly discuss a version of the model in which parents may leave physical capital bequests. This allows parents to partially control their offspring's utility. Further, we discuss changes in the costs of children and educational investments. Finally, we address health related choices.

6.1 About utility assumptions

The formulation for the dynasty utility flow function that we use here gives rise to some very useful simplifications. In this subsection we discuss several issues related to it that arise.

1. One question that is clearly very relevant for the issues raised in this paper is: Are children and their well-being really substitutes in parent's utility? From introspection, one may think that the more intuitive assumption is that if the utility of all children increases exogenously, the marginal utility of an extra child would increase (i.e. they are complements) rather than decrease. That is, parents would want more happy children, not fewer. When one thinks about the implications for relative price changes, however, it is not so obvious. For example, suppose the utility of the child is positively related to the purchase, by the parent of an investment good, say private education. And suppose the price of private education falls. When family size and utility are complements in

²⁴For plots of detailed predicted time paths for the other measures of fertility, see the Appendix.

utility, it follows that both education and family size increase, while when they are substitutes, education increases substantially, but family size falls. Testing this hypothesis could be very fruitful. For example, one could use China's one child policy to try and assess whether parents spend the freed resources from lower total child costs on their own consumption or whether they invest in the child's education. If number and quality were complements, most of the increase should be seen in parent's consumption while if they were substitutes, education should be seen to increase substantially to compensate for the lack of additional children.

2. As we have seen in Section 3, with this utility function, high elasticity is always associated with complementarity and vice versa. There are ways to combine cases of low (high) IES and complementarity (substitutability). For example, write the period utility in the dynasty problem as

$$\frac{[\mu N_t^\rho + (1 - \mu)C_t^\rho]^{\frac{1-\psi}{\rho}}}{1 - \psi}.$$

When $\rho < 1 - \psi$, C and N are complements in utility while they are substitutes if $\rho > 1 - \psi$. For this change to overturn our results, population and consumption would have to be either very complementary or very substitutable. The disadvantage of a formulation like this is that the basic interpretation of g and U_{t+1} are no longer straightforward and disaggregation becomes difficult.²⁵

An alternative formulation that would allow us to disentangle these two effects at the household level is given by:

$$u(c) = \bar{u} + \frac{c^{1-\sigma}}{1 - \sigma}.$$

By adjusting \bar{u} appropriately, one can consider $\sigma > 1$ but non-negative utility (i.e. complements) or $\sigma < 1$ and negative utility (i.e. the substitutes case) with the assumptions on $g(\cdot)$ adjusted appropriately. One disadvantage of this formulation is that there is no BGP in the sense that fertility and population growth will have a trend as long as $\bar{u} \neq 0$. Given this, we chose to derive analytical comparative statics across BGPs with $\bar{u} = 0$.²⁶

3. A related issue concerns the utility of unborn children. That is, one interpretation of the fact that children that are not born do not enter the calculation of time- t utility is that they are assigned $U = 0$. This interpretation is fine when $\sigma < 1$, but causes difficulty when $\sigma > 1$. When utility is negative we

²⁵The formulation in Section 2 assumes $\rho \rightarrow 0$, $\mu(1 - \psi) = \eta + \sigma - 1$ and $(1 - \mu)(1 - \psi) = 1 - \sigma$.

²⁶Hall and Jones (2007) use this formulation to generate a trend to the health spending share of GDP.

can assume that unborn children also get negative utility, and even less than that received by born children ($\bar{u}_{unborn} < \bar{u}_{born}$) and that parents are altruistic towards these children too. To see this, let the utility of the parent be given by:

$$U_t = u(c_t) + g(n_t)\bar{u}_{born} + h(n_p - n_t)\bar{u}_{unborn}$$

where n_p is the number of potential children and n_t is the number of children born. As can be seen from this, one interpretation of the preferences we use is that $h = 0$, not that $\bar{u}_{unborn} = 0$. Under this interpretation, parents are only weakly altruistic toward their children however.

Strict altruism with respect to the level of utilities holding the number of births fixed requires $g(\cdot) > 0$ and $h(\cdot) > 0$. Since $\bar{u}_{unborn} < \bar{u}_{born}$, strict altruism also requires that increasing n_t strictly increases U_t . This can be written as:

$$\frac{d}{dn} [g(n)\bar{u}_{born} + h(n_p - n)\bar{u}_{unborn}] > 0.$$

This condition is not necessarily satisfied. In particular, it is important that the marginal utility from the unborn increases more slowly than the marginal utility from born children decreases as children move from the unborn to the born state. To gain some intuition, consider the case where utilities are isoelastic and the same— $g(n) = h(n) = n^\eta$. Then the condition above becomes:

$$1 < \left(\frac{n_p}{n_t} - 1\right) \left[\frac{\bar{u}_{unborn}}{\bar{u}_{born}}\right]^{1/(\eta-1)}.$$

One simple way to satisfy this condition is to assume that the number of children that can feasibly be born, $\bar{n}_t = \frac{w_t}{\theta_t}$, is small relative to the number of potential children, n_p , i.e., as $\frac{\bar{n}_t}{n_p} \rightarrow 0$, the additive term representing the unborn in parent's utility is more or less independent of parent's choices. In this case, the decisions made are approximately the same as the ones made with the utility functions used throughout the paper.

4. Another concern is the fact that, for $\sigma > 1$, $U_0 = -\infty$ if for any i and t , $n_{it} = 0$. From the dynastic point of view, if at any point in time, any descendant has zero children that branch of the dynasty has $-\infty$ continuation utility implying that the time 0 decision maker also has $-\infty$ utility. When $0 \leq \sigma < 1$, this is not an issue however—the dynasty just gets $U = 0$. This is particularly relevant for questions such as those in Doepke (2005) and Kalemli-Ozcan (2003) where the probability of all born children dying is strictly positive. One can get around this problem by adding small constants to consumption and children in the utility and thereby preserve the properties of the present model to a large extent while bounding utility away from $-\infty$.

6.2 Substitutes, complements and choosing bequests

In the model laid out so far, parents have no control over children's well being or quality, U_{t+1} . This simplification allowed us to derive simple intuitions and analytic comparative statics. There are various ways, however, in which parents can affect children's initial conditions. Two ways addressed frequently in the literature using dynastic models are bequests (e.g. Becker and Barro (1988), Barro and Becker (1989)) and human capital investments (e.g. Becker, Murphy, and Tamura (1990), Manuelli and Seshadri (2007)). In this Section, we include physical capital (i.e., bequests) into the model. We first consider the partial equilibrium version as in Becker and Barro (1988). Next, we let interest rates and wages be determined in equilibrium as in Barro and Becker (1989). We find that the basic intuitions go through with this change. That is, for low values of IES, population growth rates fall when productivity growth rates increase. However, the threshold for which the sign of the productivity growth effect switches is no longer $\sigma = 1$. Quantitatively, effects are smaller. The intuition for this result is that when productivity growth increases exogenously, U_{t+1} tends to increase, but parents now have an extra margin with which to adjust U_{t+1} itself by leaving a lower bequest. This tends to dampen the complementarity /substitutability effect on fertility choice. The result suggests that even this model bypasses important effects such as increasing returns to human capital (as in Becker, Murphy, and Tamura (1990), for example).

The representative dynasty problem we are interested in is given by

$$\text{Max}_{\{C_t, N_t, K_t\}} \quad U_0(\{C_t, N_t, K_t\}) = \sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} \frac{C_t^{1-\sigma}}{1-\sigma}$$

s.t.

$$\begin{aligned} C_t + \theta_{st} N_{st} + X_t &\leq w_t N_t + r_t K_t \\ K_{t+1} &\leq (1 - \delta) K_t + X_t, \\ N_{t+1} &\leq \pi N_t + N_{st} \\ (N_0, K_0) &\text{ given.}^{27} \end{aligned}$$

Note that this problem is well defined under both assumption AI and AII derived in Section 2 as long as $\eta \neq 1 - \sigma$. The first order condition with respect to K_{t+1} and N_{t+1} together with the budget constraint, boil down to the following system of equations governing the solution to this (partial equilibrium) problem:

$$\begin{aligned} \gamma_{ct}^\sigma \gamma_{Nt}^{1-\eta} &= \beta(r_{t+1} + 1 - \delta) \\ \theta_t(r_{t+1} + 1 - \delta) &= \left[\frac{(\eta+\sigma-1)}{(1-\sigma)} \frac{C_{t+1}}{N_{t+1}} + [w_{t+1} + \theta_{t+1}\pi] \right] \\ \frac{C_t}{N_t} + \theta_t \gamma_{Nt} + \frac{K_{t+1}}{N_{t+1}} \gamma_{Nt} &= [w_t + \pi\theta_t] + (r_t + 1 - \delta) \frac{K_t}{N_t}. \end{aligned}$$

To ensure interiority in partial equilibrium, we have to (1) either rule out $\eta = 1 - \sigma$ or, (2) if $\eta = 1 - \sigma$, make the necessary parameter assumptions so that rates of

²⁷Notice that since as before one implicit assumption is that all alive adults are identical in this model, we have to assume that parents share their assets equally among all surviving family members (including themselves), so that initial conditions of all households are the same.

returns to children and capital are equalized. In general equilibrium, prices will adjust to achieve this. To close the model, wages and interest rates are determined in equilibrium by a firm hiring labor and capital to maximize profits with a constant returns to scale production function, $F(K_t, \gamma^t N_t) = AK_t^\alpha (\gamma^t N_t)^{1-\alpha}$. That is,

$$\begin{aligned} r_t &= F_K(K_t, \gamma^t N_t) \\ w_t &= F_N(K_t, \gamma^t N_t). \end{aligned}$$

On a balanced growth path, we have $\gamma_{c,t} = \gamma_c = \gamma$, $\gamma_{N,t} = \gamma_N$, $\gamma_C = \gamma_K = \gamma\gamma_N$, wages grow at γ and interest rates are constant. Denoting detrended variables by \hat{x} the above equations become:

$$\begin{aligned} \gamma_N &= \frac{[\beta(r+1-\delta)]^{\frac{1}{1-\eta}}}{\gamma^{\frac{\sigma}{1-\eta}}} \\ \hat{c}^* &= [\hat{w} + \pi\theta_s] + \gamma(r+1-\delta)\hat{k}^* - \theta_s\gamma_N - \hat{k}^*\gamma_N\gamma \\ r+1-\delta &= \frac{\gamma}{\theta_s} \frac{(\eta+\sigma-1)}{(1-\sigma)} \hat{c}^* + \frac{\gamma}{\theta_s} \hat{w} + \pi\gamma \\ r &= \alpha A \hat{k}^{*\alpha-1} \\ \hat{w} &= (1-\alpha) A \hat{k}^{*\alpha}. \end{aligned}$$

From the first equation above, it is clear that—as usual in Barro-Becker type models—higher interest rates are typically associated with higher population growth. In light of the comparative statics results above, suppose the rate of productivity growth, γ , increases. Holding the interest rate fixed (i.e., in partial equilibrium), this decreases the population growth rate. However, the interest rate may be endogenous. In response to an increase in productivity growth the BGP level of the capital-people ratio, \hat{k}^* , decreases and hence the interest rate increases which tends to increase population growth. Whether the population growth increases or decreases depends on all parameters, in particular the side of the parameter space one chooses:

- I. Under AI the positive effect from the interest rate is large since $\frac{1}{1-\eta} > 1$. However, since in this case, we also have $\frac{\sigma}{1-\eta} \geq 1$, the direct negative effect from an increase in productivity growth is also large.
- II. Under AII, the positive effect from the interest rate is small since $\frac{1}{1-\eta} < 1$. However, since in this case, we also have $\frac{\sigma}{1-\eta} \leq 1$, the direct negative effect from an increase in productivity growth is also small.

In the case where $\eta = 1 - \sigma$ a sufficient condition for the population growth rate to be decreasing in productivity growth is $\sigma > 1 - \alpha$ (this condition is necessary and sufficient in the case where there is full depreciation in physical capital and people live for only one period, i.e. $1 - \delta = \gamma\pi = 0$, the analogue without capital was $\sigma > 1$). Since $\alpha > 0$, it is possible that the net effect on population growth from an increase in productivity is negative even if $\sigma < 1$. Nevertheless, the typical

values used in the fertility literature ($\sigma \approx 0.5$) with a capital share of $\alpha \approx 0.4$ do not satisfy this condition.

Quantitatively, experiments analogous to those in Section 5 (calibrating to 1990 and changing youth mortality, longevity and productivity growth as observed in the U.S. since 1800), give results that are quite similar. Using the same parameters as before together with $\sigma = 3$, $\alpha = 0.4$ and capital depreciation of 5 percent per annum, the model predicts a fall in CBR from 30.2 to 17.19 (compared to 36.3 to 17.19 in the model without capital), while for $\sigma = 0.5$ CBR falls from only 20.3 to 17.19 (compared to an increase from 2 to 17.19 in the model without capital). For population growth rates, however, the effect of increased productivity which decreases population growth no longer dominates the effect from mortality and longevity which increases population growth rates. This produces a slight overall increase from 0.47 percent per annum to 0.65 percent per annum when $\sigma = 3$ and a large increase from -0.01 percent to 0.65 percent per annum when $\sigma = 0.5$.²⁸

Hence, the choice of σ is still crucial for qualitative and quantitative predictions of dynastic models even if parents have some control over children's initial conditions (through bequests in this case) and hence, their well-being, U_{t+1} .

6.3 Costs of children and education

There are many important extensions of the Barro-Becker model that have been considered in the literature. An important and interesting exercise suggested by the results in this paper is to revisit these channels with low elasticity and substitutability between number and well-being of children.

Throughout, we have assumed that the base costs of raising a child to adulthood have been unchanged over the period. However, when one adopts a broad view of what determines these costs—e.g., subtracting out any direct input from the child on a farm—this is clearly a strong assumption. Indeed, the relative availability of land in the U.S. and the resulting implications for the size of net costs of children may be one of the key reasons why the model predicts too low fertility for the early years (and also one of the reasons that fertility was so much higher in the U.S. than it was in the U.K.). Mateos-Planas (2002) adjusts base costs residually in order to match the entire path of population growth rates in several European countries and finds large increases in these costs since 1900. Clearly the analysis would benefit greatly from a more careful accounting of the costs of children along these dimensions.

Related to this, many of the authors who have worked on the history of fertility have emphasized the role of changes in education over the last 200 years. Examples include Becker, Murphy, and Tamura (1990), Rosenzweig (1990), Benhabib and Nishimura (1993), Galor and Weil (2000), Fernandez-Villaverde (2001), Soares (2005), Doepke (2004), Manuelli and Seshadri (2007) and many others. From a formal point of view, this is much like including a capital stock as discussed above, i.e., it is another

²⁸For more detail on quantitative result, see the Appendix.

way for parents to affect the level of utility of their children. The technology and policy issues that arise might be quite different however. Examples of the latter include the kinds of regulatory changes that have taken place, e.g., child labor laws and compulsory schooling (see Doepke (2005) for example). The necessary conditions on technology to generate a decrease in population growth and fertility in these models may well be weaker once low elasticity and substitutability are assumed.

6.4 Health investments, mortality and longevity

Finally, in our analysis, we have assumed that survival probabilities are exogenous to the decision maker. While this is probably a reasonable assumption about many of the improvements in health over the period (the development of the germ theory of disease, the advent of pasteurization of milk and vaccinations), there are also many aspects of health that affect these probabilities are in fact chosen. Indeed, some authors have explicitly modelled this choice (e.g., Fernandez-Villaverde (2001) and Kalemlı-Ozcan (2002, 2003), Manuelli and Seshadri (2007)), de la Croix and Licandro (2007), Hall and Jones (2007)). Revisiting previous results along the lines of this paper would probably also be fruitful.

7 Conclusion

In models of fertility choice based on parental altruism à la Barro-Becker, we have shown that a key model feature is the assumption on whether the number of children and utility of those children are complements or substitutes. This is important both qualitatively and quantitatively. When family size and utility of offspring are complements, as is commonly assumed, there is a strong force to increase fertility when conditions improve, either over time, or across countries. This is counter to what is seen in the data. On the other hand, when they are substitutes, as we have shown here, there is a strong force in the opposite direction. Further, we have shown that this choice is closely related to the choice of the intertemporal elasticity of substitution in consumption which through the desire to smooth consumption also affects the size of the fertility response to changes in mortality, longevity and productivity growth.

The effects laid out in this paper are purely preference driven and hence present in many extensions including formulations with the possibility of leaving bequests or educating children. Hence, the results in this paper shed new light on previous work using Barro-Becker type preferences and are qualitatively relevant for any question being addressed using these preferences.

Finally, the ability of the model quantities to match key features of the data also depends critically on the choice of the IES. These quantitative results help reconcile seemingly contradictory results in the quantitative literature addressing trends in fertility.

A Appendix

A.1 Time series used in Section 5

Table A.1: Annual Data Used for the Time Series Experiment in Section 5, U.S.

a	b	c	d	e	f	g	h	i	j	k
<i>Year</i>	γ	π_i	π_{ic}	π_{cy}	$\frac{T}{1-\pi}$	π_{ann}	<i>CBR</i>	<i>PG</i>	<i>CTFR</i>	BC
					EL	EL	data	data	data	
1800	1.000	0.771	0.895	0.916	38.41	0.964	45.40	1.027	<i>7.04</i>	—
1810	1.000	0.771	0.895	0.916	38.41	0.964	45.82	1.025	<i>6.92</i>	—
1820	1.000	0.771	0.895	0.916	38.41	0.964	45.12	1.027	<i>6.73</i>	—
1830	1.007	0.771	0.895	0.916	38.41	0.964	45.18	1.024	<i>6.55</i>	—
1840	1.007	0.771	0.895	0.916	38.41	0.964	48.17	1.023	<i>6.14</i>	—
1850	1.007	0.771	0.895	0.916	38.41	0.964	46.05	1.020	5.59	1828
1860	1.007	0.803	0.909	0.926	40.32	0.966	45.19	1.018	5.49	1838
1870	1.014	0.816	0.917	0.933	41.14	0.967	39.30	1.017	5.36	1848
1880	1.014	0.775	0.899	0.920	39.65	0.966	35.16	1.016	4.90	1858
1890	1.014	0.840	0.921	0.934	40.96	0.967	32.71	1.013	4.50	1868
1900	1.014	0.871	0.930	0.940	41.73	0.968	28.58	1.013	3.25	1878
1910	1.014	0.885	0.947	0.940	42.63	0.969	27.50	1.011	3.15	1888
1920	1.018	0.908	0.990	0.992	44.45	0.971	24.92	1.012	2.82	1898
1930	1.018	0.931	0.994	0.995	45.14	0.971	19.05	1.007	2.30	1908
1940	1.018	0.945	0.997	0.997	46.77	0.973	18.60	1.012	2.59	1918
1950	1.018	0.967	0.998	0.998	49.01	0.974	23.00	1.015	3.11	1928
1960	1.018	0.973	0.999	0.998	49.65	0.975	22.70	1.011	3.01	1938
1970	1.018	0.977	0.999	0.998	49.63	0.975	17.40	1.006	2.22	1948
1980	1.018	0.987	0.999	0.999	51.73	0.976	15.10	1.006	1.80	1958
1990	1.018	0.990	1.000	0.999	52.95	0.977	15.80	1.006	<i>1.97</i>	—

Data Sources for Table A.1:

- Column b, γ (annual productivity growth rate): is from the data on real wages in Greenwood and Vandenbroucke (2005), from 1830 to 1988, for the period from 1800 to 1830, we assumed $\gamma = 1.00$;
- Column c, π_i (survival probability from age 0 to age 1): is derived from data on Infant Mortality Rates, IMRs from:

- 1850 to 1900 are from Haines (1994a), U.S. Model, Total Population Both Sexes;
 1800 to 1840 are assumed to be the same as 1850;
 1910 is taken from U.S. Department of Commerce, Bureau of the Census (1910);
 1920 to 1990 are from National Center for Health Statistics (1998);
- Column d, π_{ic} (survival probability to age 5, conditional on surviving to age 1):
 1850 to 1900 are derived from year by year death rates in Haines (1994a), U.S. Model, Total Population Both Sexes;
 1800 to 1840 are assumed to be the same as 1850;
 1910 is derived from year by year death rates from U.S. Department of Commerce, Bureau of the Census (1910);
 1920 to 1990 are from National Center for Health Statistics (1998);
 - Column e, π_{cy} (survival probability to age 20, conditional on surviving to age 5):
 1850 to 1900 are derived from year by year death rates in Haines (1994a), U.S. Model, Total Population Both Sexes;
 1800 to 1840 are assumed to be the same as 1850;
 1910 is derived from year by year death rates from U.S. Department of Commerce, Bureau of the Census (1910);
 1920 to 1990 are from National Center for Health Statistics (1998);
 - Column f, $\frac{T}{1-\pi}$ (EL) (expectation of life at age 20): is taken from Lee (2001), Column B;
 - Column g, π_{ann} (EL) (annual adult survival rate): is derived from Column f;
 - Column h, CBR (crude birth rate, annual):
 1800 to 1930 are taken from Hacker (2003), Figure 1,
 1940 to 1990 are from Haines (1994b), Table 3;
 - Column i, PG (population growth rate, annual): are taken from Haines (1994b), Table 1, RNI=rate of natural increase (net of immigration).
 - Column j, $CTFR$ (cohort total fertility rate):
 1800 to 1840 TFR taken from Haines (1994b), Table 3, Whites
 1850 to 1980 CEB taken from Jones and Tertilt (2006), Table A6;
 1990 TFR taken from Haines (1994b), Table 3, Weighted average Whites-Blacks;
 - Column j, BC (birth cohort): birth year of mothers, year midpoint for 5-year cohorts.

A.2 Historical experiment: auxiliary figures

Here we give figures—like the one given in the main text for CBR in Section 5—for the other measures of fertility we have discussed, $CTFR$ and γ_N . They show a similar pattern overall with the model capturing significant fractions of the overall changes seen in the data—about half (see also Table 2).

Figure A.1: The U.S. experience from 1800 to 1990, $CTFR$ and TFR

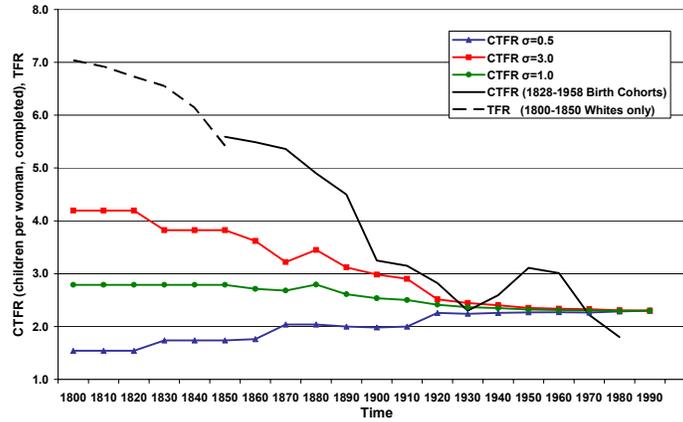
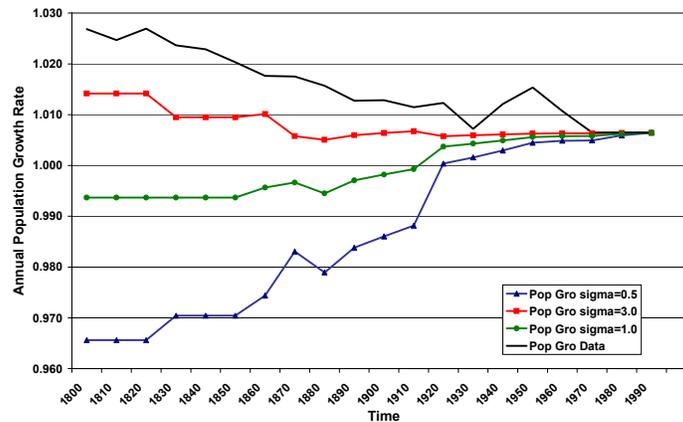


Figure A.2: The U.S. experience from 1800 to 1990, annual population growth rate



Next, we depict the decomposition exercise in Section 5 of the overall changes in these three measures into the three components, changes in γ only, changes in π only and changes in π_s only (see also Table 3).

Figure A.3: Decomposition, *CTFR*

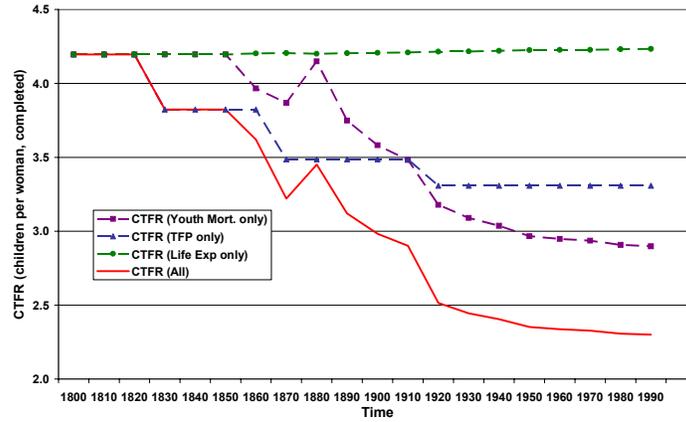


Figure A.4: Decomposition, *CBR*

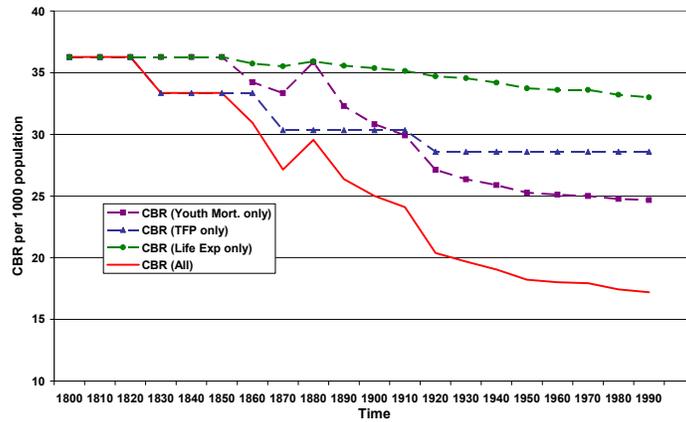
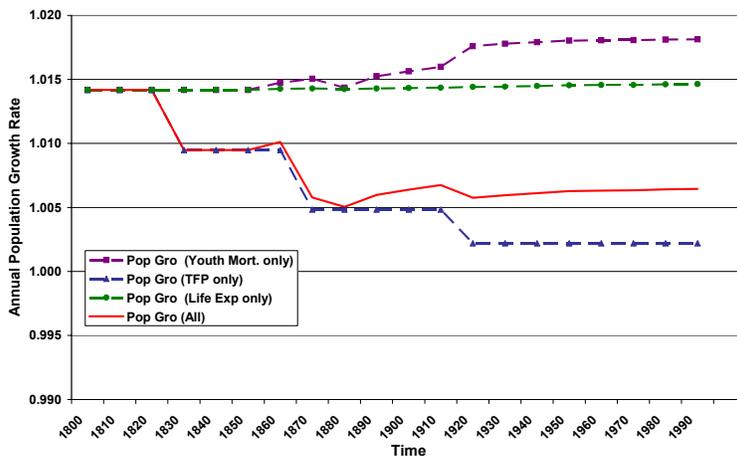


Figure A.5: Decomposition, annual population growth rate



A.3 Using EWL instead of EL

In this section we compare results when expected working lifetime instead of expected lifetime conditional on reaching age 20 is used in the experiments. The range for survival probabilities conditional on reaching age 20, π , in the main text are deduced from measures of expectation of life (EL) at age 20 (see Table A.1, column g). EL increased from 38.5 to 53 years. One issue related to this is that expected time in retirement has increased dramatically over the past 150 years (see Lee (2001)). We performed the same experiment using expected working life at age 20 (EWL) (i.e. the difference between expected lifetime and expected years of retirement) which implied an increase from 36.7 to 40.3 years (taken from Lee (2001), Column C). We obtained very similar results since the effect of longevity on births and population growth rates are small in the experiments above. In particular, we find a decrease in *CTFR* from 4.21 to 2.3, a decrease in *CBR* from 36.95 to 19.6 and a decrease in population growth from 1.44 percent per year to 0.65 percent per year. The trade-off between using either one of these measures is that on the one hand, EL overstates the benefits in terms of income from having children, while EWL understates the benefits from dynasty size (since retirement is analogous to death in this case).

A.4 Perfect Foresight Transition Paths to BGP

In this section, we discuss an alternative to the calculation given in section 5. There, the simulated data were calculated assuming that the agents believed that their current circumstances, in terms of child costs, productivity growth rates and survival probabilities, would prevail indefinitely into the future when making their decisions—i.e., the calculations are BGP to BGP. The weakness of this is that it assumes that agents act as if circumstances will not change in the future, even though they actually will. At the other extreme, one could assume that agents in a give period t , anticipate exactly all future changes that will occur—i.e., there is perfect foresight with respect to future values of γ , π and θ_s . Here we give the calculations for the model under this alternative assumption. We find that this makes very little difference in the end.

From the Planner's problem in Section 3.3, the first-order condition for N_{t+1} is:

$$\theta_{s,t} N_t^{\eta+\sigma-1} C_t^{-\sigma} = \beta \left[\frac{(\eta+\sigma-1)}{(1-\sigma)} N_{t+1}^{\eta+\sigma-2} C_{t+1}^{1-\sigma} + [w_{t+1} + \theta_{s,t+1} \pi_{t+1}] N_{t+1}^{\eta+\sigma-1} C_{t+1}^{-\sigma} \right]$$

The other equation determining the solution is:

$$C_t = [w_t + \theta_{s,t} \pi_t] N_t - \theta_{s,t} N_{t+1}.$$

After some algebra, these two equations can be rewritten as:

$$\begin{aligned} & \gamma_{Nt}^{1-\eta} \left[\frac{\left[\frac{w_{t+1}}{\theta_{s,t+1}} + \pi_{t+1} \right]^{-\gamma_{Nt+1}}}{\left[\frac{w_t}{\theta_{s,t}} + \pi_t \right]^{-\gamma_{Nt}}} \right]^{\sigma} \\ &= \beta \left[\frac{\theta_{s,t+1}}{\theta_{s,t}} \right]^{1-\sigma} \left[\frac{(\eta+\sigma-1)}{(1-\sigma)} \left[\left[\frac{w_{t+1}}{\theta_{s,t+1}} + \pi_{t+1} \right] - \gamma_{Nt+1} \right] + \left[\frac{w_{t+1}}{\theta_{s,t+1}} + \pi_{t+1} \right] \right] \\ & \frac{C_t}{N_t} \frac{1}{\theta_{s,t}} = \left[\frac{w_t}{\theta_{s,t}} + \pi_t \right] - \gamma_{Nt}. \end{aligned}$$

The first of these is a first order difference equation in γ_N . It has time varying coefficients however.

If $(\theta_{s,t}, w_t, \pi_t)$ converge in the sense that $\frac{w_t}{\theta_{s,t}} \rightarrow \frac{w}{\theta_s}$, $\frac{\theta_{s,t+1}}{\theta_{s,t}} \rightarrow \gamma$, $\pi_t \rightarrow \pi$, it can be shown that the solution to the model converges to the BGP determined by $\frac{w}{\theta_s}$, γ , and π . Further, assuming that $\frac{w_t}{\theta_{s,t}} = \frac{w}{\theta_s}$, $\frac{\theta_{s,t+1}}{\theta_{s,t}} = \gamma$, and $\pi_t = \pi$ for all $t \geq t^*$ for some t^* , it can be shown that all of the relevant variables, measured in per capita terms, are constant after date t^* . Because of this, the model can be solved backwards from t^* in this case. Thus, suppose that the sequence of exogenous parameters is given by:

$$(\theta_{s,0}, w_0, \pi_0, \dots, \theta_{s,t^*}, w_{t^*}, \pi_{t^*}, \gamma \theta_{s,t^*}, \gamma w_{t^*}, \pi_{t^*}, \dots).$$

Then, the solution to the perfect foresight infinite horizon problem is of the form:

$$(C_0, N_0, \dots, C_{t^*}, N_{t^*}, C_{t^*+1}, N_{t^*+1}, \dots)$$

where:

- 1) for $t \geq t^* + 1$, $N_{t+1} = \gamma_N N_t$ with γ_N given by the solution to:

$$\gamma_N^{1-\eta} = \beta \gamma^{1-\sigma} \left[\frac{(\eta+\sigma-1)}{(1-\sigma)} \left[\left[\frac{w_{t^*}}{\theta_{s,t^*}} + \pi_{t^*} \right] - \gamma_N \right] + \left[\frac{w_{t^*}}{\theta_{s,t^*}} + \pi_{t^*} \right] \right];$$

- 2) $\frac{C_{t^*}}{N_{t^*}} \frac{1}{\theta_{s,t^*}} = \frac{C_{t^*}^*}{N_{t^*}^*} \frac{1}{\theta_{s,t^*}^*} = \left[\frac{w_{t^*}}{\theta_{s,t^*}} + \pi_{t^*} \right] - \gamma_N$;
- 3) for $s \geq 1$,
- $$\frac{C_{t^*}^*}{N_{t^*}^*} \frac{1}{\theta_{s,t^*}^*} = \left[\frac{w_{t^*}}{\theta_{s,t^*}^*} + \pi_{t^*} \right] - \gamma_N \Leftrightarrow \frac{C_{t^*+s}^*}{N_{t^*+s}^*} \frac{1}{\gamma^s \theta_{s,t^*}^*} = \left[\frac{w_{t^*}}{\theta_{s,t^*}^*} + \pi_{t^*} \right] - \gamma_N$$
- 4) For $t < t^*$, γ_{Nt} evolves according to the difference equation:
- $$\begin{aligned} & \gamma_{Nt}^{1-\eta} \left[\frac{\left[\frac{w_{t+1}}{\theta_{s,t+1}} + \pi_{t+1} \right] - \gamma_{Nt+1}}{\left[\frac{w_t}{\theta_{s,t}} + \pi_t \right] - \gamma_{Nt}} \right]^\sigma \\ & = \beta \left[\frac{\theta_{s,t+1}}{\theta_{s,t}} \right]^{1-\sigma} \left[\frac{(\eta+\sigma-1)}{(1-\sigma)} \left[\left[\frac{w_{t+1}}{\theta_{s,t+1}} + \pi_{t+1} \right] - \gamma_{Nt+1} \right] + \left[\frac{w_{t+1}}{\theta_{s,t+1}} + \pi_{t+1} \right] \right]; \end{aligned}$$
- 5) For $t < t^*$, $\frac{C_t}{N_t}$ is given by:
- $$\frac{C_t}{N_t} \frac{1}{\theta_{s,t}} = \left[\frac{w_t}{\theta_{s,t}} + \pi_t \right] - \gamma_{Nt}.$$

A.4.1 Numerical Implementation

We keep the length of a period at $T = 20$ years. Suppose from $t^* = 1990$ on the growth rate in productivity, γ , infant, child and youth mortality ($\pi_i, \pi_{ic}, \pi_{cy}$) (and hence, detrended costs of raising surviving children, θ_s) and adult mortality (longevity), π , are constant. Then, we can use 1) above to solve for the population growth rate, γ_N , on the balanced growth path using parameter values for 1990. We can then use 4) to solve backward for γ_{Nt} , $t = 1970$ using $\gamma_{Nt+1} = \gamma_{Nt^*}$ $t^* = 1990$ and so on. To do this, we have to make one additional assumption (similar to the balanced growth assumption), namely that base costs of raising children, $(\theta_i, \theta_c, \theta_y)$ grow at the same rate as wages every period but are otherwise constant while the cost of raising a surviving child, θ_s , may vary additionally because youth mortality varies. As in Section 5, we assume that base costs are constant fractions of calibrated costs when children survive with certainty. The results from this experiment are almost indistinguishable from the BGP to BGP experiment in Section 5. This is not surprising since changes in mortality and productivity growth are very smooth. That is, knowing that mortality and productivity change slightly in the next few periods induces very similar choices to the setting in which people believe today's parameters will prevail forever. Moreover, the length of a period being 20 years implies large discounts of future utility (children's utility) and hence changes expected in the future do not affect current decisions very much.

Figure A.6: Perfect Foresight versus Balanced Growth to Balanced Growth, CBR

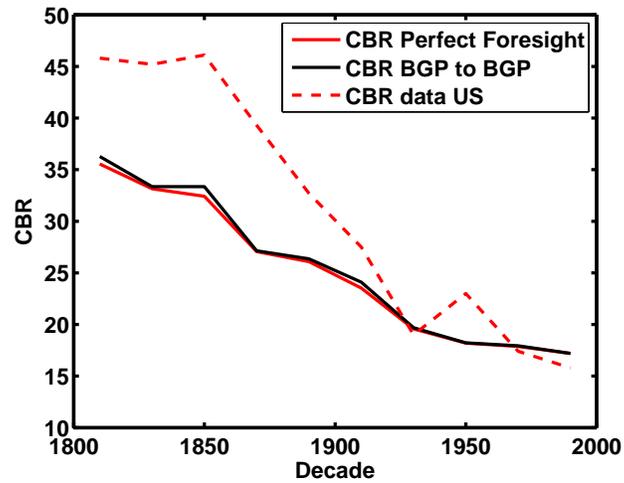
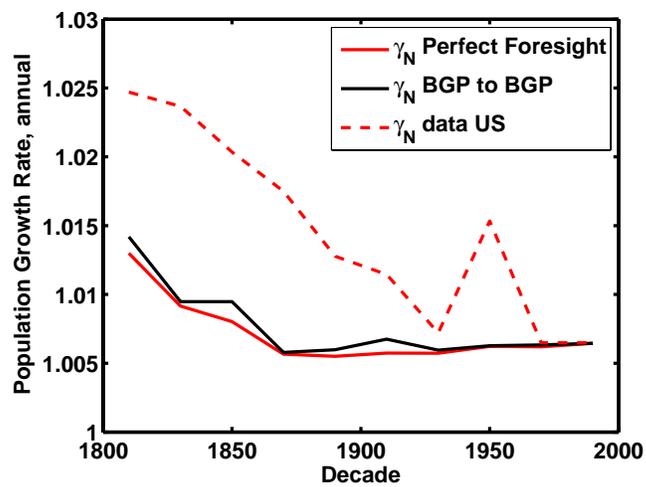


Figure A.7: Perfect Foresight versus Balanced Growth to Balanced Growth, γ_N



A.5 Quantitative comparative statics

Here we give some simple comparative statics of our measures of fertility for various values of σ for the range of relevant parameter values. We calibrate θ_s/w to match $\gamma_N = 1$ using $\gamma = 1.02$, $\pi = 1.0$, $\pi_s = 1.0$. Table A.2 shows the results of changing only γ from $\gamma = 1.00$ to $\gamma = 1.02$ while Table A.3 does the same for $\pi_s = 0.6$ to $\pi_s = 1$. Finally, Table A.4 examines changes in π corresponding to an expected lifetime at age twenty ranging from 25 to 45 years.

Table A.2: Changing productivity growth

Productivity Growth γ_{ann}	$\sigma = 0.5$		$\sigma = 1.0$		$\sigma = 3.0$	
	$\gamma_{N,ann}$	$CBR_{s,ann}$	$\gamma_{N,ann}$	$CBR_{s,ann}$	$\gamma_{N,ann}$	$CBR_{s,ann}$
1.00	0.98	5.25	1.00	15.38	1.013	21.37
1.01	0.99	10.49	1.00	15.38	1.007	18.45
1.02	1.00	15.38	1.00	15.38	1.00	15.38

Table A.3: Changing survival to adulthood (STA)

	π_{cy} (STA)	θ_s/w	$\gamma_{N,ann}$	CBR_{ann}	$CBR_{s,ann}$
$\sigma = 0.5$	0.6	0.85	0.993	19.68	11.81
	0.8	0.80	0.997	17.51	14.01
	1.0	0.77	1.00	15.38	15.38
$\sigma = 1.0$	0.6	0.65	0.996	22.46	13.48
	0.8	0.61	0.998	18.31	14.65
	1.0	0.59	1.00	15.38	15.38
$\sigma = 3.0$	0.6	0.26	0.998	24.16	14.50
	0.8	0.24	0.999	18.80	15.04
	1.0	0.23	1.00	15.38	15.38

Table A.4: Changing expected lifetime (EL)

	π_{ann}	$\frac{T}{1-\pi}$ (EL)	$\gamma_{N,ann}$	CBR_s
$\sigma = 0.5$	0.923	25	0.979	15.60
	0.959	35	0.993	15.26
	0.971	45	1.00	15.38
$\sigma = 1.0$	0.923	25	0.991	19.56
	0.959	35	0.997	17.00
	0.971	45	1.00	15.38
$\sigma = 3.0$	0.923	25	0.998	21.84
	0.959	35	0.999	18.01
	0.971	45	1.00	15.38

A.6 Including physical capital in the model: Time series experiments

Here, we show figures of the calculations for the history of U.S. fertility given in section 5 using the version of the model including physical capital discussed in Section 6. We include only the calculations for $\sigma = 3.0$ and $\sigma = 0.5$. We find that the results are similar but that, all in all, the model predicted changes in fertility are smaller for both values of σ . The one significant change is that there is no longer a reduction in predicted overall population growth rates when $\sigma = 3.0$. This is because the productivity growth rate has a much smaller effect now than it did in the version with labor income only. Mortality, however, has a very large effect—counterfactual, for population growth. Hence, overall, population growth rates increase from 0.46 percent per year to 0.66 percent per year. The crude birth rate in terms of surviving children slightly decreases from 19.1 to 17.05. This is because longevity has a negative effect on CBR_s , while positive on γ_N . Finally, total births, i.e. CBR , decreases from 30.2 to 17.2 children per 1000 population.

Figure A.8: CBR Model with K vs. Data $\sigma = 3$

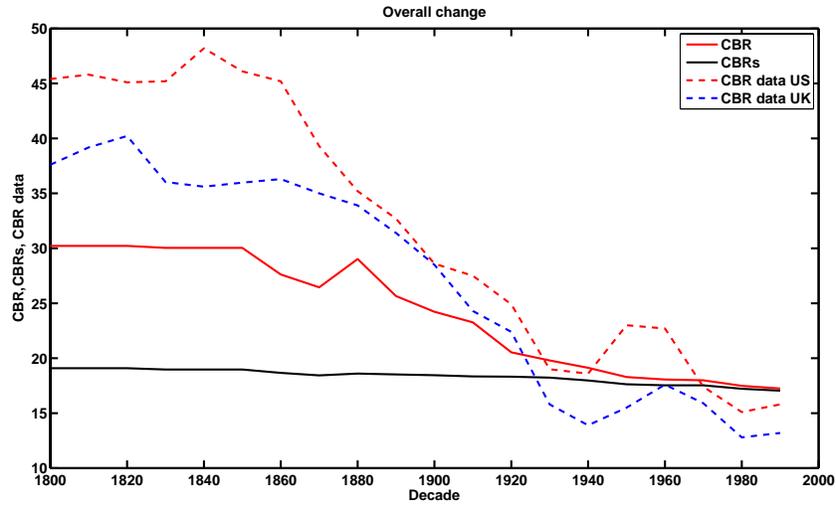


Figure A.9: Population Growth Rate Model with K , $\sigma = 3$

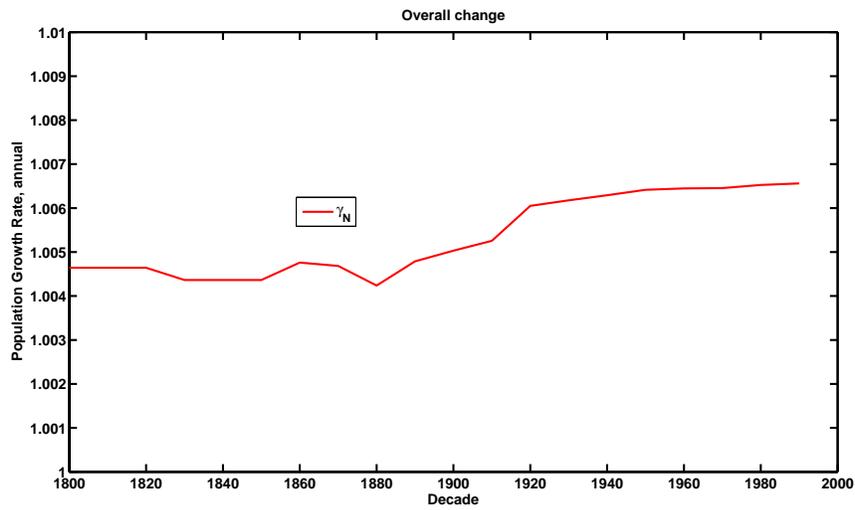


Figure A.10: CBR: Decomposition, Model with K , $\sigma = 3$

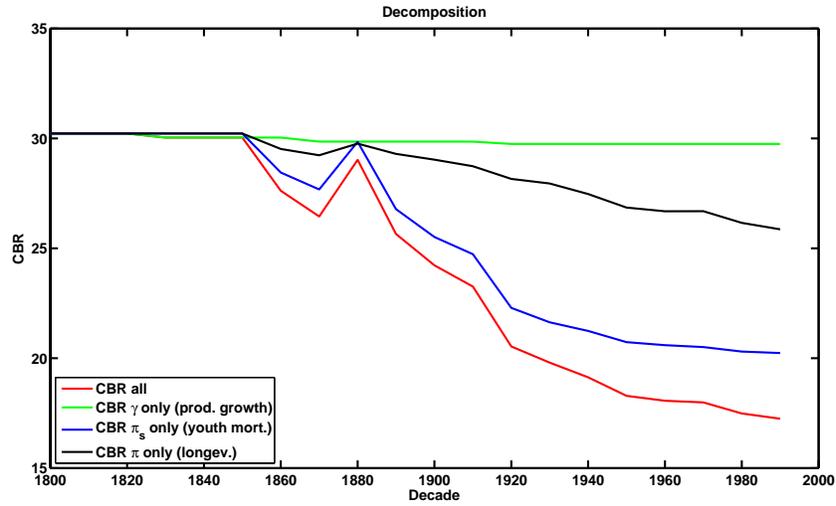


Figure A.11: Population Growth Rate: Decomposition, Model with K , $\sigma = 3$

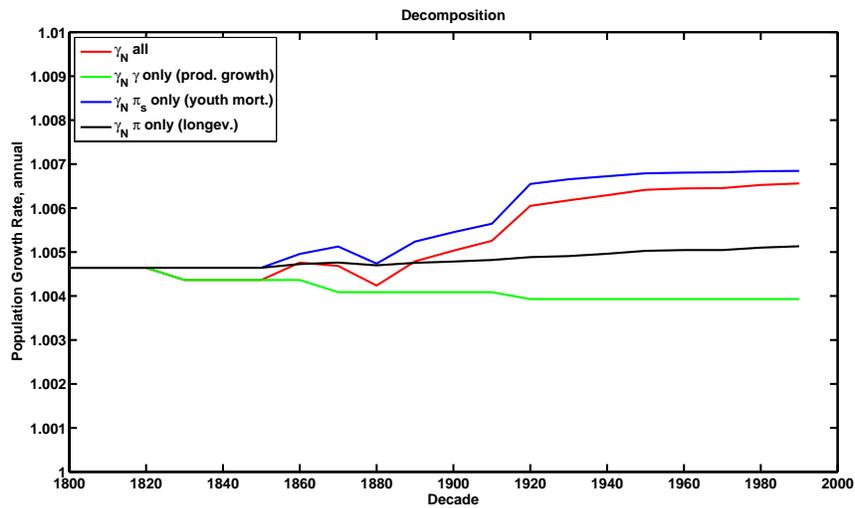


Figure A.12: CBR Model with K vs. Data $\sigma = 0.5$

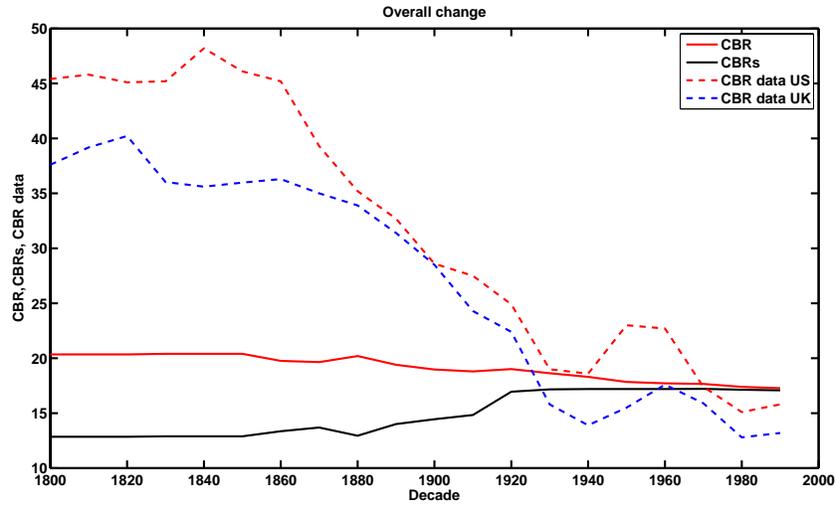


Figure A.13: Population Growth Rate Model with K , $\sigma = 0.5$

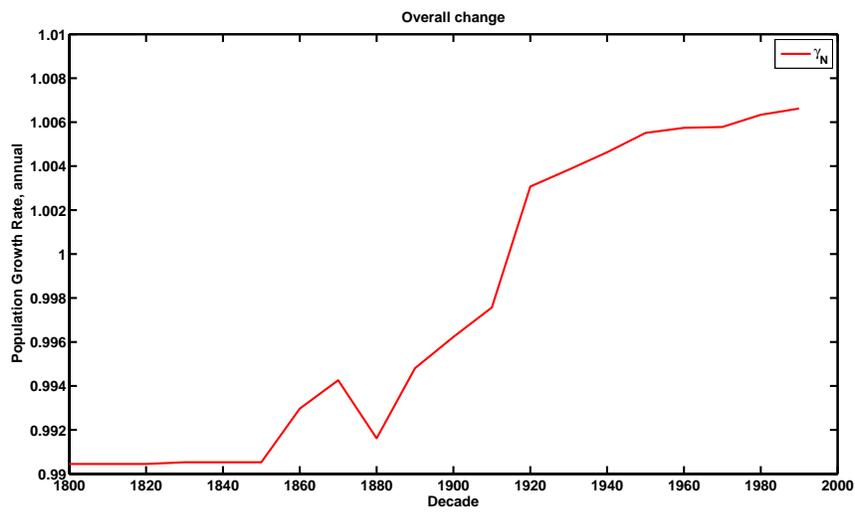


Figure A.14: CBR: Decomposition, Model with K , $\sigma = 0.5$

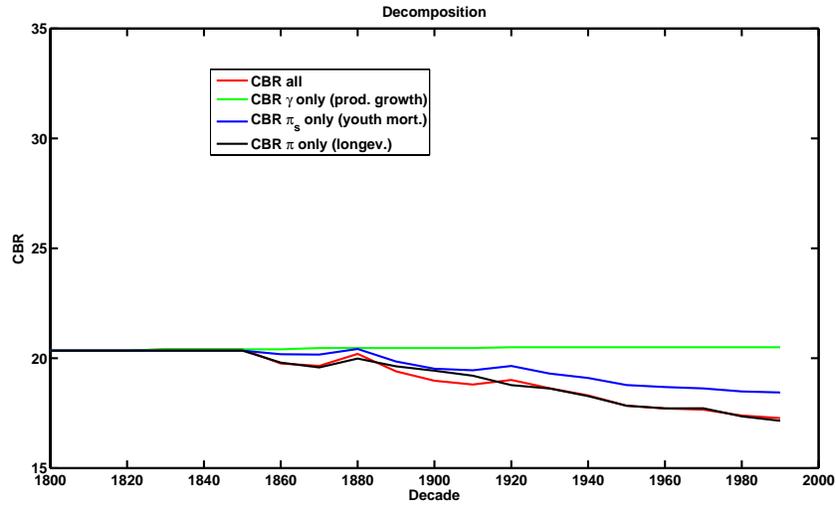
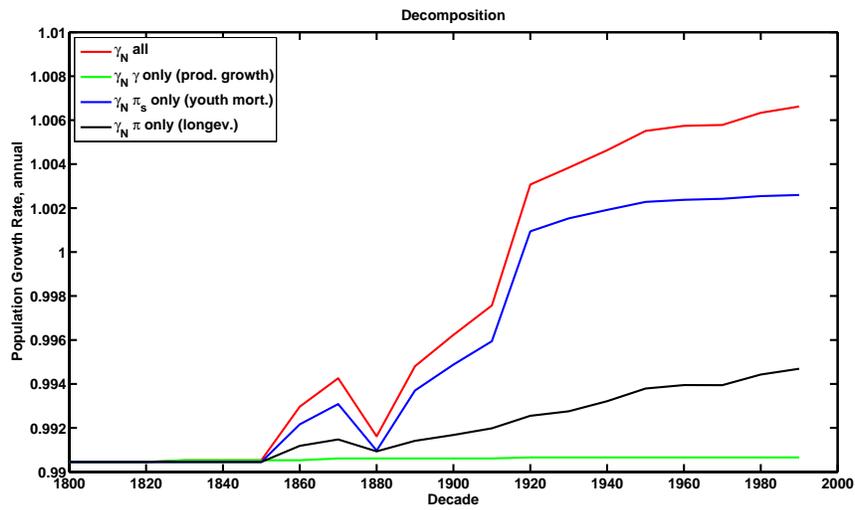


Figure A.15: Population Growth Rate: decomposition, Model with K , $\sigma = 0.5$

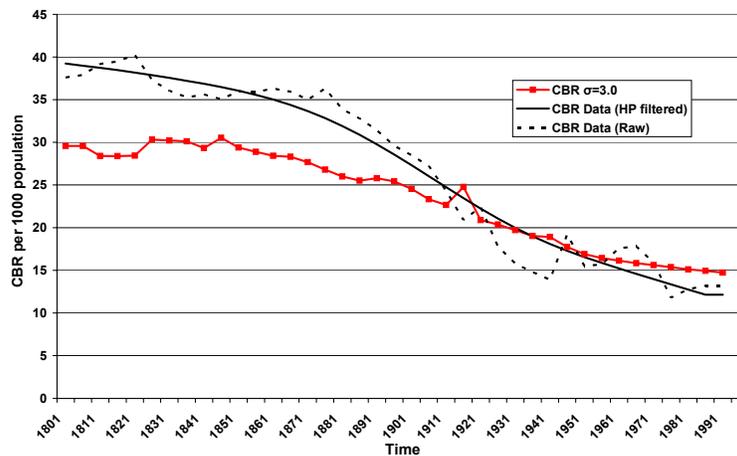


A.7 The case of the U.K.²⁹

In this section, we perform the same experiment as in Section 5 but using data for the United Kingdom (England and Wales for the most part). The results are quite similar: we capture about two-thirds of the change in CBR and one half of the change in population growth.

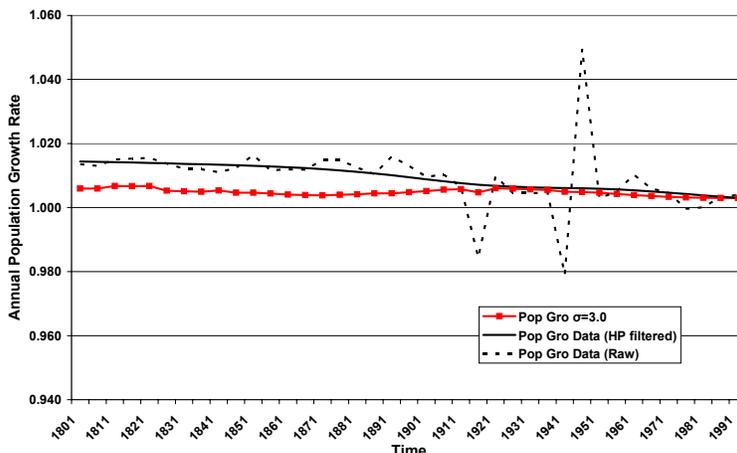
The fertility experience in the U.K. over the past 200 years is similar to that of the U.S., except that levels in 1800 were lower already. Because of this the decrease, both in CBR and population growth, was smaller. Mortality was also lower in the U.K. than it was in the U.S. around that time. Finally, our estimates of productivity growth suggest the latter was higher in 1800 as well. Since, fertility, mortality and productivity growth are very similar in the two countries in 1990, all these observations are consistent with our theory and one would expect the model to capture the same fraction of changes in fertility and population growth.

Figure A.16: The U.K. experience from 1800 to 1990, *CBR*



²⁹We thank Michael Bar and Oksana Leukhina for help with data sources.

Figure A.17: The U.K. experience from 1800 to 1990, annual population growth rate



Data Sources for Table A.5:

- Column b, γ (annual productivity growth rate) from annual growth rate in GDP per capita, (log GDP HP filtered $\lambda = 400$):
1800 to 1865 from Clark (2001),
1850 to 1990 from Maddison (1995), p. 194, rescaled to match Clark in 1850;
- Column c,d,e, $(\pi_i, \pi_{ic}, \pi_{cy})$ (survival rates from age specific mortality rates)
1800 to 1837 from Wrigley et al. (1997), Table 6.1, p.215,
1841 to 1990 from Human Mortality Database;
- Column f, $\frac{T}{1-\pi}$ (EL) (expectation of life at age 20):
1841 to 1990 from Human Mortality Database,
1800 to 1836 set constant at 39 years;
- Column g, π_{ann} (EL) (annual adult survival rate): derived from Column f;
- Column h, CBR (crude birth rate, annual):
1800 to 1871 from Wrigley et al. (1997),
1871 to 1986 from Mitchell (1998)
- Column i, CBR HP filtered (crude birth rate, annual): Column h HP filtered,
 $\lambda = 400$;
- Column j, PG (population growth rate, annual):
1800 to 1837 from Wrigley et al. (1997), Table 6.1, p.215
1841 to 1990 from Human Mortality Database;
- Column k, PG HP filtered (population growth rate, annual): Column j HP
filtered, $\lambda = 400$.

Table A.5: Annual Data Used for the Time Series Experiment in Section A.7, U.K.

a	b	c	d	e	f	g	h	i	j	k
<i>Year</i>	γ	π_i	π_{ic}	π_{cy}	$\frac{T}{1-\pi}$	π_{ann}	<i>CBR</i>	<i>CBR</i>	<i>PG</i>	<i>PG</i>
					EL	EL	data	HP	data	HP
1801	1.006	0.863	0.884	0.864	39.00	0.965	37.60	39.23	1.014	1.014
1806	1.006	0.863	0.884	0.864	39.00	0.965	37.90	38.98	1.013	1.014
1811	1.006	0.867	0.870	0.925	39.00	0.965	39.18	38.72	1.015	1.014
1816	1.006	0.867	0.870	0.925	39.00	0.965	39.48	38.46	1.015	1.014
1821	1.006	0.855	0.864	0.943	39.00	0.965	40.22	38.17	1.016	1.014
1826	1.006	0.855	0.864	0.857	39.00	0.965	37.30	37.87	1.014	1.014
1831	1.006	0.860	0.877	0.840	39.00	0.965	36.03	37.55	1.012	1.014
1836	1.007	0.860	0.877	0.840	39.00	0.965	35.27	37.21	1.012	1.014
1841	1.007	0.838	0.859	0.894	40.57	0.967	35.61	36.86	1.011	1.013
1846	1.007	0.826	0.847	0.886	38.99	0.965	35.06	36.47	1.012	1.013
1851	1.008	0.829	0.858	0.893	40.19	0.966	35.98	36.05	1.016	1.013
1856	1.008	0.830	0.856	0.898	40.63	0.967	35.89	35.57	1.012	1.013
1861	1.009	0.836	0.854	0.902	40.48	0.967	36.30	35.02	1.012	1.013
1866	1.009	0.826	0.864	0.909	39.93	0.966	35.95	34.39	1.012	1.012
1871	1.010	0.830	0.874	0.913	40.01	0.966	35.00	33.67	1.015	1.012
1876	1.010	0.839	0.882	0.926	40.29	0.966	36.30	32.84	1.015	1.012
1881	1.010	0.845	0.889	0.930	41.33	0.967	33.90	31.91	1.012	1.011
1886	1.010	0.844	0.898	0.941	41.79	0.968	32.80	30.88	1.010	1.011
1891	1.010	0.837	0.900	0.942	41.37	0.968	31.40	29.76	1.016	1.010
1896	1.010	0.825	0.907	0.950	42.85	0.969	29.60	28.57	1.013	1.009
1901	1.010	0.843	0.919	0.953	43.43	0.970	28.50	27.32	1.010	1.009
1906	1.010	0.868	0.932	0.957	44.59	0.971	27.20	26.04	1.010	1.008
1911	1.011	0.883	0.939	0.958	45.37	0.971	24.30	24.74	1.006	1.008
1916	1.011	0.900	0.936	0.919	38.3	0.964	20.90	23.47	0.984	1.007
1921	1.011	0.918	0.958	0.964	47.54	0.973	22.40	22.23	1.010	1.007
1926	1.011	0.927	0.963	0.968	47.85	0.973	17.80	21.06	1.005	1.007
1931	1.012	0.935	0.972	0.970	48.54	0.974	15.80	19.98	1.004	1.006
1936	1.013	0.942	0.981	0.975	49.32	0.974	14.80	18.99	1.005	1.006
1941	1.013	0.946	0.984	0.968	48.33	0.974	13.90	18.09	0.979	1.006
1946	1.014	0.960	0.992	0.982	50.76	0.975	19.20	17.29	1.050	1.006
1951	1.015	0.972	0.995	0.992	52.39	0.976	15.50	16.55	1.003	1.006
1956	1.015	0.976	0.996	0.993	53.17	0.977	15.70	15.87	1.005	1.006
1961	1.016	0.978	0.997	0.993	53.48	0.977	17.60	15.22	1.010	1.005
1966	1.017	0.981	0.997	0.993	53.97	0.977	17.80	14.60	1.006	1.005
1971	1.017	0.983	0.997	0.994	54.26	0.977	15.90	13.98	1.005	1.005
1976	1.017	0.986	0.998	0.994	54.79	0.978	11.80	13.36	1.000	1.004
1981	1.018	0.989	0.998	0.995	55.69	0.978	12.80	12.75	1.000	1.004
1986	1.018	0.991	0.998	0.996	56.46	0.978	13.20	12.15	1.003	1.003
1991	1.018	0.993	0.999	0.996	57.39	0.979	13.20	12.15	1.004	1.003

References

- ALES, L., AND P. MAZIERO (2007): “Accounting for Private Information,” Working paper, University of Minnesota.
- ALVAREZ, F., AND N. L. STOKEY (1998): “Dynamic Programming with Homogeneous Functions,” *Journal of Economic Theory*, 82(1), 167–189.
- ALVAREZ, F. E. (1999): “Social Mobility: The Barro-Becker Children meet the Laitner-Loury Dynasties,” *Review of Economic Dynamics*, 2(1), 65–103.
- BAR, M., AND O. LEUKHINA (2007): “Demographic Transition and Industrial Revolution: A Macroeconomic Investigation,” Unpublished manuscript, University of North Carolina, Chapel Hill.
- BARRO, R. J., AND G. S. BECKER (1989): “Fertility Choice in a Model of Economic Growth,” *Econometrica*, 57(2), 481–501.
- BARRO, R. J., AND X. SALA-I-MARTIN (1995): *Economic Growth*. New York: McGraw-Hill.
- BECKER, G. S., AND R. J. BARRO (1988): “A Reformulation of the Theory of Fertility,” *Quarterly Journal of Economics*, 103(1), 1–25.
- BECKER, G. S., K. M. MURPHY, AND R. TAMURA (1990): “Human Capital, Fertility, and Economic Growth,” *Journal of Political Economy*, 98, 812–837.
- BEN-PORATH, Y. (1976): “Fertility Response to Child Mortality: Micro Data from Israel,” *Journal of Political Economy*, 84(4), S163–78.
- BENHABIB, J., AND K. NISHIMURA (1993): “Endogenous Fertility and Growth,” in *General Equilibrium, Growth and Trade II*, ed. by R. Becker, M. Boldrin, and R. Jones, pp. 237–247. Academic Press, San Diego.
- BLANCHARD, O. J. (1985): “Debt, Deficits and Finite Horizons,” *Journal of Political Economy*, 93, 223–247.
- BOLDRIN, M., M. DE NARDI, AND L. E. JONES (2005): “Fertility and Social Security,” NBER Working Paper 11146, National Bureau of Economic Research.
- BOLDRIN, M., AND L. E. JONES (2002): “Mortality, Fertility and Saving in a Malthusian Economy,” *Review of Economic Dynamics*, 5, 775–814.
- DE LA CROIX, D., AND O. LICANDRO (2007): “The Child is Father of the Man: Implications for the Demographic Transition,” Economics Working Papers ECO2007/05, European University Institute.

- DOEPKE, M. (2004): “Accounting for Fertility Decline During the Transition to Growth,” *Journal of Economic Growth*, 9(3), 347–383.
- (2005): “Child mortality and fertility decline: Does the Barro-Becker model fit the facts?,” *Journal of Population Economics*, 18(2), 337–366.
- DOEPKE, M., AND F. ZILIBOTTI (2005): “The Macroeconomics of Child Labor Regulation,” *American Economic Review*, 95(5), 1492–1524.
- ECKSTEIN, Z., P. MIRA, AND K. I. WOLPIN (1999): “A Quantitative Analysis of Swedish Fertility Dynamics: 1751-1990,” *Review of Economic Dynamics*, 2(1), 137–165.
- EHRlich, I., AND F. T. LUI (1991): “Intergenerational Trade, Longevity, and Economic Growth,” *Journal of Political Economy*, 99(5), 1029–59.
- FERNANDEZ-VILLAYERDE, J. (2001): “Was Malthus Right? Economic Growth and Population Dynamics,” Working paper, University of Pennsylvania.
- FOGEL, R. W. (1990): “The Conquest of High Mortality and Hunger in Europe and America: Timing and Mechanisms,” Working Paper 16, National Bureau of Economic Research.
- GALOR, O., AND D. N. WEIL (2000): “Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and beyond,” *The American Economic Review*, 90(4), 806–828.
- GREENWOOD, J., AND A. SESHADRI (2002): “The U.S. Demographic Transition,” *American Economic Review Papers and Proceedings*, 92(2), 153–159.
- GREENWOOD, J., AND G. VANDENBROUCKE (2005): “Hours Worked: Long-Run Trends,” Working Paper 11629, National Bureau of Economic Research.
- HACKER, D. J. (2003): “Rethinking the “Early” Decline of Marital Fertility in the United States,” *Demography*, 40(4), 605–620.
- HAINES, M. R. (1994a): “Estimated Life Tables for the United States, 1850-1900,” Historical Working Paper 59, National Bureau of Economic Research.
- (1994b): “The Population of the United States, 1790-1920,” Historical Working Paper 56, National Bureau of Economic Research.
- HALL, R. E., AND C. I. JONES (2007): “The Value of Life and the Rise in Health Spending,” *The Quarterly Journal of Economics*, 122(1), 39–72.
- JONES, L. E., AND A. SCHOONBROODT (2007): “Baby Busts and Baby Booms: The Fertility Response to Shocks in Dynastic Models,” Discussion Papers in Economics and Econometrics, 0706, University of Southampton.

- JONES, L. E., AND M. TERTILT (2006): “An Economic History of Fertility in the U.S. 1826–1960,” *NBER Working Paper*, 12796.
- KALEMLI-OZCAN, S. (2002): “Does the Mortality Decline Promote Economic Growth?,” *Journal of Economic Growth*, 7(4), 411–39.
- (2003): “A stochastic model of mortality, fertility, and human capital investment,” *Journal of Development Economics*, 70(1), 103–118.
- LEE, C. (2001): “The expected length of male retirement in the United States, 1850–1990,” *Journal of Population Economics*, 14(4), 641–650.
- MANUELLI, R. E., AND A. SESHADRI (2007): “Explaining International Fertility Differences,” Working paper, University of Wisconsin-Madison.
- MATEOS-PLANAS, X. (2002): “The Demographic Transition in Europe: A Neoclassical Dynastic Approach,” *Review of Economic Dynamics*, 5(3), 646–680.
- NATIONAL CENTER FOR HEALTH STATISTICS (1998): “Hist 290: Death Rates for Selected Causes by 10 year Age Groups, Race and Sex: Death Registration States, 1900 to 1932, and U.S., 1933 to 1998,” in *Historical Vital Statistics of the United States: 1890 to 1998*. <http://www.cdc.gov/nchs/datawh/statab/unpubd/mortabs/hist290.htm>.
- ROSENZWEIG, M. R. (1990): “Population Growth and Human Capital Investments: Theory and Evidence,” *Journal of Political Economy*, 98(5), S38–70.
- SAH, R. K. (1991): “The Effects of Child Mortality Changes on Fertility Choice and Parental Welfare,” *Journal of Political Economy*, 99(3), 582–606.
- SOARES, R. R. (2005): “Mortality Reductions, Educational Attainment, and Fertility Choice,” *American Economic Review*, 95(3), 580–601.
- TAMURA, R. (1994): “Fertility, Human Capital and the Wealth of Families,” *Economic Theory*, 4(4), 593–603.
- (1996): “From decay to growth: A demographic transition to economic growth,” *Journal of Economic Dynamics and Control*, 20(6-7), 1237–1261.
- U.S. DEPARTMENT OF COMMERCE, BUREAU OF THE CENSUS (1910): *Mortality Statistics of the United States 1910*. Washington, D.C.
- VAN DE WALLE, F. (1986): “Infant Mortality and the European Demographic Transition.,” in *The Decline of Fertility in Europe*, ed. by S. Watkins, and A. Coale, chap. 4, pp. 201–233. Princeton University Press.

WOLPIN, K. I. (1997): "Determinants and consequences of the mortality and health of infants and children," in *Handbook of Population and Family Economics*, ed. by M. R. Rosenzweig, and O. Stark. Elsevier.