Preliminary and Incomplete

# On the Taxation of Human Capital

by

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First draft: December 1997
This draft: November, 1999

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## 1. Introduction

The incentives for the formation of human capital through taxation and spending vary widely both across time within a country and across countries. Some of the stylized facts are:

- Public spending on education is large in many countries (e.g. European and Latin American) at both the primary and secondary level, and it is provided at the federal level. In some other countries (e.g. the U.S.), principal responsibility for the public provision of primary and secondary education lies at the local level. To the extent that Tiebout's argument is correct, it is then very much like a private good.

- In some countries (e.g. some Latin American countries), privately provided elementary and secondary schooling is heavily subsidized, so that tuition covers only a fraction -- sometimes as low as 20%-- of the cost of education. In other countries (e.g the U.S.), private schooling is exempt from corporate income taxes, but tuition payments do not have a special tax treatment.

- In some countries there is substantial spending on federally provided health care (e.g Canada). In others, private expenditures in health are tax subsidized (e.g. the U.S.) and there is a small amount of government provided health care, while in some others (several Latin American countries) government provision is low (but not negligible) and private health insurance does not receive special tax benefits.

- In some countries training programs are directly provided by the federal government (e.g. some European countries and the U.S.) and there are varying --both over countries and over time-- tax incentives for private provision of training, ranging from significant subsidies to no favorable tax treatment. In some others (e.g. Latin American countries), training programs are neither provided nor subsidized.

- In some countries child care services are provided as a mix of government and heavily subsidized private goods (e.g. Sweden), in others it is privately provided and --depending on the employer-- tax deductible, while yet in others it is treated as a regular expense.

! Parental time allocated to child care is heavily subsidized in some Northern European countries; it receives an intermediate subsidy in many Latin American countries, and does not enjoy special tax benefits in others (e.g. the U.S.).

What does economic theory have to say about these different approaches in providing incentives for human capital accumulation? This is particularly relevant because many proposals of tax reform involve substantial changes in incentives that are relevant for human capital accumulation. For example, in Sweden there has been some discussion about subsidizing stay-at-home parents. In the U.S., there are proposals to increase the subsidy to privately purchased child care, and on the form of financing education which, effectively, can change the mix of public-private components. Standard models of dynamic, optimal taxation (e.g., Chamley (1986) and Judd (1985)) do not provide a clear answer to this question. The reason for this is fundamental to understanding the unique character of human capital and its implications for labor supply decisions. The Chamley-Judd results draw a clear distinction between the taxation of capital income-- income derived from a stock-- and that of labor income-- income derived from a flow- in the neo-classical treatment. Even though there are many formulations of the precise nature of human capital, there is agreement that *effective labor* --in the sense of the amount of labor input that enters the production function-- is jointly produced (at the individual level) using “human capital” and “raw time.” Thus, this stock vs. flow dichotomy from Chamley and Judd is not as useful as it might first seem. Moreover, the measurement of human capital poses difficult problems. In a narrow sense, it can be interpreted as the sum of investments in schooling, on the job training and, partially, in health (see Kendrick (1976) and Eisner (1989)). In a broad sense, it can be thought of as including general notions of “ideas” and “social knowledge” potentially quite distinct from measurable investments in education (see the discussion in Romer (1993) and Lucas (1993)).

Both the nature of the questions raised above and the concerns about the unique character of human capital also give some insights as to the features of any model likely to be useful for answering the questions raised above. First, any model of human capital formation must be dynamic in nature- there is an explicit investment character to the decision. Second, there must be a clear connection between what is considered measured labor input (i.e., "effective" labor) and the true inputs, human capital and worker time. Third, it must allow for the inclusion of both productive government spending and productive private spending. Finally, the set of instruments allowed for policy makers must be sufficiently rich on both the taxation and spending side.

Some work has been done to try and remedy this problem by explicitly treating optimal taxation in dynamic models with human capital (see Jones, Manuelli and Rossi (1993) and (1997), and Judd (1999)). In this paper, we improve on this literature by adding three new important features. The first is to allow for public spending which is both productive and chosen by the Ramsey planner. Second, we allow for a broader set of instruments for policy makers than has been considered before. Specifically, we allow for the possibility of either taxing or subsidizing the direct time used in the formation of human capital. Finally, since it is not clear whether private and public spending are substitutes or complements, we study alternative technological assumptions about the process for forming human capital.

There are two main findings. First, we find that, in all of the models that we study, optimal taxation calls for a subsidy (tax) for the time used in the formation of human capital if this input is a substitute (complement) of government goods used in the production of human capital. In the substitutes case, this is similar to Sweden's direct payment to parents described above. Second, we find that, in contrast to the Cobb-Douglas case usually studied in the literature, the fact that distortionary taxes must be used to finance all expenditures imply that there are deviations from first best proportions in the provision of private and public inputs in human capital. Moreover, the direction of these deviations depend critically on whether public and private inputs are substitutes or complements. (Very little seems to be known about this.) This last point contrasts with the intuition one derives from the work of Diamond and Mirlees (1971), where it is shown that an optimal tax system must be such that the allocation of intermediate goods --and government and private investments in human capital clearly fit this description-- should be efficient. This has important implications for decentralized program evaluation. In particular, we show that optimal allocations (in the second best sense of all indirect taxation arguments) do not imply that the productivity of public and private education must be equal and, on the contrary, it is possible that the private sector is subsidized at the same time that its productivity is lower than that of the public sector.

In section 2 we present the basic model. Section 3 contains the basic result on optimal spending and taxation, while in section 4 discusses example. Section 5 contains a preliminary discussion of the results. Finally, concluding comments are presented in section 6.

## 2. A Simple Economy with Human Capital

In this section we first describe the basic model of household and firm behavior. We consider a representative household who is infinitely lived, and has preferences given by,

(2.1) $\sum_{t=0}^{\infty} \beta^t u(c_t,\ell_t)$.

The objective if this household is to maximize utility subject to the present value budget constraint

(2.2) $\sum_{t=0}^{\infty} p_t[c_t+x_{kt}+(1-\tau_t^x)x_{ht}] = \sum_{t=0}^{\infty} p_t[(1-\tau_t^h)w_t n_{zt} h_t+\tau_t^e w_t n_{et} h_t+(1-\tau_t^k)r_t k_t]+p_0[1+(1-\tau_0^k)R_0]b_0$,

and capital accumulation constraints given by, (2.3) $k_{t+1} \leq (1-\delta_k)k_t + x_{kt}$, $t=0,1,..$

(2.4) $h_{t+1} \leq (1-\delta_h)h_t + G(x_{ht},g_t,n_{et}h_t)$, $t=0,1,..$

In this formulation $c_t$, $x_{kt}$, and $k_t$ stand for consumption, investment in physical capital, and the stock of capital at time t. Effective labor allocated to market activities --whose income can be taxed at the rate $\tau_t^h$-- is just the product of the stock of human capital, $h_t$, and raw hours, $n_{zt}$. New human capital is produced using privately purchased inputs, $x_{ht}$, government provided inputs, $g_t$, and household's effective time, $n_{et}h_t$. We use the convention that $n_{zt}+n_{et}=n_t$, and that $n_t+\ell_t=1$. We assume that G displays constant returns to scale in all three inputs. We normalize the price of consumption at t (and new capital) at one, and we let $w_t$ and $r_t$ denote the rental prices of labor and capital in terms of contemporaneous consumption. Here, $p_t$ is the price at time zero of a unit of consumption to be delivered at time t. Without loss of generality we set $p_0=1$ from now on. We take $k_0$ and $h_0$ as given.

There are several features of the specification that deserve some discussion. First, is the form of the effective labor supply function. We assume, following the early work of Heckman (1976) and Rosen (1976) and the more recent application to dynamic models by Lucas (1988), that effective labor (or effective human capital) is just the product of the stock of human capital, h, and its rate of utilization in activity i, $n_i$. The key feature in this formulation is that effective labor is a linear function of reproducible inputs (in this case just h). Under a general version of this assumption, Jones, Manuelli and Rossi (1997) show that, in the long run, the optimal tax rate on effective labor income is zero. Thus, effective labor is treated as just another capital stock. If, instead, effective labor is an arbitrary function of $(n_i,h)$, for example some homogeneous of degree one function, Judd (1999) showed that, in the long run, the optimal tax rate on effective labor is strictly positive. Moreover, the positive tax rate on effective labor is matched with an identical subsidy to educational inputs --private goods in Judd's formulation. Judd's results indicate the importance of functional form assumptions for the characterization of optimal tax codes, and suggest that empirical work is necessary to determine the "true" form of the effective labor function. In this paper, we use the simple formulation which maximizes the chances of zero taxation of effective labor, since our interest is to study whether *other* factors could account for a non-zero tax rate in the long run.

The second critical specification is the form of the function G. In this paper we assume that new human capital is produced with privately purchased market goods, $x_h$, publicly provided market goods, g, and household time, $n_e h$. We also assume that the government cannot sell g to each household. Even though we view g as private good for convenience, it is easy enough to extend the model to make it public, which would fully justify its allocation by the government. The reason for not going this route is that public goods call for Pigouvian taxes, and, although relevant, are well understood and different from the effects we are trying to capture.

Fundamentally, the justification for this more general formulation of the human capital accumulation process is that there is a fair amount of uncertainty about what human capital is and, hence, that some flexibility in its specification is desirable. For example, it is probably uncontroversial that education is one, but not the only, component of what we call human capital. In this case one could interpret g as public schooling (probably elementary and secondary), $x_h$ as private schooling ( also elementary and secondary, schooling related inputs; e.g. computers and books, and college education), and $n_e h$ as capturing student and parent time (for elementary and middle school levels parent time is probably more valuable than student time). Another interpretation is that G captures a combination of government provided and privately provided schooling, and $n_e h$ is just trainee's time. Finally, one can interpret some health related spending as belonging in G. This is particularly relevant in relative poor countries in which health related problems affect both the effectiveness of schooling and the fraction of the time that an individual can work. In most countries, health care services are a mix of government provided, and privately provided goods. In this setting, $n_e h$ corresponds to the amount of time allocated to health maintenance and improvement (e.g. exercise and fishing time). There are other possible interpretations, and section 5 includes a more extended discussion. In all cases, it is not obvious the kind of restrictions that should be imposed on G and, in particular, it is not clear that exclusion restrictions are justified.

We assume that G is homogeneous of degree one in all three inputs. The reason for this is simple: if G displayed strictly decreasing returns to scale in equilibrium there would be rents that the government will like to tax and this, in general, leads to non-zero taxes on capital stocks.

Since in the analysis of optimal indirect tax systems the results depend heavily on the nature of the tax code, it is useful to discuss the set of available taxes. As it is standard in the literature on factor income taxation, we allow for potentially separate taxes on capital and effective labor income. Moreover, we assume that the government cannot separately observe (and hence tax) the stock of human capital, h, and the number of hours worked, $n_j$, j=z,e. Of course, in this model, this amounts to the assumption that hours cannot be observed. In addition to the standard factor income taxes, we allow for the possibility of the government subsidizing (or taxing if the rate is negative) purchases of human capital inputs (this is what $\tau_t^x$ stands for) and the use of a fraction of the household's human capital (specifically, fraction $n_e$) in the production of new human capital. The subsidy (tax if negative) rate for the latter is $\tau_t^e$. Note that this is a controversial assumption as we are assuming that the planner can estimate the market value of the time that goes into human capital formation. Besides the increased flexibility, the set of taxes that we consider will allow the Ramsey planner to support an allocation in which intermediate inputs are allocated in such a way that their marginal rates of substitution agree with the best rules. Finally, our set of taxes allows for a subsidy to private education as proposed by Judd.

As is standard in aggregate models, the supply side of the model is characterized by a large number of firms with constant returns to scale production functions that rent both capital and effective labor in spot markets. The representative firm solves the following static maximization problem,

(2.5) $\pi_t = \max\{F(k_t,n_{zt}h_t,a_t) - w_t n_{zt}h_t - r_t k_t - v_t a_t\}$,

where $k_t$ is the amount of capital rented by the firm at t, $n_{zt}h_t$ is the amount of effective labor hired at t, and $a_t$ is the amount of land --or any other input in fixed supply-- used at t. The vector $(w_t,r_t,v_t)$ denote rental prices --in terms of consumption at time t-- of the productive inputs. There are two features of this formulation worth discussing. First, since F is assumed to be homogeneous of degree one, the equilibrium level of $\pi_t$ is zero. This justifies its omission from the income side of the household's budget constraint. Second, the assumption that there is a factor in fixed supply is simply a convenient way of introducing decreasing returns to scale to reproducible factors in the aggregate. In the absence of a fixed factor, this economy would display --under some conditions-- long run growth. This assumption in and of itself does not affect the results, but makes the comparability with the existing literature less transparent. In order not to artificially induce a “third best” kind of distortion we will assume that the government taxes away all the rents from land. This assumption is sufficient to obtain the zero taxation of capital income in the steady state (see Jones, Manuelli and Rossi (1997) for a discussion of the role of untaxed pure profits), and also implies that the after tax value of the rental income on land is zero, which is the reason why it was omitted from the right hand side of (2.2).

We assume that the aggregate feasibility constraint is given by,

(2.6) $c_t + x_{kt} + x_{ht} + g_t \leq F(k_t,n_{zt}h_t,a_t)$.

The only important assumption is that private ($x_h$) and public (g) inputs in the production of new human capital are assumed to be produced using the same technology used to produce “general output.” Finally, the government's budget constraint in its present value form is,

(2.7) $\sum_{t=0}^{\infty} p_t g_t + [1+(1-\tau_0^k)R_0]b_0 = \sum_{t=0}^{\infty} p_t[- \tau_t^x x_{ht} + \tau_t^h w_t h_t n_{zt} - \tau_t^e w_t n_{et} h_t + \tau_t^k r_t k_t + v_t a_t]$.

The notion of equilibrium that we use is a standard competitive equilibrium given prices, tax rates and the sequence of government spending. In order to determine the optimal sequences of government spending and the tax rates we will assume that the government solves a standard Ramsey problem.

## 3. Ramsey Problem for the Simple Economy

In order to analyze the Ramsey problem it is convenient to describe optimal choices by the private sector --both households and firms-- in terms of their first order conditions. It follows that the first order conditions for the household's problem are the constraints (2.2), (2.3) and (2.4) at equality and,

(3.1) $\beta^t u_c(t) = u_c(0)p_t$,

(3.2) $u_\ell(t) = u_c(t)\,(1-\tau_t^h)w_t h_t$,

(3.3) $u_\ell(t) = [\mu_t G_n(t) + u_c(t)\,\tau_t^e w_t]h_t$

(3.4) $u_c(t)(1-\tau_t^x) = \mu_t G_x(t)$,

(3.5) $p_t = p_{t+1}[1-\delta_k + (1-\tau_{t+1}^k)r_{t+1}]$,

(3.6) $\mu_t = \beta[\mu_{t+1}(1-\delta_h+G_n(t+1)n_{et+1}) + u_c(t+1)((1-\tau_{t+1}^h)w_{t+1}n_{zt+1}+ \tau_{t+1}^e w_{t+1}n_{et+1})]$,

where $\mu_t$ is the (discounted) Lagrange multiplier corresponding to the human capital accumulation constraint (2.3). The interpretation of these conditions is straightforward: (3.1) simply makes the marginal utility of consumption at t equal to the price of consumption at t; (3.2) is the standard equality between marginal rate of substitution between consumption and leisure and the after tax wage rate (which is $w_t h$ for an individual with h units of human capital); (3.3) equates the marginal utility of leisure to the returns of allocating one unit of time to augmenting the stock of human capital; (3.4) is the private efficiency condition for the purchase of inputs allocated to the production of human capital; (3.5) is the no arbitrage condition corresponding to physical capital; and (3.6) is the no arbitrage condition for human capital. Note that since $\mu_t$ is the shadow price of human capital in utility terms, (3.6) says that the shadow value of human capital today must equal to the discounted shadow value of human capital left over after depreciation plus the consumption value of the income generated by an additional unit of human capital. Thus, (3.6) is an arbitrage condition for human capital.

Using the first order conditions from the consumer's problem it is possible to simplify the budget constraint (2.2) substantially. First, using (2.3) and (3.5) it follows that,

(3.7) $\sum_{t=0}^{\infty} p_t[x_{kt} - (1-\tau_t^k)r_t k_t] = -[1-\delta_k+(1-\tau_0^k)r_0]k_0.$

Next, using the homogeneity of degree one of the function $G(x,g,n_e h)$, write $x_{ht}$ as,

$$x_{ht} = [h_{t+1} - (1-\delta_h)h_t - G_n(t)n_{et}h_t - G_g(t)g_t]/G_x(t),$$

and using this condition and (3.6) it can be shown that,

(3.8) $\sum_{t=0}^{\infty} p_t[(1-\tau_t^x)x_{ht} -(1-\tau_t^h)w_t h_t n_{zt} - \tau_t^e w_t n_{et} h_t] = -\{\sum_{t=0}^{\infty} p_t(1-\tau_t^x)G_g(t)g_t/G_x(t) + [(1-\delta_h)/G_x(0) + (1-\tau_t^h)w_0 n_0]h_0\}.$

Thus, the representative household's budget constraint can be written as,

(3.9) $\sum_{t=0}^{\infty} p_t[c_t - (1-\tau_t^x)G_g(t)g_t/G_x(t)] = [1-\delta_k+(1-\tau_0^k)r_0]k_0+[(1-\delta_h)/G_x(0) + (1-\tau_0^h)w_0 n_{z0}]h_0.$

The right hand side of this equation is the after tax value of wealth at t=0. The left hand side is the value of consumption minus the after tax value of the "profits" created by the presence of government provided goods, in terms of current consumption, $(1-\tau_t^x)G_g(t)g_t/G_x(t)$. Note that it is possible for the government to tax away these profits only by subsidizing private inputs in the production of human capital at the rate of 100% --effectively a full tax credit. However, in this case, the private sector would choose $x_h$ so that its marginal product is zero (because in this case it is a free good). It is clear that, in equilibrium, the subsidy rate will fall short of 100% and, as we will show later, it is the presence of untaxable profits that creates an incentive for the government to tax or subsidize human capital.

From the representative firm's problem, it follows that,

(3.10) $F_k(k_t,n_{zt}h_t,a) = r_t,$

(3.11) $F_z(k_t,n_{zt}h_t,a) = w_t,$

(3.12) $F_a(k_t,n_{zt}h_t,a) = v_t$,

where we have already imposed that the equilibrium value of $a_t$ equals a, its exogenous level per firm.

Following Lucas and Stokey (1983), it is convenient to model the government in the Ramsey problem as choosing an allocation. However, since the allocation must be supportable as a competitive equilibrium, it is necessary to describe the class of restrictions that equilibrium considerations impose on candidate allocations.

It can be verified that not all allocations can be made to satisfy the first order conditions for the consumer problem. Define an *extended* allocation as one in which the government is choosing a collection of sequences $[\{c_t\},\{x_{kt}\},\{x_{ht}\},\{h_{t+1}\},\{k_{t+1}\},\{n_{zt}\},\{n_{et}\},\{g_t\},\{\mu_t\}]$, where the last term is just the consumer's marginal valuation of human capital in terms of utility. Even if the government picks an extended allocation, it is not automatically guaranteed that it will satisfy the consumer's (and firms') first order conditions (and, hence, that it is supportable as an equilibrium). To see this, suppose that the government has picked a feasible extended allocation, where feasibility refers to the real allocation (i.e. no restrictions on the $\{\mu_t\}$ sequence other than non-negativity). Given such an extended allocation, (3.10)-(3.12) can be used to determine factor prices; (3.1) can be used to determine the time zero price of consumption at time t, (3.2) pins down the tax rate on labor income the tax rate on labor income, $\tau_t^h$, (3.3) determines $\tau_t^e$, and (3.4) pins down $\tau_t^x$. Finally, (3.5) and (3.1) jointly determine $\tau_t^k$ (we arbitrarily set the initial tax rates equal to zero to prevent lump sum taxation). The problem is that nothing guarantees that the extended allocation will satisfy (3.6) and the budget constraint (3.9). Since these are necessary (and given our concavity assumptions sufficient as well) conditions for an allocation to be a competitive equilibrium, the need to be imposed as additional constraints on the planner's problem. Using (3.1) and (3.2) in (3.6) it is possible to eliminate all taxes form this Euler equation, and to write it as,

$$\mu_t = \beta[\mu_{t+1}(1-\delta_h) + u_\ell(t+1)n_{t+1}/h_{t+1}],$$

where $n_t = n_{zt} + n_{et}$. Thus, the previous equation says that the value of an additional unit of human capital today must equal to the discounted value of the undepreciated portion next period plus the full return in both the market and the production of additional human capital.

Given this discussion, the Ramsey problem in this economy is,

**(RP)** $\max \sum_{t=0}^{\infty} \beta^t u(c_t,1-n_t),$

subject to,

$$\sum_{t=0}^{\infty} \beta^t[u_c(t)c_t - \mu_t G_g(t)g_t]=u_c(0)\{[1-\delta_k+(1-\tau_0^k)F_k(0)]k_0+[(1-\delta_h)/G_x(0) + (1-\tau_t^h)F_z(0)n_{z0}]h_0\},$$

$$F(k_t,n_{zt}h_t,a_t) \geq c_t + x_{kt} + x_{ht} + g_t,$$

$$(1-\delta_h)h_t + G(x_{ht},g_t,n_{et}h_t) \geq h_{t+1},$$

$$(1-\delta_k)k_t + x_{kt} \geq k_{t+1},$$

$$\mu_t - \beta[\mu_{t+1}(1-\delta_h) + u_\ell(t+1)n_{t+1}/h_{t+1}]=0,$$

where the maximization is over an extended allocation, in the sense defined above.

### *3.1 Optimal Steady State Taxes*

As is standard in the literature, we will concentrate on the long run properties of optimal tax and spending codes. Since, as we will show later, the qualitative features of the optimal tax code will depend on the degree of substitutability between the inputs in the production of new human capital, it is convenient to introduce notation for the (Allen-Uzawa) partial elasticity of complementarity. Let $\kappa_{ij} \equiv G_{ij}G/(G_iG_j)$ be the partial elasticity of complementarity. It can be shown that (see Sato and Koizumi (1973)) that for any input y, sign$(\partial\nu(y)/\partial g) \gtreqless 0$ if and only if $\kappa_{yg} \gtreqless 1$, where $\nu(y)$ is input y's share defined as $\nu(y) \equiv G_y y/G$, $y=x_h,g,n_eh$. It is always difficult to interpret partial measures of substitution (or complementarity) but we will interpret a $\kappa_{ij}>1$ as evidence of complementarity, and a value of $\kappa_{ij}<1$ as indicating substitutability. This interpretation is precise when the G function is of the CES variety. It is also reasonable in other, more general, settings discussed in section 4.[2] The basic result of this section is,

**Proposition 3.1**. Assume that the solution to the Ramsey problem is interior and converges to a steady state. Then,

i) The steady state tax rate on capital income, $\tau^k_\infty$, is zero.

ii) The steady state subsidy on household time, $\tau^e_\infty$, is positive (negative) if household time and government provided goods are partial substitutes; that is, $\tau^e_\infty \gtreqless 0 \Leftrightarrow \kappa_{ng} \lesseqgtr 1$.

iii) The tax rate on effective labor, $\tau^h_\infty$, exceeds, equals or falls short of the subsidy rate on private purchases of goods to produce human capital, $\tau^x_\infty$, depending on whether the partial elasticity of complementarity between government goods, g, and private goods, $x_h$, $\kappa_{xg}$, is greater, equal or less than one. More formally,

$$\tau^h_\infty \gtreqless \tau^x_\infty \Leftrightarrow \kappa_{xg} \gtreqless 1.$$

**Proof**: See appendix.

First, as it is argued in the appendix (see also Jones, Manuelli and Rossi (1997) for an extended discussion on this), it is possible to view the Ramsey problem as a modified optimal growth problem, with a pseudo utility function given by $v(c,n_z,n_e,h,g,x_h,\mu;\Phi) \equiv u(c,1-n_z-n_e) + \Phi[u_c(\bullet)c_t - \mu G_g(\bullet)g]$, and capital stocks given by $(k,h,\mu)$. (Here $\Phi$ is the Lagrange multiplier corresponding to the budget constraint and it is a measure of the marginal welfare cost of distortionary taxation.) Direct inspection of this problem reveals that k, physical capital, enters in a way that is similar to its role in a standard Cass-Koopmans problem: it appears in the feasibility constraint and in its law of motion. Since it is well known that the steady state efficiency condition in a Cass-Koopmans problem does not depend on the form of the utility function, it is not surprising that the Ramsey problem (a "modified" Cass-Koopmans problem) shares the same

[2] For a discussion of the properties of the partial elasticity of complementarity see Blackorby and Russell (1989)

property. Thus, in the Ramsey problem the long run efficiency condition that the marginal product of capital equal the sum of the discount rate and the depreciation factor, also holds. This condition implies that capital cannot be taxed, since taxation induces a deviation from this first best.

Thus, as far as capital is concerned, the model predicts that the dynamic efficiency condition will be satisfied at the steady state.

The second interesting feature of the optimal long run tax code is the tax treatment of non-market labor input. If effective labor and government goods are partial substitutes ($\kappa_{ng} < 1$), the optimal code subsidizes the allocation of raw time to the formation of human capital relative to working in the market. To see this, use (3.2)-(3.4) to obtain,

(3.13) $u_\ell/u_c = (1-\tau^h)wh = [(1-\tau^x)(G_n/G_x) + \tau^e w]h.$

(3.14) $(1-\tau^x)/G_x = \beta\{((1-\tau^x)/G_x)[1-\delta_h+G_n n_e] + w[\tau^e n_e+(1-\tau^h)n_z]\}.$

The first condition simply equates the market value of the marginal hour devoted to work, $(1-\tau^h)wh$, with the value of the marginal hour allocated to human capital accumulation, $[(1-\tau^x)(G_n/G_x) + \tau^e w]h$. The latter is given by the value of the subsidy plus the consumption value of an extra unit of human capital available tomorrow, $(1-\tau^x)/G_x$, multiplied by the marginal product of time in the production of human capital, $G_n$. Thus, a subsidy on private purchases of human capital producing inputs, $\tau^x$, acts like a tax on the shadow value of time allocated to the production of human capital, while the direct subsidy to student time, $\tau^e$, acts like a tax on the market value of time. More precisely, since (3.13) can be written as, $(1-\tau^h-\tau^e)w = (1-\tau^x)(G_n/G_x)$, it is clear that, for the static decision on allocating labor between the two sectors, it is the sum of two taxes, $\tau^h+\tau^e$, that plays the role of the "effective" tax rate on labor, while the subsidy to private purchases of education, $\tau^x$, acts as a tax on time allocated to the formation of human capital.

Why is it that, in the substitutes case, the Ramsey planner wants to encourage the formation of human capital? There are two places where effective labor in human capital formation appears: the pseudo utility function, and the implementability constraint for the shadow price of h. The effect of h on these two concepts is, potentially, quite different. First, note that a higher level of h reduces, in the steady state, the shadow value of capital, $\mu$ which is given by $\mu[1-\beta(1-\delta_h)]=\beta u_\ell n/h$. Also, a higher level of h increases the planner's pseudo-utility if it reduces $\mu G_g(\bullet)g$. Even if $G_{gn}$ is positive, it is necessary to compare the positive impact of relaxing the $\mu$-constraint with the potentially negative impact --from the Ramsey planner's point of view-- upon profits induced by the provision of g. The first effect, the relaxation of the implementability constraint, dominates in the substitutes case. One interpretation of substitutability is that the impact of changes in h on pure profits is small and, hence, this effect is less important. The opposite holds when time and government provided goods are complements ($\kappa_{ng}>1$) and then the use of household time is taxed; finally, there is exact cancellation in the case of unitary partial elasticity of complementarity.

The optimal tax system calls for relatively “large” ($\tau^x_\infty > \tau^h_\infty$) subsidies to privately purchased inputs, $x_h$, when they are substitutes for government provided --and hence tax financed-- inputs, g. Thus, whenever $x_h$ and g are partial substitutes, the Ramsey planner would rather increase investment in human capital through subsidies that encourage private investment in $x_h$ rather than through direct expenditures in g. Conversely, whenever private and public inputs are complements, the optimal policy is to offer a subsidy that falls short of the tax rate on labor income (it could even be a tax) since increases in private investment in $x_h$ have to be matched (for efficiency considerations) with increases in g, and these, in turn, indices additional distortions.

One problem with the measure of complementarity (or substitutability) that we use, the Allen-Uzawa elasticity of substitution, is that its value is, in general, dependent on the optimal (in the sense of solving the Ramsey problem) allocation. In section 4, we present a series of examples in which this elasticity is either constant or particular cases can be discussed without knowledge of the full allocation.

### *3.2 The Allocation of Intermediate Inputs*

Ever since the work of Diamond and Mirlees (1971), the standard intuition in the literature on indirect commodity taxation is that, under some conditions, intermediate goods should not be taxed or, if they are taxed, the tax rates have to be the same. This result manifests itself in the prescription that the marginal conditions to allocate intermediate goods (in interior solutions) in the second best problem coincide with their first best counterparts. Even though there is no presumption that the Diamond and Mirlees result will hold whenever the set of taxes is not a “complete” set of optimal commodity taxes, or when --as in this paper-- there are optimally chosen publicly provided inputs, it has been standard in applied work to proceed “as if” the results are applicable. This is in some cases justified since in environments that do not fit the Diamond-Mirlees assumption, it is possible to prove versions of their result (see Chari and Kehoe (1997) and Judd (1997)).

In this section, we study whether the Ramsey allocation respects this “first best” flavor for intermediate goods in the problem at hand. Why is it that the result might not hold? First, the existence of unpriced publicly provided goods results in the presence of partially taxable rents which, effectively, makes the Ramsey problem a “third best” problem. Second, we consider what can be viewed as a realistic set of taxes, but this set may fall short of the taxes assumed by Diamond-Mirlees since they had access to a rich set of commodity taxes that we rule out.

The major finding is that --in general-- the optimal plan creates distortions in the allocation of intermediate goods, and that the nature of those distortions is closely related to the qualitative features of optimal factor taxes as described in the previous section.

To discuss these departures from first best rules, it is useful to begin by characterizing the first best conditions for the allocation of intermediate goods. From the solution of a Cass-Koopmans (first best) planner's problem for the economy at hand, it can be shown that the steady state is characterized by the following set of conditions (among others),

(3.15) $G_x = G_g$,

(3.16) $G_n/G_x = F_z$.

(3.17) $\beta^{-1} - (1-\delta_h) = G_xF_zn$,

The first condition simply says that since both $x_h$ and g are market goods (produced with the same technology) their marginal products in the production of human capital have to be the same. The second condition is an efficiency condition in the allocation of raw hours across the two activities --working in the market and producing more human capital. The marginal product of an additional hour allocated to market work is (proportional to) $F_z$, measured in market goods. The marginal product of an additional hour allocated to investing in human capital is (proportional to) $G_n$, measured in units of human capital, while the shadow price of new human capital (available for productive use next period) in terms of consumption is just $1/G_x$. Thus, (3.16) is an efficiency condition determining the allocation of labor to two sectors. Finally, (3.17) equals the user cost of any stock at the steady state (given by the interest rate plus the depreciation factor, $\beta^{-1} - (1-\delta_h)$, to the marginal product of an additional unit of h in the production of human capital, $F_zG_x$ or $G_n$, times the full utilization rate n (which equals $n_e+n_z$).

What are the features of the tax system that make these conditions hold? From the household's first order conditions, we get that in a steady state equilibrium,

(3.18) $(1-\tau^h- \tau^e)F_z = (1-\tau^x)(G_n/G_x)$

(3.19) $(1-\tau^x)[\beta^{-1}-(1-\delta_h)] = (1-\tau^h)G_xF_zn$.

Thus, a tax code that equates the tax rate on labor income with the subsidy on purchases of market goods used to produce human capital ($\tau^x = \tau^h$), and that it does not subsidize the use of household labor in the production of new human capital ($\tau^e = 0$), has a chance of replicating the first best (in the narrow sense used above). Proposition 3.1 shows that, in general, $\tau^e \neq 0$ and, thus, that there is no presumption that the first best marginal conditions will be satisfied by the

Ramsey allocation. It is possible to relate the direction of the distortion to the pattern of substitutability and complementarity that underlies our findings for the optimal tax code. The results are summarized in,

**Proposition 3.2**. Assume that the solution to the Ramsey problem is interior and converges to a steady state. Then,

i) The steady state efficiency condition for the physical capital stock, $\beta^{-1} - (1-\delta_k) = F_k$, is satisfied.

ii) The steady state dynamic efficiency condition, equation (3.17), is such that the discounted value of the marginal product of an additional unit of human capital exceeds, equals or falls short of its cost, depending on whether the partial elasticity of complementarity between government goods, g, and private goods, $x_h$, $\kappa_{xg}$, is greater, equal or less than one. More formally,

$$\beta[(1-\delta_h) + G_x F_z n] \gtreqless 1 \Leftrightarrow \kappa_{xg} \gtreqless 1.$$

iii) The marginal product of an additional hour in production exceeds, equals or falls short its value in producing new human capital if market and government provided goods are more (partial) complements than household time and government goods. Formally,

$$G_x F_z \gtreqless G_n \Leftrightarrow \kappa_{xg} \gtreqless \kappa_{ng}.$$

iv) The marginal product of privately purchased market goods in the production of human capital exceeds, equals or falls short of the marginal product of government goods if and only if the share-weighted average of the elasticity of complementarity between $x_h$ and g and $n_e h$ is greater than one. In symbols,

$$G_x \gtreqless G_g \Leftrightarrow \kappa_{xg}\,(\upsilon(x_h)+\upsilon(g)) + \kappa_{ng}\,(\,1-(\upsilon(x_h)+\upsilon(g)) \gtreqless 1 \Leftrightarrow (\kappa_{xg}-\kappa_{gg})\upsilon(g) \gtreqless 1.$$

**Proof**: See appendix.

Thus, the strength of complementarity and substitutability between private and government inputs in the production of human capital determines whether, relative to the first best, public and privately provided non-time inputs will be over or under provided.

Let's consider the dynamic allocation of human capital. In a first best, this dynamic allocation costs one unit of consumption to produce and has a total return of $(1\text{-}\delta_h)$ units left next period, plus the value of human capital, which is given by its marginal value in production, $F_z n_z$ divided by the price of new human capital $1/G_x$, and its value in creating more human capital, $G_n n_e$. At the first best, this value is just $G_x F_z(n_z+n_e)$. Thus, in the Ramsey solution, this discounted value exceeds the cost if private and public inputs are complements, and falls short of it if they are substitutes.

The third part of the proposition shows that, using marginal rates of transformation as relative prices, the marginal product of raw time in the production of goods, $F_z$, is high relative to its value in human capital production, $G_n/G_x$, if private and government goods are "more complementary" than time and government goods ($\kappa_{xg} > \kappa_{ng}$), with the obvious change when the relative complementarity changes. Thus, when $x_h$ and g are better complements than $n_e$h and g, the optimal plan discourages allocating too many market goods resources to the production of new human capital and this is, in part, accomplished via an increase in the number of hours allocated to this activity. Of course, this results in the equilibrium marginal product of labor in the human capital sector to be "low" relative to its marginal product in goods production. It is easy to check that $F_z>G_n/G_x$ corresponds to the case $\tau^h+\tau^e > \tau^x$, and, hence, that the total effective tax on market allocation of labor, exceeds the subsidy to goods investment in human capital.

Finally, the proposition gives some sense of the cases in which the marginal products of $x_h$ and g will not be equal (that is, the static efficiency condition (3.15) will not hold). Even though the details vary --and the formulation is somewhat difficult to interpret-- it seems reasonable to interpret condition iv) in Proposition 3.2 as saying that if $x_h$ and g are complements, in the sense of $\kappa_{xg}$, large then $G_x > G_g$. Thus, complementarity with government provided goods results in an equilibrium in which the marginal product of the private good is higher than the marginal product of a technologically similar publicly provided good because of the additional distortions involved in direct provision.

## 4. Examples

The results in Propositions 3.1 and 3.2 highlight the role of the degree of substitutability between inputs in determining the long run qualitative properties of the optimal tax code. In this section we discuss special cases of the function G that illustrate and give more economic content to those results.

### *4.1 The Cobb-Douglas Case*

In the Cobb-Douglas case the G function is given by,

$$G(x_h,g,n_eh) = A\ (x_h)^{\alpha}(g)^{\gamma}(n_eh)^{(1-\alpha-\gamma)}.$$

For this function it is easy to calculate the partial elasticities of complementarity, and they are all equal to one ($\kappa_{ij}=1$ for all $i,j = x_h,g,n_eh$). This is the case closest to the one studied by Milessi-Ferretti and Roubini (1994) and (1998) (although they consider only $\gamma = 0$) and Corsetti and Roubini (1996) (who allow for $\gamma > 0$), and it implies that privately purchased inputs should be tax deductible ($\tau^h = \tau^x$) and that non-market effective labor allocated to increasing the stock of human capital is neither taxed nor subsidized ($\tau^e = 0$). In this case, all the first order conditions (3.15)-(3.17) hold. Thus, in this case, the second best solution is such that properly measured marginal products of private and government inputs in the production of human capital equal their marginal costs. This implies that cost benefit analysis to determine the appropriate amount of g can be conducted ignoring the fact that --at the margin-- additional levels of government spending must be financed using distortionary taxes.

This case then delivers the analogue of the Diamond and Mirlees result: the allocation of intermediate --both government provided and privately purchased-- goods satisfies the first best equality between marginal rates of transformation across different uses.

*4.2 The CES Case*

The next example is one in which all inputs are equal substitutes. The specific form of G is,

$$G(x_h,g,n_e h) = A\,[\alpha(x_h)^{-\rho} + \gamma(g)^{-\rho} + (1-\alpha-\gamma)(n_e h)^{-\rho}]^{-1/\rho}.$$

In this case the elasticity of complementarity for all pairs of inputs is $1+\rho$, the inverse of the elasticity of substitution. For simplicity, our comments consider the substitutes case ($\rho < 0$), with all inequalities reversed in the case of complements. In this case, it follows that $\tau^x > \tau^h$, and private purchases of market goods are subsidized, relative to the tax rate of income. Moreover, it can be checked that $\tau^e = \tau^x - \tau^h > 0$. This condition implies (see (3.18)) that the tax code does not distort the static allocation of labor between the two sectors, production of output and production of human capital. and the optimal tax code requires more than a tax deduction --i.e. a partial tax credit-- for private inputs.

The optimality in the static allocation of time does not extend to other intermediate goods. The results of Proposition 3.2 can be used to verify that the measured return to market goods in the production of human capital is "too low" ($\beta[(1-\delta_h) + G_x F_z n] < 1$), and the static condition for the allocation of $x_h$ and g indicates an "inefficiently small" level of public spending, in the sense that the marginal product of an additional unit of g exceeds that of an additional unit of $x_h$ ($G_g > G_x$).

In the CES case $\kappa_{ij} = 1+\rho$ for all pairs of inputs and, hence, part iv) of Proposition 3.2 implies that the optimal fiscal policy (in the case of substitute inputs) is such that the marginal $G_x < G_g$. Thus, this case is consistent with the paradoxical result that government goods are "underprovided," at the same time that competing privately purchased inputs are heavily subsidized.

### *4.3 A Mixture of CES Case*

In this section we consider and intermediate case in which the function G is given by a mixture of a CES production functions. The advantage of this formulation is that it does not constrain the elasticity of substitution among pairs of inputs to be the same (or even constant). It turns out that this case can be summarized by three subcases where, in each formulation, we allow for constant elasticity of substitution between a pair of inputs, and between some aggregate of those and the third input. In general, using the notation $(y_1,y_2,y_3)$ to denote our three candidate inputs we can describe the technology by,

$$G(M,y_3) = [\gamma M^{-\varphi} + (1-\gamma)y_3^{-\varphi}]^{-1/\varphi},$$

$$M(y_1,y_2) = [\alpha y_1^{-\rho} + (1-\alpha)y_2^{-\rho}]^{-1/\rho}.$$

Following the notation in the previous section we will use $\kappa_{ij}$ to denote the partial elasticity of complementarity between inputs i and j in the function G, $\upsilon(i)$ to denote the share if input i in G, and $\kappa_{M,ij}$ and $\upsilon_M(i)$ to designate the equivalent concepts for the M function. As it turns out, some results do not depend on the CES specification for M. Those cases will be indicated in the analysis.

The three different specifications, depending on the identity of the $y_i$ variables, are summarized in the following table,

| Specification | Variables | | |
|---|---|---|---|
| | $y_1$ | $y_2$ | $y_3$ |
| I | $x_h$ | $n_e h$ | g |
| II | g | $n_e h$ | $x_h$ |
| III | $x_h$ | g | $n_e h$ |

It is convenient to consider each specification separately. For specification I, none of the results hinge on the specific functional form of M. All that is required is that M is a concave homogeneous of degree one function. This specification assumes that private inputs (both goods and time) are combined with each other to produce some intermediate good, and this intermediate good is combined with government inputs to produce human capital. In this case it follows that $\kappa_{xg} = \kappa_{ng} = (1+\varphi)$ . Thus, if the private intermediate good M and g are substitutes, ($\varphi<0$), the optimal policy calls for an income tax credit on expenses related to the production of human capital at a rate exceeding that of the income tax rate, and a subsidy to household non-market time allocated to the production of human capital. As in the standard CES, the rate of return on market goods used in the production of human capital is low ($\beta[(1-\delta_h) + G_x F_z n] < 1$), and the static condition for the allocation of $x_h$ and g is also consistent with a view that there is excess spending on private goods ($G_g > G_x$). One interpretation is that, in the substitutes case, it is "cheaper" to induce the private sector --through subsidies-- to invest in additional human capital then to provide additional inputs directly.

The second specification, II, can be interpreted as formalizing the idea that household time and government inputs are used to produce some intermediate input and this input, in turn, is combined with privately purchased market inputs to produce new human capital. In this case, and without any assumptions about the functional form of M, $\kappa_{xg}$ also equals $(1+\varphi)$. Thus, the results that depend on these two quantities (relative taxes and subsidies and the rate of return on $x_h$) coincide. When it comes to determining the existence of a subsidy (or a tax) to household non-market time, it is possible to show that $\tau^e_\infty \gtreqless 0 \Leftrightarrow \rho \lesseqgtr \varphi(1-\upsilon(M))$. Thus, if household time and government inputs are more substitutes, in the sense of the above condition, than the intermediate input M and $x_h$, non-market labor is subsidized. It is clear that it is possible for the optimal policy to consist of a subsidy to private purchases of education, and a tax on student (or parent time). This would occur when $\varphi < 0 < \rho$.

In terms of the marginal rates of transformation, the total return on $x_h$ is low (high) when $x_h$ and M are substitutes (complements). It can be checked that the static allocation of time is skewed toward human capital formation (i.e. $F_z > G_n/G_x$) if $\varphi > \rho$. This condition roughly says that there is more substitutability between $n_e h$ and g than between M and $x_h$. As with other results, this can be interpreted as saying that good substitutes are "encouraged" by the Ramsey planner through the provision of tax incentives. Finally, it can it can be shown that $G_x \gtreqless G_g \Leftrightarrow (1+\varphi - \kappa_{M,gg})\upsilon_M(g) \gtreqless 1$. This condition is, in general, difficult to interpret. In the special case in which M is CES, it reduces to $G_x \gtreqless G_g \Leftrightarrow \varphi\upsilon_M(g) + \rho(1-\upsilon_M(g)) \gtreqless 0$. Thus, if both functions display complementarity between privately purchased market inputs and an aggregate of government provided inputs and household time, M, the equilibrium allocation displays higher marginal product of $x_h$ (relative to g), while in the case in which both are substitutes, $G_x$ will be lower than $G_g$. It is possible for the rate of return on $x_h$ to be high, which could be interpreted to mean that the absolute level of $x_h$ is low (this happens whenever $\varphi > 0$), and, at the same time, the marginal product of $x_h$ is low relative to that of g (this is likely to happen if $\rho$ is sufficiently negative), which could be interpreted as indicating that the composition of the total amount of market goods in the production of h is skewed toward privately provided goods.

The third specification looks at a case that can be interpreted as a situation in which market inputs --both privately purchased and government provided-- are used to produce an intermediate good which, when combined with household time, produces new human capital. In this case, household non-market labor is subsidized (taxed) if it is a substitute (complement) for M, independently of the specific functional form of M.

In the case of a CES aggregator for M, it follows that $\tau_\infty^h \gtreqless \tau_\infty^x \Leftrightarrow \rho \gtreqless \varphi(1\text{-}\upsilon(M))$. Thus, more complementarity between the two types of goods (private and public) than between their (intermediate) input and $n_e h$ results is small subsidies for private purchases of human capital augmenting inputs. A version of this condition ($\rho > \varphi$) implies that static allocation of labor favors the human capital sector ($F_z > G_n/G_x$). Finally, the static distortion in the allocation of $x_h$ and g, depedns just on the elasticity of substitution of the M aggregator; with the marginal product of private goods exceeding (falling short) that of public goods if they are complements (substitutes).

Overall, our examples suggest that productive efficiency for intermediate inputs --in the sense that their marginal rates of transformation across sectors and time must agree with those of the first best allocation-- is the exception. Even though the results are difficult to summarize it is possible to identify a general pattern: if privately purchased market goods, $x_h$, and publicly provided goods, g, are substitutes (complements) then the optimal fiscal policy is such that, in the long run, the marginal product of privately purchased goods in the production of human capital, $G_x$, falls short (exceeds) that of publicly provided goods. This can be interpreted as the outcome of a policy that "relatively encourages" investment in $x_h$. One reason for this is that substitutability (complementarity) between $x_h$ and g pushes the government in the direction of decreases (increases) in g whenever $x_h$ is increased. Given the cost of distortionary taxes, this has first order negative effect on welfare. In addition, the tax treatment of household non-market time depends on its substitutability with publicly provided goods: if these two concepts are substitutes then non-market time is subsidized, while if they are complements it is taxed.

Our examples also indicate that popular functional forms, like Cobb-Douglas, that have proven very useful in many applications, are somewhat limiting for this taxation and spending exercises. The reason is simple: a key determinant of the qualitative features of optimal tax and spending policies is the degree of substitution in production, and the Cobb-Douglas functions fixes this at one.

## 5. Discussion

One of our main, and to some surprising, findings is that depending on properties of the technology to produce human capital, the optimal tax and spending regime can exhibit a complex array of taxes and subsidies and, in general, implies that the Ramsey allocation will not satisfy simple cost benefit calculations. What are the implications for the structure of taxation in more concrete terms? What goods or services should be taxed or subsidized?

Unfortunately, the basic problem is that, as indicated in the introduction, the notion of human capital is both natural, and very difficult to pin down with any precision. In this section we offer several interpretations of the concept and we discuss the implications for tax codes.

One potential problem in trying to implement the tax code derived in Proposition 3.1 is that it assumes that the value --at market wages-- of the time allocated by the household to the formation of human capital is observed. In some instances, e.g. high school students, it is possible to estimate this concept with some precision. However, when parental time is involved, it is more difficult to see how the taxing authority can determine this foregone income. In section 6 we explore the implications of setting the subsidy rate equal to zero.

We can now discuss alternative notions of human capital, and the implications of our model in those cases. Our analysis is not meant to be exhaustive. Consider first the case in which human capital is interpreted as being mostly determined early in life. In this scenario, the household time input is interpreted as parents' time allocated to child rearing and $x_h$ as privately purchased market goods also "used" in child rearing activities, e.g. day care and health care services. The government input g could be interpreted as community programs or publicly provided health services. In this case, the long run optimal tax code calls for the subsidization of the "stay home" parent at a rate proportional to his/her opportunity cost, wh if parent time is a substitute for government services. Thus, parents with higher potential earnings receive a larger subsidy. If $x_h$ and g are partial substitutes --as they seem to be in some applications-- then the subsidy rate to privately purchased market goods exceeds, at the long run optimum, the tax rate on labor income. Thus, in practical terms, this can be implemented with a tax credit for purchases of day care or health care, with the tax credit rate exceeding the income tax rate.

If the "horizon" is extended beyond early childhood to include elementary schooling, the main implication is not that student time should be subsidized or taxed, since an elementary student's h is likely to be close to zero, but that parents' time and private schooling --if substitutes for public schooling-- should be tax deductible at rates exceeding 100%. At least qualitatively, this prescription seems to resemble some features of the proposals for tax reform recently debated in Sweden (subsidization of stay at home parent) and some Latin American countries that heavily subsidize private education. Moreover, proposals to subsidize (or, equivalently, give tax credits) for day care services, which imply a tax credit at a rate that exceeds the income tax rate, can be rationalized if these privately provided day care services are viewed as substitutes for government provided inputs.[3]

Another possible interpretation of human capital is that it is produced with student/worker time, public education, g, and privately provided training, $x_h$. Again, the implication is that student (in this case high school and college) time should be taxed if, at is often argued, is a complement to public education. Moreover, the degree of subsidization of private training, if any, again

[3] Subsidization of day care services is common in many Northern European countries, and that have recently been considered in the U.S. (see Murray (1997)).

depends on the degree of substitutability with public education: if they are complements it receives a “small” subsidy, and if they are substitutes a “large” one.

In our analysis, we have restricted ourselves to the use of market goods, as well as household time, to increase the stock of human capital. In a series of papers, Milesi-Ferretti and Roubini (1994) and (1998), have explored the implications of alternative specifications of leisure (raw time, quality time and home production) as well as different Cobb-Douglas specifications for F and for a version of G that includes capital and effective labor as inputs. They consider the case in which growth is endogenous, and they study a large number of cases. The majority of their results confirm the Jones, Manuelli and Rossi (1997) finding that all taxes --including, of course--taxes on effective labor-- are zero in the long run.

The work that is closest to ours is contained in a recent paper by Corsetti and Roubini (1996). They introduce endogenous government spending that can affect the accumulation of human capital. Their setting is different from the one described in this paper in the sense that they introduce externalities --and hence there is an a priori case for Pigouvian taxation--, they allow factor income from different sectors to be taxed differently and do not consider the possibility of directly subsidizing the purchase of goods that are used in the production of human capital. Their main focus seems to be to determine the conditions under which the Ramsey allocation is unconstrained first best or not. Corsetti and Roubini restrict themselves to Cobb-Douglas production functions and, hence, they do not address the effects of the elasticity of substitution between public and private goods on optimal spending and taxation. Not surprisingly, they find that intermediate goods are allocated according to first best rules.

## 6. Concluding Comments

In this paper we investigated the long run properties of the optimal fiscal (spending and taxation) policy regime in a setting in which human capital accumulation is, jointly with physical capital, a source of growth. The two key features are that the government is unable to separately tax human capital and pure time, and that fiscal policy can affect the accumulation of human capital. In order to concentrate on the issues raised by the notion of human capital, we deliberately ignored the possibility of externalities.

Overall our results suggest that the zero limiting taxation of physical capital income (a pure stock concept), identified in the early analysis of factor income taxation in dynamic settings as an important qualitative property the long run optimal tax code, is a much more robust finding than the zero limiting taxation of human capital income (a mix of stocks and flows).

In a setting in which human capital is produced using privately purchased and government provided market inputs (e.g. private training and public education) and household non-market time, the optimal tax code is such that, in the long run, household time allocated to the creation of human capital is subsidized (taxed) in proportion to its opportunity cost if it is a substitute (complement) for government spending.

The optimal tax code calls for a tax on effective labor income and a subsidy to the purchases of market goods used in the production of new human capital. One of the implications of the model is that the size of the rate of subsidy to purchases of market goods relative to the income tax rate --effectively, the rate at which these expenses receive an income tax credit-- depends on the degree of substitution between privately purchased and government provided goods in the production of new human capital. The general sense of the results is that if the two inputs are substitutes then the subsidy is generous --tax credit rate in excess of the income tax rate-- while if the inputs are complements the tax credit on expenditures on private inputs used to produce new human capital falls short of the income tax rate.

The model also has implications for the allocation of public spending, and for project evaluation in the area of human capital accumulation. The optimal second best allocation requires, in general, that intermediate inputs be allocated in a distortionary manner, in the sense that their static and dynamic marginal rates of transformation are not the same across sectors. Thus, there is no presumption that a useful generalization of the Diamond-Mirlees result is applicable to these economies.

In general, the results do not imply simple rules for the allocation of public spending. The general flavor is that if privately purchased inputs and government provided inputs used in the production of human capital are substitutes, then the optimal allocation is such that, in the long run, the marginal product of private inputs falls below that of government provided inputs. In concrete terms, it means that if government spending is viewed as substitute for private spending, then government projects should be selected in such a way that the marginal project has a higher rate of return than the marginal private project.

The results also have implications for recent work on the optimal amount of government productive investment. It is common to evaluate whether there has been over or underinvestment in government provided goods by looking at the rates of return of public and private investment. In general, a finding that the rate of return on public investment is higher than the rate of return on private investment is interpreted as evidence of a suboptimal allocation. At least for the case of investment in human capital, our results indicate that the second best allocation can be consistent with differences in rates of return (or marginal products). Thus, findings that capital allocated to educational activities falls short of some growth (or employment) maximizing level, as documented in Aschauer (1997), should not be necessarily interpreted as evidence of underinvestment.

Even though our model is aggregate in nature, its implications would apply to settings in which micro considerations are reflected in a more a more detailed specification of the function G. For example, if in the production of education different inputs can be identified as belonging to our $x_h$ or g categories (e.g. some inputs provided by parents in public schools and others paid for by the school budget) then it is not clear that equality in the marginal products is a desirable outcome. This result greatly complicates cost-benefit analyses of educational programs, and suggests that changing the mix of expenditure categories as advocated, for example, by Pritchett (1997) need not result in an improvement. Similarly, interpreting G as a measure of human capital related health activities, our arguments suggest caution in interpreting productivity differentials between public and private provision of health care (especially when the types of services are different) as indications of the need to reallocate resources (for a discussion see Filmer, Hammer and Pritchett (1997)). In recent work, Anderson and Martin (1998), also emphasize the idea that the shadow price of publicly provided inputs depends on their elasticity of substitution with privately purchased goods; their framework, however, is sufficiently different from ours that an exact comparison is difficult.

On the positive side, our theoretical model suggests a workable simple framework to use in evaluating alternative policies, and points applied researchers in the direction of estimating elasticities of substitution as the critical elements in determining the qualitative features of optimal tax and spending policies.

## *Appendix*

In this appendix we prove Propositions 3.1 and 3.2.

*Proof of Proposition 3.1*: We first describe the Lagrangean for the Ramsey problem. It is given by, $\sum_{t=0}^{\infty} \beta^t$ $\{u(c_t,1\text{-}n_t) + \Phi[u_c(t)c_t - \mu_t G_g(t)g_t] + \lambda_t[F(k_t,n_{zt}h_t,a_t) - (c_t + x_{kt} + x_{ht} + g_t)] + \eta_t[(1-\delta_h)h_t + G(x_{ht},g_t,n_{et}h_t) - h_{t+1}] + \chi_t[(1-\delta_k)k_t + x_{kt} - k_{t+1}] + \theta_t[\mu_t - \beta(\mu_{t+1}(1-\delta_h) + u_\ell(t+1)n_{t+1}/h_{t+1})] + V_0$, where $V_0$ involves endogenous variables dated at t=0. Although an important component of the solution, the t=0 choices have no impact --other than affecting the marginal welfare cost of taxation, $\Phi$-- on the steady state results.

It is useful to define the pseudo-utility function v as,

$v(c,n_z,n_e,h,g,x_h,\mu;\Phi) \equiv u(c,1-n_z-n_e) + \Phi[u_c(\bullet)c_t - \mu G_g(\bullet)g]$.

Then, using v as the objective function, the Ramsey problem is a modified optimal growth problem with one feasibility constraint, and three laws of motion for the stocks (k,h,$\mu$). Unlike standard optimal growth problems, the Ramsey problem could --potentially-- have non-convex preferences (in general v is not a concave or quasi concave function) and one one non-convex constraint (the law of motion for $\mu$). However, if the solution is interior --which is easy to guarantee-- and if it converges to the steady state (and this should be taken as an assumption, as we do not have a proof of convergence), the steady state conditions are,

(A.1) $u_c[1+\Phi(1+(u_{cc}c)/u_c)] = (\theta/h)u_{c\ell}n + \lambda$,

(A.2) $u_\ell[1+\Phi(u_{c\ell}c)/u_\ell] = (\theta/h)(u_{\ell\ell}n - u_\ell) + \lambda F_z h$,

(A.3) $u_\ell[1+\Phi(u_{c\ell}c)/u_\ell] + \Phi\mu G_{gn}gh = (\theta/h)(u_{\ell\ell}n - u_\ell) + \eta G_n h$,

(A.4) $\Phi G_g g = \theta\delta_h$,

(A.5) $\Phi\mu[G_g + G_{gg}g] = \eta G_g - \lambda$,

(A.6) $\Phi\mu[G_{xg}g] = \eta G_x - \lambda$,

(A.7) $1= \beta(1-\delta + F_k)$,

(A.8) $\eta = \beta[\eta(1-\delta_h + G_n n_e) + \lambda F_z n_z - \Phi\mu G_{gn}gn_e + (\theta/h)u_\ell(n/h)]$,

(A.9) $\mu = \beta[\mu(1-\delta_h) + u_\ell n/h]$.

First note that (A.7) and the steady state version of (3.5) implies i). To prove ii) use (3.18) and (3.19) to obtain, $\tau^e =((1-\tau^x)/(\beta G_x F_z n))[1-\beta(1-\delta_h +G_n n)]$. Thus, to determine the sign of $\tau^e$ it suffices to determine the sign of $[1-\beta(1-\delta_h +G_n n)]$. Note that (A.2) and (A.3) imply that,

(A.10) $\lambda F_z = \eta G_n - \Phi\mu G_{gn}g$.

Using (A.10) in (A.8) we obtain,

$\eta = \beta[\eta(1-\delta_h + G_n n) - \Phi\mu G_{gn} gn + (\theta/h)u_\ell(n/h)]$,

or,

(A.11) $\eta[1- \beta(1-\delta_h + G_n n)] = \beta n[(\theta/h)u_\ell(1/h) - \Phi\mu G_{gn} gn]$.

Next, using the fact that (A.4) and the steady state assumption imply that $\theta/h = \Phi G_g g/G$, and (A.9) we get,

$\eta[1- \beta(1-\delta_h + G_n n)] = \Phi\mu[(G_g g/G)(1- \beta(1-\delta_h)) - \beta G_{gn} gn]$,

or,

$\eta[1- \beta(1-\delta_h + G_n n)] = \Phi\mu(G_g g/G)[1- \beta(1-\delta_h) - \beta G_n n(G_{gn}G/G_g G_n)]$,

or,

$\eta[1- \beta(1-\delta_h + G_n n)] = \Phi\mu(G_g g/G)[1- \beta(1-\delta_h + G_n n) + \beta G_n n(1- \kappa_{ng})]$,

which implies,

(A.12) $[1- \beta(1-\delta_h + G_n n)][\eta - \mu(\theta/h)] = \mu(\theta/h)\beta G_n n(1- \kappa_{ng})$.

Thus, if $\eta - \mu(\theta/h) > 0$ and $\mu(\theta/h) > 0$, (A.12) proves ii. Since the marginal welfare cost of distortionary finance $\Phi$ is strictly positive, (A.4) implies that $\theta > 0$. Equation (A.9) implies that $\mu > 0$. To show that $\eta - \mu(\theta/h) > 0$, consider (A.8) again and use (A.10) to eliminate the term $\eta G_n n_e - \Phi\mu G_{gn} gn_e$, which equals $\lambda F_z n_e$. Then, (A.8) is,

$\eta = \beta[\eta(1-\delta_h ) + \lambda F_z n + (\theta/h)u_\ell(n/h)]$,

or, using $\beta u_\ell(n/h) = \mu[1-\beta(1-\delta_h)]$, we get

(A.13) $[1-\beta(1-\delta_h)][\eta - \mu(\theta/h)] = \beta\lambda F_z n$.

Since $1-\beta(1-\delta_h) > 0$, and $\beta\lambda F_z n > 0$, it follows that $\eta - \mu(\theta/h) > 0$. This proves ii). To prove iii) note that (3.19) imply that $(1-\tau^h)/(1-\tau^x) = [\beta^{-1}-(1-\delta_h)]/G_x F_z n$. Thus, it follows that,

$\tau^h_\infty \gtreqless \tau^x_\infty \Leftrightarrow [\beta^{-1}-(1-\delta_h)]/G_x F_z n \lesseqgtr 1$.

We now show that $[\beta^{-1}-(1-\delta_h)]/G_x F_z n \lesseqgtr 1 \Leftrightarrow \kappa_{xg} \gtreqless 1$. To this end, use (A.13) to write,

$[1-\beta(1-\delta_h)]/\beta G_x F_z n = \lambda/[\eta G_x - \mu\Phi G_x G_g g/G]$.

Next, use (A.6) to obtain,

$[1-\beta(1-\delta_h)]/\beta G_x F_z n = [\eta G_x - \mu\Phi G_{xg} g]/[\eta G_x - \mu\Phi G_x G_g g/G]$.

Direct calculations show that $[\eta G_x - \mu\Phi G_{xg} g]/[\eta G_x - \mu\Phi G_x G_g g/G] \lesseqgtr 1 \Leftrightarrow \kappa_{xg} \gtreqless 1$, where $\kappa_{xg} \equiv G_{xg}G/G_x G_g$.

■

*Proof of Proposition 3.2*: First note that i) follows trivially from (A.7). Also note that ii) was proved in the course of proving part iii) of Proposition 3.1. To prove iii) use (A.10) and (A.6) to get,

$$\eta(G_xF_z - G_n) = \mu\Phi g(G_{xg}F_z - G_{gn}),$$

or,

$$\eta(G_xF_z - G_n) = \mu\Phi G_g g/G(\kappa_{xg}G_xF_z - \kappa_{ng}\, G_n) = \mu(\theta/h)(\kappa_{xg}G_xF_z - \kappa_{ng}\, G_n),$$

where the last equality uses (A.4). Manipulation of this last expression implies,

$$G_xF_z(\eta - \mu(\theta/h)\kappa_{xg}) = G_n(\eta - \mu(\theta/h)\kappa_{ng}),$$

or,

$$\text{(A.14)}\quad G_xF_z/G_n = (\eta - \mu(\theta/h)\kappa_{ng})/(\eta - \mu(\theta/h)\kappa_{xg}).$$

Thus, if $\eta - \mu(\theta/h)\kappa_{xg} > 0$, (A.14) implies the desired result. We next verify that $\eta - \mu(\theta/h)\kappa_{xg} > 0$. To this end, observe that (A.6) implies that,

$$G_x(\eta - \mu\Phi G_{xg}g/G_x) = \lambda > 0.$$

The left hand side can be written as,

$$G_x(\eta - \mu(\theta/h)\kappa_{xg}),$$

using (A.4) and the steady state assumption. This completes the proof of iii).

Finally, to prove iv) use (A.5) and (A.6) to show that,

$$G_x \gtreqless G_g \Leftrightarrow G_{xg} \gtreqless G_g g/G + G_{gg},$$

where we used the fact that $\Phi\mu > 0$. Direct calculations show that the above condition is equivalent to $(\kappa_{xg}-\kappa_{gg})\upsilon(g) \gtreqless 1$. Finally, homogeneity of degree one of G and the properties of the partial elasticity of complementarity imply that $\kappa_{xg}\,(\upsilon(x_h)+\upsilon(g)) + \kappa_{ng}\,(1-(\upsilon(x_h)+\upsilon(g)) = (\kappa_{xg}-\kappa_{gg})\upsilon(g)$.

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