Baby Busts and Baby Booms:
The Fertility Response to Shocks in Dynastic Models*

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Abstract

Economic demographers have long analyzed fertility cycles. This paper builds a foundation for these cycles in a model of fertility choice with dynastic altruism and aggregate shocks. It is shown that under reasonable parameter values, fertility is pro-cyclical and that, following a shock, fertility continues to cycle endogenously as subsequent cohorts enter retirement. Quantitatively, the contemporaneous elasticity of fertility to productivity shocks lies between 1 and 1.7 while the lagged elasticity lies between -0.9 and -1.6, generating dampened oscillations. Applied to the U.S. experience in the 20th century, in the model, the Great Depression generates a large baby bust – between 56% and 91% of that seen in the U.S. in the 1930s – which is followed by a baby boom – between 64% and 113% of that seen in the U.S. in the 1950s. The model can also account for much of the fertility experience of the American South during and after the Civil War. Statistical analysis across countries in the 1930s and 1950s and across U.S. states in the most recent recession further supports our theory.

Keywords: Fertility cycles, stochastic growth models, dynastic models.
JEL Classification Numbers: J11, J13, E13, O11.

1 Introduction

Demographers have long analyzed fertility cycles. One particularly dramatic example is the U.S. experience over the 20th century. During the first part of the century, fertility rates continued a drop, that demographers associate with the Demographic Transition. This drop started in the mid 19th century: Total Fertility Rates (TFR) fell from 5.7 children per woman to 3.0 over the period from 1850 to 1925. In the years that followed there was an even more abrupt drop during the years of the Great Depression, from 3.0 to 2.1. Indeed, women in their prime fertility years during the Great Depression had, on average, only 2.2 children in their entire lifetime. These women, born between 1905 and 1915, had fewer children than any previous cohort. Following this, fertility rebounded significantly during the 1950s and early 1960s, the baby boom. At its peak, TFR reached 3.6 while Cohort Total Fertility (CTFR) showed a similar, but smaller, increase up to 3.2. Only in the 1970s did fertility go back down to the level seen in the 1930s.

One of the key hypotheses put forward by economic demographers for these large and opposite swings is known as the 'Easterlin hypothesis', see Easterlin (1961, 1968, 1978, 1987). In a nutshell, the idea behind this hypothesis is that fertility was exceptionally low during the Great Depression because of the large negative shock to incomes. Then, in substantial part, due to the fact that fertility had been so low during the 1930s, the baby boom occurred. The mechanism through which low fertility in the past leads to high fertility one generation later has usually been attributed to the feedback effects of the resulting unbalanced age-structure on relative wages of young fertile workers to older workers in combination with differences between expected lifetime income relative to 'material aspirations formed in childhood.'[1]

In this paper, we build a model of fertility choice combining features of a stochastic growth model with a model of fertility choice with dynastic altruism à la Barro-Becker (Becker and Barro, 1988, and Barro and Becker, 1989).[2] In addition to the stochastic component, the model extends existing Barro-Becker type models to include multiple periods in the working life. For simplicity, we assume that labor is the only factor of production (except in Section 5.1) and that different age-groups are perfect substitutes in production (up to age-

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[1]The so called 'Easterlin hypothesis' has developed over Easterlin's own work and has been interpreted in more than one way by other authors. For an excellent overview until 1998, see Macunovich (1998). She 'focuses just on the fertility aspects of the Easterlin hypothesis', which is our focus as well. For a more general overview, see Macunovich and Easterlin (2008).

[2]Some have put forth the idea that the baby boom was a consequence of low fertility during the Great Depression – i.e., the baby boom was 'catching up.' However, completed fertility (CTFR) was low for both the women immediately preceding and immediately following the baby boom and hence, this cannot be true at the level of the individual mother. It can, in principle, hold across cohorts in a dynastic model. This distinction is relevant in our analysis and is one of our motivations for studying a dynastic model.

specific productivity levels). Thus, we abstract from feedback effects of relative cohort sizes on relative wages. In this model, we study, both qualitatively and quantitatively, fluctuations in fertility, triggered by (large) income shocks using preference parameter configurations in line with the standard growth and business-cycle literatures.

We show that both of the effects originally discussed by Easterlin, the large reactions of fertility to large income shocks, and the oscillations thereafter, are present in the model (despite the absence of feedback effects through wages). In particular, we find that fertility is procyclical as long as most of the costs of children are in terms of goods, or as long as there is sufficient curvature in the period utility function. In addition, the policy function for current fertility as a function of past fertility that comes out of our model is negatively sloped around the steady state. We find that the dynamics of adjustment following a movement off a balanced growth path is one of dampened oscillations. Putting these two effects together, in the model, a large negative income shock is met with a contemporaneous reduction in fertility followed by a baby boom a generation later. Thus, in the model the Great Depression (or any other similarly large negative income shock) would trigger a baby bust, followed by a baby boom a generation later.

In our model, children are partly a consumption good (Barro-Becker preferences) and partly an investment good (future labor force) for the dynasty. Comparing our results with those standard in growth models gives substantial intuition. The role of the capital stock(s), is played by the age distribution of the population, while the number of births plays the role of investment. Then, as is standard in growth models (for standard consumption smoothing reasons), periods when productivity is particularly high are times when investment is high. Here this translates to: fertility is high during periods of high productivity – an income boom generates a baby boom. Correspondingly, an income bust generates a baby bust. In addition, the larger is the desire to smooth consumption, the larger the elasticity of fertility to income shocks. There is one subtlety that needs to be added with this interpretation. This is that, when the cost of a new child is primarily in terms of time, good times are both good times to save for the future (invest) and times when that investment is most expensive. As a result, in this case, fertility is only procyclical when there is sufficient desire to smooth consumption (i.e., more curvature than log utility).[3]

A similar intuition gives us insight as to why the model exhibits dampened oscillations. This has to do with the specification of the implicit depreciation rates of the age-specific

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[3]Notice that one important feature here is that income is *temporarily* high or low, *relative* to trend. As shown by Jones and Schoonbroodt (2010), a *permanent* increase in productivity growth has a permanent *negative* effect on fertility under the parameter values that generate pro-cyclical fertility here. Thus, the important distinction between temporary and permanent changes in productivity shocks is just as important here as it is in standard growth and business cycle models.

populations. When the stock is the number of people, the counterpart of depreciation is movement out of the work force. Thus, fertility (i.e., investment) will be higher in periods when a relatively large share of workers are nearing retirement age. Because of this effect, a baby bust now means that the next generation's workforce is primarily older workers – more workers will be retiring. Consequently, investment will be higher in response – a baby boom will occur.

On the quantitative side, we use a calibrated version of the model to simulate the size of the response of fertility to productivity shocks, both contemporaneous and delayed. After calibrating to U.S. averages, we find that the contemporaneous response to a 1 percent deviation in productivity lies in the range of 1 to 1.7 percent depending on the nature of the costs of children (generally, responses are smaller when the costs of children are primarily in terms of parental time as opposed to goods) while the elasticity one period later lies between 0.9 and 1.6. This implies that the response of (completed) fertility to a 'standard recession' (say, productivity is 5 percent below trend for 2 years) is relatively small – of the order of 0.02 to 0.04 children per woman with a subsequent baby boom of similar size.

However, we find that the reduction in fertility implied by the model as a response to the 12 percent decrease in productivity during the Great Depression is between 56 and 91 percent of the observed pre-WWII baby bust in Total Fertility Rates (TFR). Moreover, the subsequent endogenous fluctuations in fertility triggered by this bust, in conjunction with the productivity boom in the 1950s and 1960s, captures between 64 and 113 percent of the post-WWII baby boom in TFR.[4]

Another important episode in American history is the U.S. Civil War. In this period, the South was hit particularly hard – estimates of the drop in GDP are 36 percent by 1867. Fertility rates also fell, dropping 10 percent below trend. This was followed by a moderate boom in the late 1870s and 1880s. Taking the effects of the war, unrelated to productivity shocks, into account, we derive bounds suggesting that productivity dropped between 11 and 21 percent in the 1860s and either remained 10 percent below or just reached their 1860 level in the 1880s. The resulting model predictions for contemporaneous and lagged fertility movements around trend are similar, quantitatively, to those observed in the data.

There is also a large statistical literature on the cyclicality of fertility. Sobotka et al. (2010) extensively review this literature and analyze several major recessions in detail. We provide two additional pieces of statistical evidence. First, since both the Depression and the baby boom are phenomena that occurred in many different countries, we go on to study

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[4]Realistically, it seems unlikely that fertility decisions are affected by quarter to quarter fluctuations in productivity (as addressed in the business cycle literature). This is consistent with our quantitative findings. Rather, a prolonged boom or bust is required for the fertility effect to be large.

one obvious implication of our theory. If the mechanisms that we study are important, it should be true that countries that had deeper Depressions in the 1930s should also have larger baby busts in the 1930s, and that those with larger baby busts should experience larger baby booms in the 1950s. In Section 6.1, we show that this is indeed true, conditional on the economic circumstances in the 1950s. These observations further support our theory. Second, we provide evidence on the recent recession across U.S. states. We find that states with a larger drop in income per capita tend to have a larger reduction in fertility in 2008. Whether this drop will have permanent effects on completed fertility and will trigger further dampened oscillations depends on the length of the recession and remains to be seen.

Many other demographers and economists have studied fluctuations in fertility but they have focused on different channels. First, operationally, the mechanisms that are the driving forces behind our results are considerably different than those emphasized by Easterlin in his work. For example, he emphasizes differences between expected lifetime income relative to 'material aspirations formed in childhood.' Here, the basic mechanisms are best understood as variants of standard effects of growth models – the desire to smooth consumption, etc.[5]

Second, following Easterlin many authors have *assumed* that the relationship between fertility today and past fertility has a negative slope. Based on this assumption, they analyzed the resulting dynamics of age-structured populations, see Lee (1974) and Samuelson (1976) for early analysis of dynamic population systems of this kind.[6] In contrast, we provide a choice-theoretic foundation for the negative slope and, hence, the cycles.

Third, we assume that different ages of workers are perfect substitutes in production. Nevertheless, fertility cycles are generated through the curvature of preferences. This shows that feedback effects from the age-structure on wages are not necessary to generate population cycles. This is an alternative to Easterlin's feedback effect which, as suggested by Lee (2008), can be generated by using a CES production specification, where different age-groups are not perfect substitutes.

Further, the mechanisms that we highlight here are also complementary to those explored in other papers that have used choice-theoretic dynamic macroeconomic models to study the baby boom in the 1950s and 1960s. Three key examples are Greenwood et al. (2005), Doepke et al. (2007) and Albanesi and Olivetti (2010). In Greenwood et al. (2005), the channel that is highlighted is the effects of the drastic improvement in technologies of home production that took place in the post WWII period. In essence, these improvements temporarily made having children cheaper generating the boom. In Doepke et al. (2007), they emphasize

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[5]Feichtinger and Dockner (1990) assume a positive relationship between births and the difference between actual and expected consumption, which is more in line with Easterlin's original argument.

[6]Feichtinger and Sorger (1989) extend Samuelson's initial model to continuous time.

the relatively high female participation rate that took place during WWII. This, because of learning on the job, made that specific cohort of women relatively high productivity. Because of this, the cohort of women entering their 20s in the 1950s faced a relatively tough job market causing them to delay entering the workforce and have larger than normal families. Albanesi and Olivetti (2010) emphasize the decrease in maternal mortality as a cause of the baby boom. This decrease in mortality also increased the incentive to invest in human capital, triggering the return to low fertility in the 1970s. Thus, the primary channels emphasized in those papers differ from ours. One advantage of our approach is that it is also able to capture the sizable downward movement, relative to trend, in fertility as seen in both the 1930s and the Civil War.

Finally, our findings can rationalize the observations in Butz and Ward (1979) as follows. In brief, they find that fertility, while procyclical, has become less responsive to business cycle frequency fluctuations over time. In our model, this can happen for two reasons. The first comes from the fact that the size of the effects that we find are smaller when the costs of children are in terms of time. Since there has been a large increase in labor force participation among married women, this causes the time cost of children to be increasingly important. Second, due to continuing fluctuations triggered by the Great Depression, the fertility response to productivity shocks is mitigated. In our data analysis, we show that, controlling for productivity shocks in the past is crucial to find a positive correlation of fertility with contemporary shocks.

In Section 2, we review the the data for the U.S. since 1900 in detail, focusing on both the size and timing of movements in fertility and productivity. The model is laid out in Section 3 and the analytic results are presented. In Section 4, we explore the quantitative implications of a calibrated version of the model compared to U.S. data, for the periods following both the Great Depression and the Civil War. In Section 5 we study the sensitivity of our results to some basic changes in our assumptions. Finally, we present statistical evidence in Section 6 while Section 7 concludes.

2   Twentieth century U.S. data

In this section, we lay out the basic facts about the time paths of productivity and fertility in the U.S. over the 20th century. We begin with the facts pertaining to the growth in productivity using a consistent measure for total factor productivity (TFP) and labor productivity (LP) from Chari et al. (2007). As most economists know, this period is one of more or less continued growth in productivity with a few interruptions. The most significant of these is the Great Depression. Figure 1 shows the natural logarithms of TFP and LP over the period

from 1901 to 2000.

The facts about productivity over this period can be described as follows:

1. the continual upward trend;

2. the marked decline below trend that took place in the 1930s and early 1940s;

3. the return to trend in the early 1950s;

4. the significant increase above trend that took place in the 1950s and 1960s;

5. the productivity slowdown since the 1970s.

Figure 1: Total Factor Productivity and Labor Productivity, 1901-2000 (1929=100)
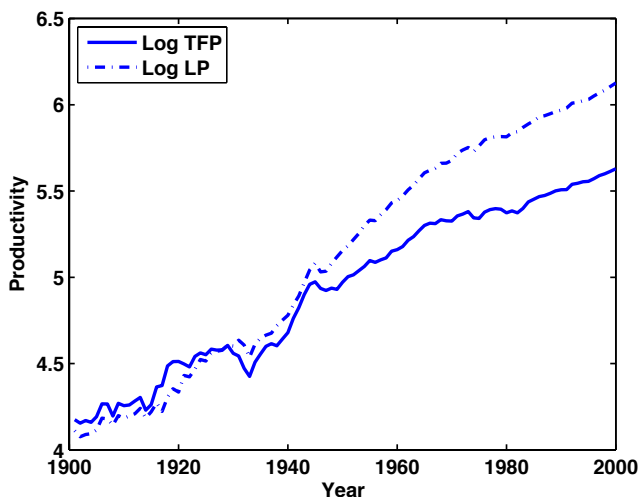
This timing of the movements of productivity around trend fits well with the movements in fertility seen in the data. Figure 2 shows the time path of the Total Fertility Rate (TFR) and Cohort Total Fertility Rates (CTFR) (by birthyear of mother +25 years) over the period from 1850 to 2000. We have two time series for TFR, which calculates how many children a woman would have over her lifetime if current age-specific fertility rates were to prevail in the future. The first series is the one prepared by Haines (1994) using Census data and hence is available only every 10 years.[7] The second TFR series comes from the Natality Statistics Analysis from National Center for Health Statistics. It is available at annual frequencies, but only since 1917. The CTFR series comes from Jones and Tertilt (2008) and counts how many children were born to a particular cohort of women at the end of their fertile period.

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[7]Since the data on TFR is only available decade by decade and since it covers the U.S. as a whole, it is not apparent in this figure that there was a significant drop in births in the South during and after the Civil War. See Section 4.4 where we show annual data on Crude Birth Rates (CBR) for this period for the South.

Implicitly, it is equivalent to adding age-specific fertility rates pertaining to a particular cohort of women over time. Its frequency is five-year birth cohorts.

At the beginning, fertility is still in the midst of what is known to demographers as the Demographic Transition, the marked fall in fertility (and mortality) that has occurred in all developed countries. This fall accelerates from the late 1920s to the mid 1930s. Fertility then increases to reach its peak in the baby boom of the 1950s and 1960s. It appears that a good description would be:

1. high, and fairly constantly decreasing fertility from 1850 until 1925, when it reaches a TFR of about 3.0 children per woman;

2. an acceleration of the rate at which fertility is falling between 1925 and 1933 (from TFR=3.0 to TFR=2.1);

3. constant, but low, fertility over the period from 1933 to 1940, with the level at about TFR=2.2;

4. rapidly rising fertility from 1940 to 1957, with TFR going from 2.2 up to 3.7;

5. high, stable fertility from 1957 to 1961 at about TFR=3.6;

6. a rapid decrease from 1961 to 1976, with TFR going from 3.6 down to 1.7;

7. a slight increase and then stable low fertility over the remainder of the period, with the level at about TFR=2.

Figure 2: (Cohort) Total Fertility Rate, 1850-2000
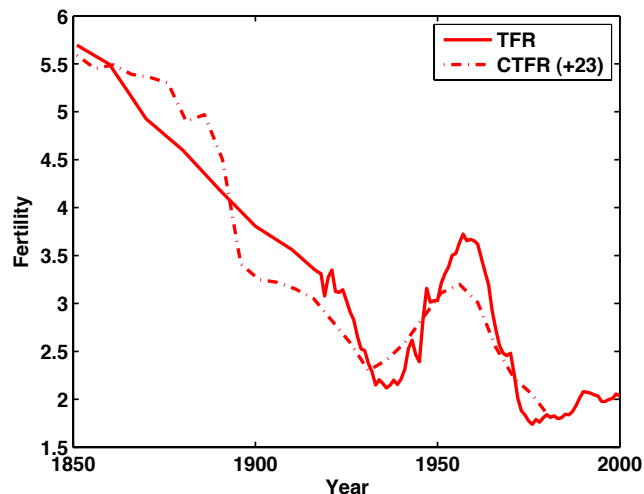
We will refer to 2 and 3 as the pre-WWII baby bust, 4 and 5 as the post-WWII baby boom and 6 as the baby bust of the 1970s. The exact sizes of these features of the data

depend on how one treats the trend growth in productivity and trend decrease in fertility over the period. For example, was there a common, exogenous growth rate in productivity over the entire period with higher frequency (albeit highly autocorrelated) fluctuations around this trend? Or, were there several regimes of growth? For fertility, one can see that while TFR decreases smoothly over time, the early CTFR data shows that the largest decrease happens for cohorts of women born between 1858 (4.9 children per women) and 1878 (3.25 children per woman). The fluctuations thereafter, however, look very similar in both series, though somewhat larger in TFR than CTFR.

For fluctuations, we fit a linear trend to the (ln) TFP and LP series from 1901 to 2000 ($\ln P_t = \alpha_0 + \alpha_1 t + \epsilon_t$, where $P = \{TFP, LP\}$), and detrend TFR using an HP filter (smoothing parameter, $w = 20,000$). We obtain annual percent deviations over this period plotted in Figure 3. Several alternative detrending methods were studied with very similar results.

Figure 3: TFR, TFP and LP Percent Deviations From Trend, 1901-2000
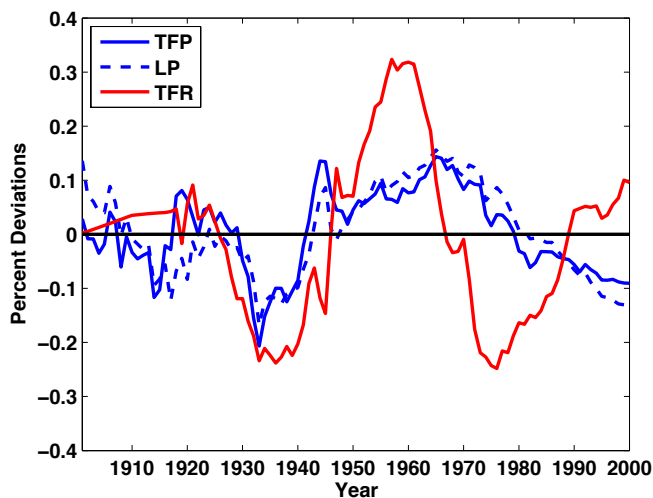
Although it is not perfect, there is an impressive coincidence in timing. The coefficient of correlation between annual TFP and TFR deviations for the years 1901 to 2000 is 0.4, with a coefficient of 0.7 from 1901 to 1940 and 0.2 from 1941 to 2000. This suggests that the U.S. TFR is procyclical during the early time period but the correlation is much weaker thereafter (see also Butz and Ward, 1979). As suggested by the model below, one reason for the decrease in the correlation may be the increase in female labor supply which made the opportunity cost of children, women's wages, procyclical.

What our model simulation is also going to capture is the large downward deviation in the 1930s due to the large negative shock during the Great Depression and a baby boom following endogenously as a response to the baby bust itself, one generation later. We therefore also

run the following regression. Let $\widehat{X}_t$ denote the percent deviation from trend in variable X in period t.

$$\widehat{TFR}_t = \lambda_0 + \lambda_1 \widehat{P}_t + \lambda_2 \widehat{P}_{t-l} + \varepsilon_t$$

where $P = \{TFP, LP\}$ and $l \in \{20, ..., 25\}$. The results for $l = 20$ are given in Table 1. Similar results go through for larger values of l.

These regressions show that the coefficients on contemporaneous productivity (λ_1) are both positive and significantly different from zero, while the coefficients on productivity a generation ago (λ_2) are both negative and significantly different from zero. Also, λ_1 and λ_2 are of similar magnitude in absolute value, while the constant (λ_0) is likely to be zero.

Table 1: U.S. TFR and Productivity: Regression results

Indep. Var.	Coefficient	Indep. Var.	Coefficient
Constant	-0.0051	Constant	-0.0058
$\widehat{TFP}_t$	0.8363**	$\widehat{LP}_t$	0.6004*
$\widehat{TFP}_{t-20}$	-0.8401**	$\widehat{LP}_{t-20}$	-0.6218*

*, **: Significantly different from zero at 5% and 1%, respectively.

3 The model

In this section, we lay out a model of the response of fertility to period by period stochastic movements in productivity. To do this, we use a model of fertility based on that developed in Becker and Barro (1988) and Barro and Becker (1989) (Barro-Becker henceforth). The simplification that we make is to assume that there is no physical (or human) capital in the model. Thus, the flow of income is solely due to wage income.[8] On the other hand, we add a stochastic component as well as an explicit age-structure to the basic Barro-Becker model.

3.1 Model setup

A period is 20 years. Every person lives for four periods, one as a child and $T = 3$ periods as an adult. There is an initial age distribution of the population given by (N_0^3, N_0^2, N_0^1) where N_0^3 is the number of initial old (i.e., their age in period $t = 0$ is $a = 3$). We normalize by assuming that $N_0^3 = 1$.

Children (age $a = 0$) do nothing. At age $a = 1$, young workers consume, have children and supply one unit of labor (net of time spent to raise children) to earn a wage, w_t^1. At age

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[8]We relax this assumption in Section 5.1.

$a = 2$, old workers consume and supply one unit of labor inelastically to earn a wage, w_t^2, but are no longer fertile. In the last period of their lives, age $a = 3$, agents are retired and only consume.

Adults care about consumption, the number of children and their children's future utility. Following the original Barro-Becker formulation, we assume that the utility of a person who was born in period $t - 1$ and whose first period as an adult is in period t is given by:

$$(1) \qquad U_t^1 = V_t^1 + \phi \beta g(n_t) U_{t+1}^1$$

where U_t^1 represents the full value of utility of an age $a = 1$ adult in period t looking from that point forward, V_t^1 is the utility this person gets from his own path of consumption, n_t is the number of children that he has and U_{t+1}^1 is the utility that his typical child will receive.

Let c_s^a be the amount of consumption for the typical person in period s that is age a. We assume that utility from the time path of own consumption $((c_t^1, c_{t+1}^2, c_{t+2}^3)$ – young worker, old worker, retirement) is of the form:

$$(2) \qquad V_t^1 = \sum_{a=1}^{T} \beta^{a-1} u(c_{t+a-1}^a).$$

We distinguish between time preference as measured by the discount factor, β and the degree of altruism between generations within a period, ϕ. That is, $\phi = 1$ means that a person cares as much about the utility of his children as he cares about his own (see Manuelli and Seshadri, 2009).

Similarly to equation (1), we can define the continuation utility for a person who was a young adult in period t (i.e., he was born in period $t - 1$) looking forward from the point when he reaches age $a > 1$. Of particular interest is the continuation utility of an adult of age T in period 0 – i.e., the initial old. These are the only agents in the model who care about all agents of all ages in all periods. Sequentially substituting V and U for later generations and grouping terms by period instead of generations, utility for the initial old, U_0^T, is given by:

$$(3) \qquad U_0^T = \sum_{t=0}^{\infty} (\beta \phi)^t \left[\sum_{a=1}^{T} \phi^{1-a} \left[\Pi_{k=0}^{t-1-a} g(n_k) \right] u(c_t^a) \right]$$

Assuming $g(n) = n^\eta$, simplification occurs in equation (3) because $g(n_t)g(n_{t+1}) = g(n_t n_{t+1})$. To see this, let N_t^a be the number of descendants of the initial age T agent, that are of age

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a in period t. Then the laws of motion for population are given by:

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(4) \qquad N_t^1 = n_{t-1}N_{t-1}^1 \text{ is the number of births in period } t-1;
$$

$$
N_t^a = N_{t-1}^{a-1} \text{ for } a = 2, ..., T;
$$

$$
N_t^a = 0 \text{ for } a > T;
$$

$$
N_0^T = 1.
$$

Using the functional form assumption for g and the laws of motion for population, U_0^T can be written as:

$$
(5) \qquad U_0^T = \sum_{t=0}^{\infty} (\beta\phi)^t \left[\sum_{a=1}^{T} \phi^{1-a} g(N_t^a) u(c_t^a) \right]
$$

Hence, as is standard in dynastic models à la Barro-Becker, the utility of the initial old can be written in terms of the number of descendants of age a in period t, N_t^a. Assuming $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, following Jones and Schoonbroodt (2010), there are two sets of parameter restrictions that satisfy the natural monotonicity and concavity restrictions of U_0^T:

AI. $\qquad 0 < 1 - \sigma \le \eta < 1;$

AII. $\qquad 0 > 1 - \sigma \ge \eta.$[9]

Under AI., g is increasing and utility is positive, while under AII. g is decreasing but utility is negative, so that, in both cases, ancestor's utility is increasing in the number of descendants. Also, under AI. the desire to smooth consumption over time and across generations is low, while under AII., it is high. These distinctions are relevant for the results below. We often concentrate on the case where of $\eta = 1 - \sigma$, allowed under both configurations. There are at least two advantages to this choice. First, simplification occurs because the direct utility benefit of increasing population as well as the indirect utility cost (of diluting per capita consumption) exactly cancel out in this case. Second, changing the side of the parameter space (AI. versus AII.) also involves changing η *relative* to $1-\sigma$, except in the case where $\eta = 1 - \sigma$. When comparing the two sides of the parameter space, this assumption allows for only an absolute change without the relative change.

We assume that the cost of children born in period t is in terms of period t consumption but allow this cost to depend on the wage of young workers – $\theta_t(w_t^1)$. Thus, this allows for the two most common ways of modeling child costs: goods costs – $\theta_t(w_t^1) = \theta_t$ – and time

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[9] The knife-edge case where $\sigma \to 1$ and $\eta = \delta(1 - \sigma)$ can also be analyzed. See Jones and Schoonbroodt (2010) or Schoonbroodt and Tertilt (2010) for details.

costs $- \theta_t(w_t^1) = bw_t^1$. Thus, feasibility for the dynasty in period t is given by:

$$(6) \qquad \sum_{a=1}^{3} N_t^a c_t^a + \theta_t(w_t^1) N_t^b \leq \sum_{a=1}^{2} w_t^a N_t^a \equiv W_t$$

where $N_t^b = n_t N_t^1 = N_{t+1}^1$ is the total number of births in period t. If the costs are in terms of time, $\theta_t = bw_t^1$, there is an additional constraint, namely that $0 \leq bN_t^b \leq N_t^1$.

We assume that wages grow at a constant rate, γ, on average. Moreover, there is an aggregate shock so that the entire age-specific wage profile shifts up and down by s_t in period t. Thus, in period t, wages are given by:

$$(w_t^1, w_t^2) = (\gamma^t s_t w^1, \gamma^t s_t w^2).$$

We assume that s_t is a first-order Markov Process.

To ensure the existence of a balanced growth path, we also assume that the costs of children grow at rate γ. That is, we assume that, $\theta(w_t^1) = \gamma^t \theta$ in the goods cost case and $\theta(w_t^1) = \gamma^t b s_t w^1$ in the time cost case.

To complete the specification of the model we need to make a decision about which agents get to choose what and when. There are many different ways to do this. A useful benchmark is a Planner's problem approach. Accordingly we analyze the problem that maximizes the overall utility of the initial old. This has two advantages as a benchmark. First, this is the only group that cares, either directly or indirectly about all agents in the model. Second, it has the feature that the resulting allocation can also be supported dynamically as the equilibrium of a game in which in each period, the oldest individuals make choices for all variables for all of their descendants. This equilibrium can further be decentralized through state contingent transfers. That is, if parents can impose a (possibly negative) transfer on their children every period they are both adults and alive after the current shock is observed, then the equilibrium allocation in which every person decides on their own consumption and fertility in addition to the two transfers is the same as the Dynastic Planner's allocation.[10]

Given this, the Planner's Problem, $P(\gamma, \beta; \{N_0^a\}, s_0)$, is to choose $\{\{c_t^a(s^t)\}_{a=1}^{3}, N_{t+1}^1(s^t)\}_{t=0}^{\infty}$ to maximize utility in (5) subject to feasibility in (6) and laws of motion of population in (4) where $s^t = (s_0, s_1, ..., s_t)$ is the history of shocks up to and including period t.

As is usually true in models with exogenous, trend growth, solutions can be obtained by solving a related model with no growth and a different discount factor. Thus, the solution to $P(\gamma, \beta; \{N_0^a\}, s_0)$ can be obtained directly from the solution to $P(1, \hat{\beta}; \{N_0^a\}, s_0)$ – i.e., $\gamma = 1$ (no growth) and the discount factor, $\hat{\beta}$, depends on γ, β, σ, and η. Because of this

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[10]See Appendix A.1 for details.

result, we abstract from trend growth through most of the remainder of the paper. In those cases where the solution to the model depends on the discount factor, we will use this result to calibrate to the appropriate discount factor in the detrended model.

Let us introduce one additional piece of notation. Let $n_t^b \equiv \frac{N_t^b}{0.5N_t^1}$ be the number of births per woman. This is the model quantity that we will identify with the Cohort Total Fertility Rate (CTFR) in the data, while TFR will be a weighted average over several cohort's fertility.

3.2   Consumption across the age distribution

First, we analyze how consumption within a given period, t, is distributed across the different ages of agents alive at the time. The relevant term in a typical period, t, is:

$$\sum_{a=1}^{3} \phi^{1-a} g(N_t^a) u(c_t^a) = \sum_{a=1}^{3} \phi^{1-a} g(N_t^a) u\left(\frac{N_t^a c_t^a}{N_t^a}\right) = \sum_{a=1}^{3} \phi^{1-a} g(N_t^a) u\left(\frac{C_t^a}{N_t^a}\right),$$

where $C_t^a = N_t^a c_t^a$ is the total consumption of the age a cohort in period t.

Given any level of aggregate consumption in a period, C_t, the planner will choose a distribution across ages to maximize the above subject to $\sum_{a=1}^{3} C_t^a = C_t$. It follows that this is done by equating the marginal utility of a unit of aggregate consumption across the different ages.

To gain more intuition, consider the Planner's objective within a period:

$$\sum_{a=1}^{3} \phi^{1-a} (N_t^a)^{\eta+\sigma-1} \frac{(C_t^a)^{1-\sigma}}{1-\sigma}$$

As can be seen, the marginal utility of C_t^a is affected by three things. First, is g increasing (AI) or decreasing (AII) in N? Second, is N growing over time? Third, what is ϕ?

For example, suppose $\phi = 1$. In this case, the age a term is $g(N_t^a) u\left(\frac{C_t^a}{N_t^a}\right)$. If population is growing (the case typically of empirical interest), then N_t^a is decreasing in a – there are less people in older generations. Thus, if g is increasing (decreasing) in N, $g(N_t^a)$ is decreasing (increasing) in a. Thus, other things equal, the marginal value of an increase in per capita consumption within a period is decreasing (increasing) in age. On the other hand, in this case a given level of aggregate consumption in a cohort is split across fewer people in older groups increasing per capita consumption. This leads to a lower value of u'.

Thus if g is increasing in N, whether the Planner will want consumption to be increasing or decreasing in age within a particular period will depend on which of these two effects is larger. If g is decreasing in N, then if population is growing, per capita consumption is increasing in a for sure. For example, specializing further to the case where $\eta = 1 - \sigma$ (allowed under both configurations), and $\phi = 1$, period t utility becomes:

$$\sum_{a=1}^{3} \frac{C_t^{a1-\sigma}}{1-\sigma}.$$

Thus, aggregate consumption of all age groups within a period will be equalized, $C^a = C^{a'}$ for all a, a', and hence, larger age groups (younger cohorts if N is increasing) will have smaller per capita consumption – $c_t^a < c_t^{a'}$ for $a < a'$.

3.3   Procyclical fertility and endogenous oscillations

In this section we study the properties of the solution to the Planner's Problem outlined above. In particular, we characterize how the policy functions from this problem depend on both the current shock and the initial state.

To gain some intuition about the working of the model, notice that if $\eta = 1 - \sigma$, then N does not enter the period utility function except in aggregate consumption, and hence, if people are productive for only one period ($w^a = 0$, $a \neq 1$), N plays exactly the same role in this model as k does in a stochastic Ak model.[11] Similar to Ak models, without exogenous growth (i.e., $\gamma = 1$), aggregate consumption, C, grows at the same rate as N. However, in the absence of shocks, per capita consumption is constant. There is one important twist to the Ak-analogy. This is that, at least in the case where child-rearing is modeled as a time cost, the cost of the investment good is also stochastic. In that case, since $\theta_t = bw_t^1$, periods when productivity is high are also periods when children – the analog of the investment good in the Ak model – are expensive. Other than that, the analogy is very close.

We now derive comparative statics of current fertility with respect to productivity shocks and last period's fertility. To do this, we first simplify the problem to one with only one state variable. We then take first-order conditions and analyze comparative statics across steady states/balanced growth paths therein.

Denote by $V(N^1, N^2, N^3; s)$ the maximized value obtained in the problem $P(1, \widehat{\beta}; \{N_0^a\}, s_0)$ when the initial conditions are $(N_0^1, N_0^2, N_0^3) = (N^1, N^2, N^3)$ and $s_0 = s$. Because of our assumptions on the functional forms for the utility function, it is straightforward to show that the value function is homogeneous of degree η in (N^1, N^2, N^3), i.e., $V(\lambda N^1, \lambda N^2, \lambda N^3; s) = \lambda^\eta V(N^1, N^2, N^3; s)$. The problem $P(1, \widehat{\beta}; \{N_0^a\}, s_0)$ is therefore a standard, stationary dynamic program as long as s is first-order Markov. Because of this result, we can characterize the solution through Bellman's Equation.

Under the additional assumptions that $\eta = 1 - \sigma$, and $\phi = 1$, as discussed in Section 3.2, $C^a = C^{a'}$ and V satisfies:

(7) $\quad V(N^1, N^2, N^3; s) \equiv \max_{N^{a'}, C, N^b} \left[T \frac{C^{1-\sigma}}{1-\sigma} + \beta E \left[V(N^{1'}, N^{2'}, N^{3'}; s') | s \right] \right]$

---

[11] See Jones and Manuelli (1990) and Rebelo (1991), seminal papers on this model.

subject to:

$$TC + \theta N^b \le s \sum_{a=1}^{2} w^a N^a;$$
$$N^{a\prime} = N^{a-1}, \text{ for } a = 2, 3;$$
$$N^{1\prime} = N^b.$$

As can be seen from this, since $w^3 = 0$ by assumption, we have that $V(N^1, N^2, N^3; s) = V(N^1, N^2, N^{3*}; s)$ for any N^3, N^{3*}. Assuming further that shocks are *i.i.d.* and using the homogeneity property of V, the Bellman Equation in (7) is equivalent to one where fertility per young person, $n' = N^b/N^1$, is the choice variable and last period's fertility per person, the current stock of young to old workers $n = \frac{N^1}{N^2}$ is the state variable. That is, let $\widehat{V}(n; s) = V(N^1/N^2, 1; s)/T$, then (7) becomes:

$$(8) \quad \widehat{V}(n; s) \equiv \max_{n'} \left[\left(\frac{s\,[w^1 n + w^2] - \theta(s) n' n}{T} \right)^{1-\sigma} / (1 - \sigma) + \beta n^{1-\sigma} E\left[\widehat{V}(n'; s') \right] \right].$$

Taking the first-order condition with respect to n' and rearranging gives:

$$(9) \quad LHS(n') \equiv \frac{\theta(s)}{TE\widehat{V}_1(n', s')} = \left(\frac{s\left[w^1 + \frac{w^2}{n}\right] - \theta(s) n'}{T} \right)^{\sigma} \beta \equiv RHS(n').$$

$LHS(n')$ is increasing in n', with $LHS(0) = 0$ if $E\widehat{V}_1(0, s) = \infty$. Also, $RHS(n')$ is decreasing in n' with a positive intercept at $n' = 0$. Thus, there is a unique solution.

To see the behavior of n' as a function of the shock, we consider the two extreme cases, $\theta(s) = \theta$, a goods cost, and a time cost, $\theta(s) = bsw^1$. In the first case, $\theta(s) = \theta$, RHS shifts up when s goes up while LHS is unchanged. Thus, n' is increasing in s – fertility is procyclical. This is a pure income effect which is larger, the larger σ, i.e. the desire to smooth consumption. Also, it follows that n' is linear in s. On the other hand, when $\theta(s) = bsw^1$, both LHS and RHS shift up when s increases. The effect on RHS is still the pure income effect, which tends to increase fertility and is increasing in σ, s^{σ}. The effect on LHS is the substitution effect because when s is high, fertility is temporarily more expensive which tends to decrease fertility. This effect is linear in s. If $\sigma > 1$, the income effect dominates, i.e. RHS shifts up by more than LHS. Thus, again n' is increasing in s – fertility is procyclical. If, $\sigma < 1$ the substitution effect dominates – fertility is countercyclical.

To see how current fertility (per young adult), n', depends on past fertility, n, notice that RHS shifts down when the current state, n, increases while LHS is independent of n. Thus n' decreases when n increases, generating cycles. This crucially depends on $w^2 > 0$. If there is only one period of working life (as in standard Barro-Becker models), this effect is zero.[12]

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[12]Note that since the number of births *per woman* (CTFR) is $n^b = n/0.5$, all these comparative statics go through without change.

We summarize these results in the following proposition:

Proposition 1. *Current fertility, $n'(n,s)$, is*

1. a. *procyclical if $\theta(w^1) = \theta$, or if $\theta(w^1) = bsw^1$ and $\sigma > 1$;*
   b. *countercyclical if $\theta(w^1) = bsw^1$ and $\sigma < 1$;*
2. a. *independent of last period's fertility, n, if $w^2 = 0$;*
   b. *decreasing in last period's fertility, n, if $w^2 > 0$.*

Thus, if $w^2 > 0$ the model generates endogenous cycles, triggered by productivity shocks.

The intuition for why fertility is procyclical in this model is similar to that in many growth models. Here, fertility plays the role of an investment good and the usual consumption smoothing logic implies that when the shock is high, investment should be high so as to offset the effects of future shocks on consumption. This argument is direct when the cost of children is a goods cost. It is tempered when the cost is a time cost by a second effect. This is that the cost of the investment good is also higher than average when the shock is high. Thus, whether or not it is a good idea to invest in those periods depends on how strong the desire is to smooth consumption. When this force is strong – σ is large – the consumption smoothing effect is large relative to the cost effect and fertility is procyclical. Thus, the more important the time cost in raising children, the more procyclical is the cost of children itself. That is, whenever productivity is low, the cost of children is also low and vice versa. This dampens the procyclicality of fertility and, indeed, when the desire to smooth consumption is very low, fertility actually is countercyclical.

Some intuition for the fertility cycles in point 2 of the proposition can also be obtained by analogy with growth models. Here, what we find is a source of endogenous cycles. The relevant analogy from capital theory here is to consider a model in which depreciation is not constant. Here, we have an extreme version. Capital (i.e., people) that is built in period $t-1$ has full productive capacity in period t and period $t+1$ – there is no depreciation between periods t and $t+1$ – but has zero productive capacity in period $t+2$. Thus, age-specific depreciation rates here would be $\delta_1 = 0$ and $\delta_2 = 1$ – no depreciation after one period, full depreciation after two. In a situation like this, when $n = N^1/N^2$ is lower than usual, the planner expects next periods depreciation rate to be higher than usual (because N^2 is relatively high). Because of this, to smooth consumption, current investment – $n' = N^b/N^1$ – will be relatively high. Thus, n' is high when n is low. When there is only one period of working life, the depreciation rate is always 100 percent and hence this effect is not present. Because of this, fertility is independent of last period's fertility. The example with one period working life, $w_t^2 = 0$, is interesting because it corresponds most closely to that of the

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original Barro-Becker model while the model with more than one period productive life is more realistic.[13]

Using the decentralization with (possibly negative) state-contingent transfers outlined at the end of Section 3.1, some more intuition can be gained. First, fertility, in most cases, is procyclical. To understand the intuition note that the parent cares about a composite of per capita consumption of descendants and number of descendants, $\frac{n_t^\eta (c_{t+1}^1)^{1-\sigma}}{1-\sigma}$. The way to increase the future composite is to increase either the number of children, chosen today, or per capita consumption, chosen out of resources tomorrow. If today income is lower than average, the way to smooth own consumption today with the composite is to have fewer children today, given that resources tomorrow are expected to be higher (i.e. expected to be on average). This is the intuition for the baby bust in the 1930s.

The second question is: why would young workers have more children if they have relatively few siblings? The intuition for this result comes from the way transfers from age 2 workers to their children change when past fertility changes. Suppose for some reason people chose low fertility last period. Suppose this period, income is back on trend/average. Then, to equalize marginal utility from own consumption and the composite of children and per capita consumption of children, old workers who are no longer fertile decide to give a relatively high transfer today so that children choose high per capita consumption. Since this relatively high transfer is a pure (temporary) income effect to the age 1 children, they will have many children today. This is independent of the type of costs of children because the transfer does not increase the opportunity cost of children while a productivity shock does in the time cost case.

Thus, according to our theory, while the baby bust in the 1930s may be explained by the Great Depression, the baby boom in the 1950s is first and foremost a response to low fertility in the past.

4 Quantitative results

In this section, we use the facts about the time paths of productivity and fertility in the U.S. to perform quantitative experiments on the model. To do this, we calibrate parameters to selected moments of our data. We then use this model to explore two kinds of questions. First, what does the calibrated model say about the size of responses to shocks to productivity – e.g., what is the elasticity of fertility with respect to a productivity shock? In keeping with the theoretical results of the previous section we study both the current response to a shock and also the lagged response one generation later due to the misalignment of the age structure

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[13]In Section 5.2 we add a third period of working life to show that results, though qualitatively more complicated, are very similar quantitatively.

of the workforce. Second, based on the estimated policy functions from the calibrated model we study the predicted response to productivity shocks like those seen in U.S. history – e.g., the Great Depression and the American Civil War in the South.

We find that the answers to these quantitative questions also depends on the nature of the costs of children. Because of this, we give results for two alternative specifications. These are: (1) all costs are time costs ($\theta(s) = bw^1 s$); (2) all costs of raising children are goods costs ($\theta(s) = \theta$). As noted above, these two specifications are qualitatively different in that with a time cost, the costs of children are higher in productivity booms than in busts. Reality probably lies somewhere in-between these two extreme cases.[14]

We find that the quantitative responses in the model are economically significant in all cases. For example, the elasticity of fertility to a contemporaneous productivity shock lies between 1 and 1.7,[15] while the elasticity one period later lies between 0.94 and 1.6. Standard recessions will have rather modest effects on fertility, however. Large recessions, such as the Great Depression or the Civil War, on the other hand, have important and long lasting effects.

We therefore turn to the historical record of the United States and study the predicted response of fertility to the productivity booms and busts over the 20th century and the second half of the 19th century. To do this, we must first construct a series of shocks to feed into the model. It is not obvious how to do this realistically. On the one hand, in the model, it is assumed that the shock for the current period is realized at the beginning of the period. Effectively, this means that individuals know, at the beginning of the period, what the sequence of annual shocks will be over the next 20 years. Even with highly correlated shocks at annual frequency, this assumption seems extreme at best. Another alternative is to decrease the length of a period. This necessarily increases the size of the state space. For example, even decreasing the period length to 10 years and maintaining the *i.i.d.* assumption increases the size of the state space from 2 dimensional to 5.[16] Relaxing the *i.i.d.* assumption which might be required for 10- as opposed to 20-year periods, adds another state variable. In addition, one would have to address timing of births (between age 20-30 versus 30-40)

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[14]Indeed, the time cost may have become more relevant over time as women entered the labor force. Since women tend to be the parent taking care of children, the relevant opportunity cost is only procyclical if the latter are actually working whenever they are not busy raising children.

[15]As will be seen below, the model response to a shock is smaller when the cost is in terms of time than when it is in terms of goods. Thus, 'between 1 and 1.7' corresponds to 1 with a time cost and 1.7 with a goods cost.

[16]The state space is three-dimensional in our problem: N^1, N^2 and s. Using the homogeneity results, this can be reduced to two state variables, $N = N^1/N^2$ and s. With 10-year periods (and age-groups) the problem becomes six-dimensional: N_b^2, $N^{1,1}$, $N^{1,2}$, $N^{2,1}$, $N^{2,2}$ and s, which using the same methods can be reduced to a five-dimensional problem.

more seriously. This is not an easy problem (see Caucutt et al. (2002), Doepke et al. (2007) and Sommer (2010) for examples). Thus, there are technical difficulties with following the strategy of decreasing the period length.

Because of this, we present results for several alternative methods for constructing the relevant series of productivity shocks for TFP and LP for the 20th century. First, since the data shows that over the period 1940 to 1980 at least 60 percent of all births are to women age 20 to 30,[17] we assume that women have all their children between age 21 and 30. As our baseline experiment, we therefore assume that the relevant productivity shock for them is the average one in the data for that 10 year period. We conduct sensitivity analysis on this specification in Section 4.3.2. We also have to choose which productivity measure to use, total factor productivity (TFP) or labor productivity (LP). We use TFP in our baseline case and discuss LP in Section 4.3.3.

In our baseline experiment, a shock to productivity (or labor income) the size of the Great Depression gives rise to a contemporary baby bust that accounts for 56 to 91 percent of the reduction in TFR seen in the data. Further, the model prediction of the lagged response to such a shock is a baby boom. Combined with the productivity boom in the 1950s, the predicted size of the baby boom lies between 64 and 113 percent of the actual size of the baby boom observed in the TFR data.

For the Civil War, additional data issues arise, some of which are related to data availability. For instance, there is no labor productivity series going far enough back. Instead, we have to use GDP per capita estimates. GDP may have decreased due to factors unrelated to productivity which are absent from our model. In particular, the large death rate for men of military age in the early 1860s and the large decrease in labor supply by blacks due to emancipation. We therefore consider two extreme assumptions, a large and a small productivity shock case.

4.1 Parameterization

Preference Parameters: Throughout, we assume that $\eta = 1 - \sigma$ and set $\sigma = 3$ following Jones and Schoonbroodt (2010) among others.[18] The discount factor is set to $\beta = 0.96^{20}$ to match an annual interest rate of about 4 percent.

Wage and Productivity Parameters: From the model, wages are given by $w_t^a = s_t \gamma^t w^a$, where w^a is the base wage (in period 0) for workers of age $a = 1, 2$; γ is the trend productivity growth rate for a 20-year period and s_t is the productivity shock which we assume to be

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[17]See Vital Statistics of the United States, Table 1-7.

[18]e.g., Mateos-Planas (2002) and Scholz and Seshadri (2009) also use a value of $\sigma = 3$.

i.i.d. over time.[19] Further, for computational reasons, it is convenient to assume a functional form for the distribution of productivity shocks, s_t. We assume that $\ln \hat{s}_t \sim N(0, \sigma_s^2)$ where $\hat{s}_t = s_t e^{\sigma_s^2/2}$ so $E(s_t) = 1$. Thus, the parameter values to be determined are w^1, w^2, γ and σ_s.

We normalize $w^1 = 1$ and choose $w^2 = 1.25$. This is in line with life-cycle earnings profiles from Hansen (1993) and Huggett (1996). The growth rate of productivity, γ, and the standard deviation of productivity shocks, σ_s, are calculated from the TFP series plotted in Figure 1. First, we compute the linear trend in productivity by running the following ordinary least-squares regression on annual data:

$$(10) \qquad \ln TFP_t = \alpha_0 + \alpha_1 t + \epsilon_t.$$

We find that $\alpha_1 = 0.0159$ and therefore set $\gamma = (e^{\alpha_1})^{20} = 1.0161^{20}$. That is, productivity grows at an average of 1.61 percent per year over the 20th century.[20]

To pin down the value of σ_s several steps are required. First, in our baseline experiment, we use the 10 most fertile years of each cohort within a dynasty, namely age 20 to 30, to determine the productivity shock this cohort's fertility choice is affected by. Thus, the shock we are interested in is $\ln \hat{s}_t = \ln \left(\sum_{t=1}^{10} e^{\epsilon_t} \right) - \mu$ where $\mu = E(\ln(\sum_{t=1}^{10} e^{\epsilon_t}))$. To approximate its standard deviation, σ_s, we assume that ϵ_t follows an AR1 process and estimate

$$(11) \qquad \epsilon_t = \rho_0 + \rho_1 \epsilon_{t-1} + \nu_t,$$

where $\nu_t \sim N(0, \sigma_\nu)$, simulate a long series of $\{\epsilon_t\}$, compute a series $\{\ln s_t\}$ and calculate its standard deviation to get $\sigma_s = 0.07$. Of course, the series, $\ln s_t$, so constructed is not exactly normally distributed. However, for all simulations, the Kolmogorov-Smirnov test produces a *p*-value of about 0.4. This means that, at any reasonable significance level, the test would accept the null hypothesis that the distribution of $\ln s_t$ is normal and hence this should provide a good approximation.

Costs of Children: Given preference and productivity parameters, we then calibrate child costs, $\theta(\cdot)$, to match an annual population growth rate of 0.645 percent per year (see Haines, 1994, Table 1). In the model, population growth corresponds to $\frac{N_b + N^1 + N^2}{N^1 + N^2 + N^3}$. In a steady state with no uncertainty, this is given by the level of fertility choice (per person), n^{ss},

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[19]This is a reasonable approximation for long movements in labor productivity across generations, which within a dynasty are 20 years apart.

[20]An alternative detrending method would be to use a Hodrick-Prescott filter, instead. However, it is not clear what value to use for the smoothing parameter since we are interested in long fluctuations, rather than quarter-to-quarter or annual deviations from trend. In any case, for low enough smoothing parameter resulting productivity shocks would typically be smaller. Assuming that the growth rate of productivity is constant in the model but using HP productivity shocks would result in smaller fertility responses in the experiments at the end of this and the next section.

satisfying $n'(n^{ss}, 1) = n^{ss}$. Thus, the target is $n^{ss} = 1.00645^{20}$. This corresponds to a steady state CTFR level of $n^{ss}/0.5 = 2.27$ children per woman. In the goods cost case, we find $\theta = 0.1932$, while with time costs we find $b = 0.1927$. Since the wage for young workers is normalized to 1, these costs imply that it takes about 20 percent of a young worker's income or time to produce a child. This means that a two person household can at most have ten children.

Summary: These parameters are summarized in Table 2.[21]

Table 2: Parameter Values, Baseline

Parameter	σ	β	w^1	w^2	γ	σ_s	θ (goods cost)	b (time cost)
Value	3.00	0.96^{20}	1.00	1.25	1.0161^{20}	0.07	0.1932	0.1927

4.2 Model impulse responses

Given the parameter values from the previous section we calculate the decision rules from the model. These can then be used to estimate the model responses to different size productivity shocks. The results are summarized in Table 3. The rows of the table correspond to the two alternative types of cost structures for children, goods and time. The first two columns report the elasticities (at the steady state) of (a) current and (b) lagged fertility to productivity shocks. The next two columns give the (c) contemporaneous and (d) lagged change in fertility levels that the model predicts will result from a one percent increase in productivity. Column (e) recalls the steady state level of completed fertility (CTFR).

Table 3: Impulse Response (in percent and levels), Baseline

Cost Type	% Deviations		CTFR Levels		
	Initial (a)	Lagged (b)	Initial (c)	Lagged (d)	Steady St. (e)
Goods	1.741	-1.561	2.314	2.239	2.274
Time	1.039	-0.940	2.298	2.253	2.274

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[21]We provide sensitivity to the parameter values in the Supplementary Appendix B.5, Table B.11.

As can be seen from this table, in the goods cost case the fertility response to a 1-percent productivity shock generates a 1.74 percent contemporaneous increase in fertility (column (a)) with a lagged decrease of 1.56 percent one period later (column (b)). In the time cost case, the corresponding percentage changes are a 1.04 percent contemporaneous increase and a 0.94 decrease one period later.

These magnitudes can be compared to the regression results shown in Table 1. In particular, column (a) is to be compared with λ_1 and column (b) with λ_2. Thus, if the model fit the data perfectly we should see 0.83 in column (a), and -0.84 in column (b). Although the model quantities are higher, they are remarkably similar in the time cost case.

Comparing the two cases, the initial effect is 70 percent larger in the goods cost case than it is in the time cost case. The reason for this difference is that when child costs are primarily in terms of time there are two offsetting effects. First, when times are good there is a natural tendency to save for the future to equalize marginal utilities. Here, the only way to do this is by increasing family size. On the other hand, since the cost of raising children is positively related to the wage, when times are good, children are relatively more expensive – a force in the opposite direction. When the costs of children are in terms of goods, this second effect is not at work and hence, the overall effect is larger in this case. As can be seen in the table, as the theory predicts, the first effect dominates if the intertemporal elasticity of substitution is low enough ($\sigma > 1$, see Proposition 1).

Further, since in both cases, the ratio of the initial response to the lagged response is about -90 percent, the half-life (in absolute value) of the effect is 7 periods (or generations) in both cases. In particular, the effect 7 periods later is a decrease in fertility of 0.8 percent in the goods cost case and 0.5 percent in the time cost case (i.e., half of the initial responses).

To get a sense about the size of these effects note that a one-percent decrease in productivity for a 10-year period roughly corresponds to a recession where GDP is 5 percent below trend for two years. This would decrease fertility by about 0.02 to 0.04 children per woman with a subsequent baby boom of similar size. Thus, the quantitative effects on (completed) fertility of a normal sized recession would be quite modest. As we shall see below, the model predicts that a recession the size of the Great Depression gives a much larger response.

4.3 The U.S. Great Depression

In this section we use the historical record of the actual series of productivity shocks in the U.S. to study the predicted response of fertility to the productivity busts and booms that have occurred over the last 100 years.

4.3.1 Baseline

In our baseline case, we assume that the relevant productivity shock is the average one in the data for that 10 year period. We conduct sensitivity analysis on this specification in Section 4.3.2. Further, we use TFP in the baseline case and discuss LP in Section 4.3.3.

To focus on the effects of the Great Depression, we start the model simulation so that fertility decisions are made in the 1910s, 1930s, 1950s, 1970s and 1990s.[22] We assume that the economy is at its steady state age-distribution.

The relevant series of productivity shocks, s_t, is shown in Table 4. The results are shown in Figure 4. With a goods cost, the model predicts that fertility is 19.1 percent below trend during the 1930s; with a time cost the prediction is 11.8 percent below trend. In the data, TFR is actually 21 percent below trend in the 1930s. Thus, the model captures a significant fraction of this movement in either case. This is primarily due to the large, negative shock of 12.4 percent during the Great Depression of the 1930s – the dynasty's young-to-old worker ratio is almost at its steady state value in the 1930s and hence effects from this are minimal.

Table 4: Productivity shocks (in percent), Baseline

Decade	1910s	1930s	1950s	1970s	1990s
Productivity Shock	-1.8	-12.43	6.89	3.90	-8.97

As noted above, there is also a lagged effect in the model from the Great Depression due to the misalignment of the age structure that results from low fertility during the 1930s. This implies, a baby boom for the next cohort in this dynasty which takes place in the 1950s. With a goods cost, the model prediction is an increase in fertility above trend of 34.9 percent, while with a time cost, the corresponding model prediction is 19.7 percent above trend. These model predictions result from a combination of the fertility response to the low young-to-old worker ratio due to the baby bust and the 8.4 percent productivity boom in the 1950s. For the purposes of comparison, the peak of the baby boom in the TFR data is 31 percent above trend in the late 1950s.

Hence, the model accounts for 56 to 91 percent of the baby bust and for 64 to 113 percent of the baby boom in TFR. In subsequent periods, TFR in the model keeps fluctuating – 11.3

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[22]This dynasty is the most affected by the Great Depression. Note that, in our model, TFR and CTFR are equivalent since one period exactly corresponds to fertile ages. We therefore choose to plot our model simulations against the TFR measure of fertility. In the Supplementary Appendix B.1, we present an alternative mapping between model and data by constructing overlapping dynasties in order to distinguish between a measure of CTFR and TFR.

Figure 4: Percent Deviations in TFR, Baseline
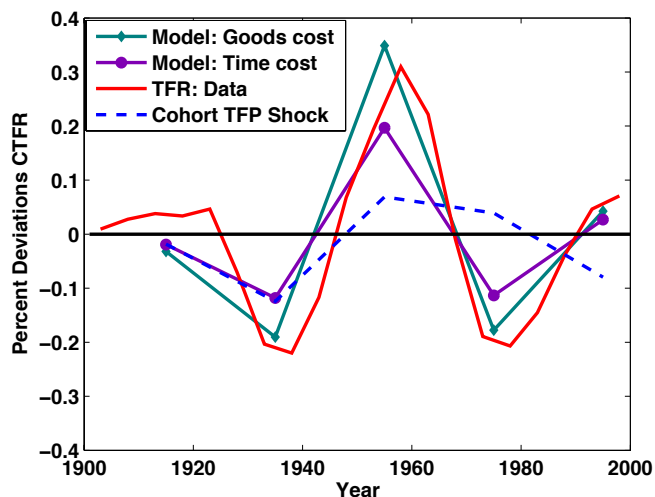
to 17.8 percent below trend in the late 1970s (due to the prior baby boom but dampened by the 3.9 percent increase in TFP). The corresponding movements in the data on TFR is 20.7 percent below trend. The increase in fertility in the 1990s of 7.1 percent above trend in the data is also partly captured by the model which predicts a 2.7 to 4.3 percent increase.

4.3.2 Varying the age range of productivity shocks

One problem with the assumption that the only relevant shock for the dynasty is when their youngest cohort is age 20 to 30 is that it implicitly assumes that the shock lasts for the entire period of 20 years. In this section, we describe model results when the shock is set to the average productivity shock over the period where the cohort is age 20 to 40.

Figure 5(a) shows productivity shocks for this experiment compared to the baseline. Overall shocks are smoother and smaller because some busts and booms are smoothed out due to the longer time period. Therefore, the baby bust is smaller than in the baseline model as can be seen in Figure 5(b). Because of the smaller baby bust in the 1930s and smaller shocks, the baby boom is also smaller and so are subsequent fluctuations.

The problem with this alternative is that it implicitly assumes that fertility is uniform between age 20 and 40 when in reality most of them had completed their fertility by age 30.

Since this alternative is at the opposite extreme of the baseline, the two experiments together should give us reasonable bounds on how much of the fluctuations in fertility over the 20th century in the U.S. the model can account for.[23]

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[23]In Supplementary Appendix B.3 we perform an experiment where TFP shocks are weighted by average age-specific fertility rates. This is an intermediate case between the baseline and the present experiment.

Figure 5: Percent Deviations in TFR, Age Range and Labor Productivity
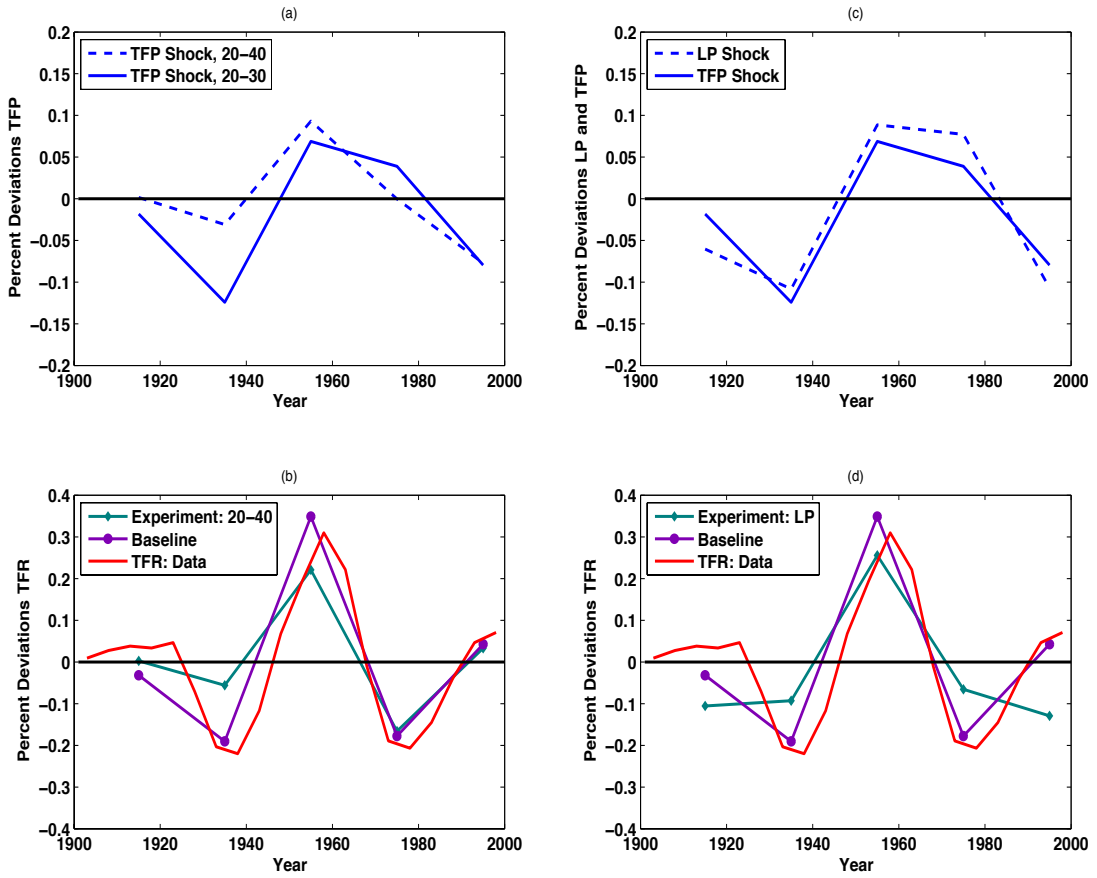
Note: Tables B.3 to B.5 in Supplementary Appendix B.2 show the exact sizes of shocks, as well as TFR responses for the two experiments compared to the baseline.

4.3.3 Labor productivity versus total factor productivity

Since there is no capital in our model, it is not clear wether we should use labor productivity (LP) or total factor productivity (TFP) to infer productivity shocks. The argument for using TFP is that if capital were included in the model, fertility would respond to TFP alongside investment. The argument for using LP is that most people don't own much capital (besides durable goods and houses) so that their fertility response is to changes in LP.

Here we perform the same experiment as in the baseline case but use deviations from trend in LP instead of TFP. Again, we fit a linear trend through the LP data. The resulting productivity shocks for each cohort are compared to the baseline in Figure 5(c). As can be seen in the figure, there are two main differences between the LP compared to the TFP series of shocks. The very first cohort making fertility decisions in the 1910s experience a significant negative shock. Therefore, as shown in Figure 5(d), despite the similar size of the shock in the 1930s, fertility decreases less in the LP case than in the TFP case. This is because of the mitigating effect of the low young-to-old worker ration due to low fertility in the 1910s. Subsequently, the baby boom is of similar size as the baseline due to the larger LP shock in the 1950s and 1960s compared to TFP. Overall, however, the results are quite similar.

Note that the growth rate of LP in the data is 2.23 percent per year on average, while it is only 1.61 percent per year for TFP. Also, the estimated standard deviation of shocks is 0.08 in most simulations while it is only 0.07 for TFP. In the experiment above, we held these two parameters constant so that we perform only one change at a time and this may have some effect on the outcomes.

4.4 The American Civil War

A second example of a major economic collapse in the U.S. is that of the South after the Civil War. While economic historians disagree about the size, duration and causes of this depression, they all agree that it was a significant downturn, particularly during the period from 1860 to 1870.

Figure 6 shows fertility, as measured by Crude Birth Rates (CBR, number of births per 1000 population) from Hacker (1999), in both the North and the South over this period.

As can be seen from the figure, there was a sizable reduction in fertility in both regions with that in the South both larger and longer lasting. It also shows that by the mid 1870s, fertility had recovered in the South and was even above trend. Thus, this episode is similar to the baby bust/baby boom cycle that was seen in the country as a whole over the period from 1930 to 1960 as discussed above. There are differences however.

Figure 6: CBR During The American Civil War
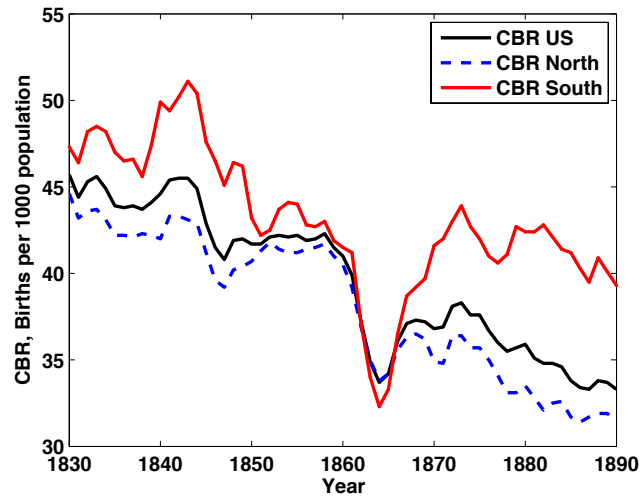
As an example, the size of the post war boom in this episode is much smaller than the original bust in the 1860s. This is in contrast with what we saw in the Great Depression. Similarly, the timing of the subsequent boom is quite different in this episode than it was during the Great Depression. Here, the boom begins around 1870, only 10 years after the beginning of the Civil War while in the later episode it was a full 20 years from the bottom of the 1930s baby bust to the baby boom.

There are also differences in circumstances that may account for some part of these differences. First, it is not clear what part of the reduction in fertility that occurred between 1860 and 1865 should be attributed to the effects of economic difficulties instead of a wartime effect. As can be seen in the figure, there was also a large, temporary, reduction in fertility in the North over this period even though best estimates of the size of the downturn in the North are quite small.[24] Thus, it is likely that at least part of this reduction in the South is due to the war and not the economic downturn. Second, in contrast to the Great Depression, there was a significant loss of life in the South during the Civil War and this was concentrated among prime age males. Indeed, estimates are that up to 20 percent of all males between the ages of 13 and 43 in the South were killed during the war. Third, emancipation led to a major, and permanent, change in the workings of the labor market in the South. Estimates are that labor supply of freedmen fell by up to 37 percent.

All of these considerations should have an impact on how we should view this episode

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[24]Similar temporary fertility reductions during periods of war can also be seen in other examples – e.g., in Germany, the U.K., France, Belgium and Italy during WWI (see Section 6.1 for data sources) and in Italy, Belgium and France during WWII.

through the lens of the model we have developed above. In what follows, we conduct a quantitative assessment of the model using this episode while trying to make adjustments for the differences in circumstances between this event and the Great Depression.

4.4.1 The size of the downturn

As noted, there is some disagreement about the exact size and composition of the economic downturn that took place in the South beginning in 1861. To use the model to estimate the impact of the downturn on fertility, we need a measure of the 'shock' that took place. Ursua (n.d.) has constructed data on per capita GDP for the South over the period from 1861 to 1869. According to his calculations, per capita GDP fell 32 percent from 1861 to 1867 before starting to recover. His estimates imply that on average over that 10 year period, GDP per capita was 21 percent below its 1860 level. Engerman (1966) estimated that per capita GDP was 39 percent below its 1860 level in 1870 and still 21 percent below in 1880. Using Census of Manufacturing data, Ransom and Sutch (2001) argue that by 1870 both total and per worker output levels had regained their 1860 levels by 1870 in manufacturing in large parts of the South (Ransom and Sutch, 2001, Tables 3.1 and 3.2). They use this evidence to argue that a large part of the reduction in agricultural output, particularly on plantations, after the war (and perhaps during it) was due to a permanent change in per worker hours among emancipated slaves – a reduction of between 28 and 37 percent according to their estimates (Ransom and Sutch, 2001, Chapter 3) – along with other institutional changes in labor markets.[25]

Thus, deciding on the size of the 'shock' to use in the model is not straightforward, either for the 1860s, or for the subsequent decades. At one extreme, using Ursua's findings followed by Engerman would lead us to shocks on the order of -21 percent for the period from 1860 to 1869 and -10 percent from 1880 to 1889[26] – the large shock case. At the opposite extreme, we use Ursua's decrease in GDP plus 10 percent to account the permanent decrease in labor supply by blacks and adopt the Ransom and Sutch measure of productivity in manufacturing – productivity is back on track beginning in 1870. This gives us a shock of -11 percent for the 1860s, and back on trend thereafter – the small shock case.

Since it is not obvious which of these is more accurate, we will conduct two versions of our quantitative exercise, one for each extreme case.

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[25]Goldin and Lewis (1975) also use this as an upper bound to compute the cost of the Civil War in terms of consumption.

[26]Engerman stops in 1880 where he finds that output is still 20% below its 1860 level. We assume that output is back to its 1860 level by 1890 and hence use the average – -10% – for the 1880 to 1889 period.

4.4.2 The direct impact on fertility from the war

A second problem hinted at above is that there is likely a direct impact of the war on fertility over and above any effect due to the 'shock' to productivity and income (e.g., absence of men of fertile age, etc.). Since this is not included in the model, we will attempt to control for this effect by using the fertility reduction in the North. Since the recession in the North was small and short at best, we attribute the baby bust in the North exclusively to the war effect. Thus, the relevant reduction in fertility in the South is not the 9.8 percent fall seen in the data (averaging over the 1860s, after detrending), but rather 7.3 percent – the reduction in the South minus that in the North.

4.4.3 Wartime deaths

As noted above, there was significant loss of life in the South as a direct consequence of the war. This was concentrated in males between age 13 and 43, prime candidates for parenthood. As noted, approximately 20 percent of the men in this age group were killed during the war. We can partially account for this in the model by adjusting the starting value of N^1, the number of young working adults. There is a difficulty however in that, in the model, we don't differentiate between men and women. Thus, one could either use the fact above by reducing N^1 by 20 percent, or by 10 percent (the fraction of the overall population in this age group that died). Below we report results using a reduction in N^1/N^2 of 10 percent and briefly refer to the case where this number is higher.

4.4.4 Some illustrative calculations

Based on the considerations above, we compute the model response in fertility for four cases – big and small shock and goods versus time costs. To do this, we use the same calibration as in Section 4.1, Table 2. For all four experiments, we assume that our state variable, N^1/N^2, is on average 10 percent below trend in the 1860s due to deaths.

Percent deviations in fertility (to which we compare model predictions) as well as percent deviations in productivity for big and small shock are reported in Table 5 under "Adjusted Data" followed by model predictions for percent deviations in fertility in the four cases.

As can be seen, the overall pattern of model predictions and data agree – a fertility reduction during the 1860s and a subsequent boom. However, the model predictions are typically larger than what is seen in the data. The most extreme cases come from the large shock in the goods cost case on the high end and the small shock in the time cost case on the low end. Both the baby bust in the 1860s and the baby boom in the 1880s observed in the data falls into the range of our four cases, though the boom is very close to the low end

of the range.[27,28]

Table 5: The American Civil War: Model versus Data

	Data			Model			
	Productivity		Fertility	Goods		Time	
	Large	Small	(S-N)	Large	Small	Large	Small
1860-69	-20.93	-10.93	-7.33	-28.55	-10.02	-14.38	-2.24
1880-89	-10.00	0.00	2.45	15.35	10.15	3.75	2.09

5   Sensitivity to changes in model features

In this section, we discuss the sensitivity of our results to changes that involve altering some of the basic features of the model. Here we study the robustness of our results both to including capital and having more (i.e., shorter) time periods and hence, more active generations. We find that our basic results are robust to these two qualitative changes to the model.

5.1   The effects of including physical capital

In this section we show that including physical capital as an alternative investment into the model does not change the contemporaneous elasticity to productivity shocks for reasonable calibrations. To do this while keeping the size of the parameter space small, we consider the special case where $w^2 = 0$ and $\theta(s^t) = \theta$ (i.e. the goods cost case) and compare two specifications of the production function for the dynasty:

1. $F(K, N^1) = w^1 N^1$, the simplified version of the baseline case without capital;

2. $F(K, N^1) = K^\alpha (N^1)^{1-\alpha}$, the case including capital.

Case 1 is a special case of the model laid out in Section 3 where $w^2 = 0$, and, as such, the results of that section apply automatically. The important equation describing the elasticity

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[27]The average baby boom in the 1880s comes from a boom of the order of 4.5 percent in the first half of the decade and 0.4 percent in the second half. Moreover it doesn't take the 1870s baby boom into account, 4.7 and 2.3 percent in the first and second half, respectively. It is therefore not entirely clear which magnitude the model predictions ought to be compared to.

[28]We have also experimented with different assumptions on the 1860s value of N^1/N^2. Assuming that it was 15 instead of 10 percent below trend leads to model predictions between 5 and 7 percent smaller baby busts and smaller baby booms. This reverses some of the signs of the predictions.

of fertility with respect to current productivity shocks is:

$$\frac{n'(s) - E(n'(s))}{E(n'(s))} = \frac{\psi s - \psi}{\psi} = s - 1$$

where $\psi = \left(\frac{w}{\theta + \frac{\theta}{\beta(1-\sigma)E[V(s')]}}\right)$ is independent on s.

When capital is included (case 2), the problem the dynasty solves is given by:

$$P(\gamma, \beta; \{N_0^a\}, K_0, s_0) \qquad \max \qquad U_0^T = E\left[\sum_{t=0}^{\infty} (\beta\phi)^t \left[\sum_{a=1}^{T} \phi^{1-a} g(N_t^a(s^{t-a})) u(c_t^a(s^t))\right] | s_0\right]$$

subject to:

$$\sum_{a=1}^{3} N_t^a(s^{t-a}) c_t^a(s^t) + \theta N_t^b(s^t) + K_{t+1}(s^t) \leq s_t (K_t(s^{t-1}))^\alpha (\gamma^t N_t^1(s^{t-1}))^{1-\alpha};$$
$$N_t^1(s^{t-1}) = N_{t-1}^b(s^{t-1}) \text{ is the number of births in period } t-1;$$
$$N_t^a(s^{t-a}) = N_{t-1}^{a-1}(s^{t-1-(a-1)}) \text{ for } a = 2, ..., T;$$
$$N_t^a(s^{t-a}) = 0 \text{ for } a > T;$$
$$N_0^a \text{ given, } a = 1, ..., T.$$

where $s^t = (s_0, s_1, ..., s_t)$ is the history of shocks up to and including period t.

Under the assumptions that $\phi = 1$, $\eta + \sigma = 1$, that costs are in terms of goods, $\theta_t = \gamma^t \theta$, and shocks are *i.i.d.* the solution for fertility is given by:

$$(12) \qquad n'(s) = \frac{(\beta\gamma^{1-\sigma}(1-\sigma)Ev(k', s'))^{\frac{1}{\sigma}}}{\left((\gamma k' + \theta)^{\frac{1}{\sigma}} + (\gamma k' + \theta)(\beta\gamma^{1-\sigma}(1-\sigma)Ev(k', s'))^{\frac{1}{\sigma}}\right)} [sf(k) + (1 - \delta)k]$$

where $f(k) = F(K/N, 1)$.

Some useful intuition can be gained by examining the case where capital fully depreciates after one 20-year period, $\delta = 1$. In this case, since s' and k' are independent of s, we can write $n'(s) = \Psi s$, where Ψ independent of s, and hence:

$$\frac{n'(s) - E(n'(s))}{E(n'(s))} = \frac{\Psi s - \Psi}{\Psi} = s - 1$$

as above. Thus, the elasticity of fertility with respect to a productivity shocks is the same in the two models.

If $\delta < 1$, this is not the case, but under realistic assumptions about the annual rate of depreciation the effects are quantitatively similar. To see this, we use the same parameters as in the baseline, except $w^2 = 0$. Further, we follow Jones and Schoonbroodt (2010) and set $\alpha = 0.33$ and $\delta = 1 - (1 - 0.088)^{20}$. Finally, we calibrate $\theta = 0.0214$ to get an annual population growth rate of 0.65. The wage, w^1, is not fixed in this model but moves with the marginal product of labor. Therefore, the relevant magnitude to be compared to $\theta = 0.19$ in

the baseline model, where w^1 was normalized to $w^1 = 1$, is the cost of children as a fraction of one person's income, $\theta/w^1 = 0.1609$.

We find that, with these parameters, the fertility response to a 1 percent deviation in productivity decreases from 1.0 to 0.9938. Hence, introducing capital into the model with one period working life tends to dampen the effects of productivity shocks ever so slightly.[29]

5.2 Three Period Working Life (3PWL)

While the initial fertility response to productivity shocks is very similar whether $w^2 > 0$ or $w^2 = 0$, dampened oscillations only occur when $w^2 > 0$. Since this property of the model is important both qualitatively and quantitatively, in this subsection we study whether this feature is robust to more realistic decision periods. To do this, we add one more period of working life and show that similar dampened oscillations occur in this case. Thus, we assume that $w^3 > 0$. Since this increases the number of state variables by one, we abstract from productivity shocks thereby reducing the size of the state-space back to two. In the experiments, we take the initial fertility deviation in and around the 1930s as given and evaluate the oscillations that follow.

Under the assumptions that $\eta = 1 - \sigma$, and $\phi = 1$, as discussed in Section 3.2, $C^a = C^{a\prime}$, and the BE in (7) becomes:

$$(13) \qquad V(N^1, N^2, N^3) \equiv \max_{C, N^b} T \frac{C^{1-\sigma}}{1-\sigma} + \beta V(N^{1\prime}, N^{2\prime}, N^{3\prime})$$

subject to:

$$TC + \theta N^{b\prime} \leq \sum_{a=1}^{3} w^a N^a;$$
$$N^{a\prime} = N^{a-1}, \text{ for } a = 2, 3;$$
$$N^{1\prime} = N^b.$$

The relevant first-order condition here is:

$$(FOC) \qquad LHS(n') = \frac{\theta}{T \widehat{V}_1(n', 1, \frac{1}{n})} = \beta \left(\frac{w^1 + \frac{w^2}{n} + \frac{w^3}{nn^-} - \theta n'}{T} \right)^{\sigma} = RHS(n'),$$

where $n' = N^b/N^1$, $n = \frac{N^1}{N^2}$ and $n^- = \frac{N^2}{N^3}$.

Then, $LHS(n')$ is increasing in n', with $LHS(0) = 0$ if $E\widehat{V}_1(0, s) = \infty$. Also, $RHS(n')$ is decreasing in n' with a positive intercept at $n' = 0$. Thus, there is a unique solution.

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[29]One can show that in a model with stochastic ageing (useful to obtain reasonable working life lengths while keeping the state space small), the elasticity is larger (smaller) than 1 if the adult survival rate is larger (smaller) than $(1 - \delta)$. Since in the present model, survival in the workforce is zero, this is an extreme special case. Thus, introducing capital in general does not change the fertility response to productivity shocks very much.

As before, RHS shifts down when last period's fertility per capita, n, or fertility two periods ago, n^-, increase. Since LHS is independent of n^-, it follows that n' decreases when n^- increases and hence, there will necessarily be some oscillatory behavior. This is complicated however since LHS depends on n. Which way LHS shifts following an increase in n depends on the cross-partial derivative with respect to N^1 and N^3, V_{13}. In particular, if $V_{13} > 0$ then LHS shifts up when n increases and, hence, n' unambiguously decreases. However, if $V_{13} < 0$, then LHS shifts down when n increases and, hence, the overall effect on n' is ambiguous.

In what follows, we compare simulations of this model to simulations in the baseline as well as a non-stochastic version of the baseline model (i.e., a special case of the model presented in this section where $w^3 = 0$). This intermediate simulation allows us to disentangle how much of the difference comes from having a third period of working life and how much comes from ignoring productivity shocks.

We assume the model period is 15 years. Thus, people work from age 15 to age 60. We keep all the parameters the same as in the baseline, except for w^3 and the cost of children, θ. We set $w^3 = 1.125$. That is, 45-60 year old workers earn 25 percent less than workers age 30-45. Calibrated costs don't change much from the baseline ($\theta = 0.1932$ or $b = 0.1927$). Here, we find $\theta = 0.1946$ to match the steady state population growth rate of 0.65 percent per year as before. In the intermediate deterministic version with $w^3 = 0$, $\theta = 0.1901$. Incidentally, note that in all these simulations $V_{13} < 0$ so that the effects of last period's fertility on current fertility are ambiguous.

The simulation consists of off balanced growth path dynamics taking the fertility deviation in and around the 1930s from the baseline as given. That is, everything plotted until 1930 is exogenous here and we are interested in the baby boom and the potential for dampened oscillations thereafter. Note that in the deterministic version of the model, whether costs are in terms of time or goods is irrelevant – the only reason why the two cases would differ here is that we use different baby busts from the baseline results as initial conditions.

Figure 7 shows TFR for the goods cost case. As can be seen, magnitudes don't differ much between the deterministic model with two versus three periods of working life. Hence, most of the difference to the baseline results are due to the lack of additional productivity shocks, reinforcing the baby boom (and hence subsequent fluctuations). However, the baby boom happens slightly earlier. This is less of an issue of length of the period as it is one of length of generations. In the three period model, parents and children are 15, instead of 20 years apart. The analogous results for the time cost case are similar and hence are not included.

In sum, the dampened oscillations we find in the baseline model are not just an artifact

Figure 7: Percent Deviations in TFR, 3PWL, Goods Cost
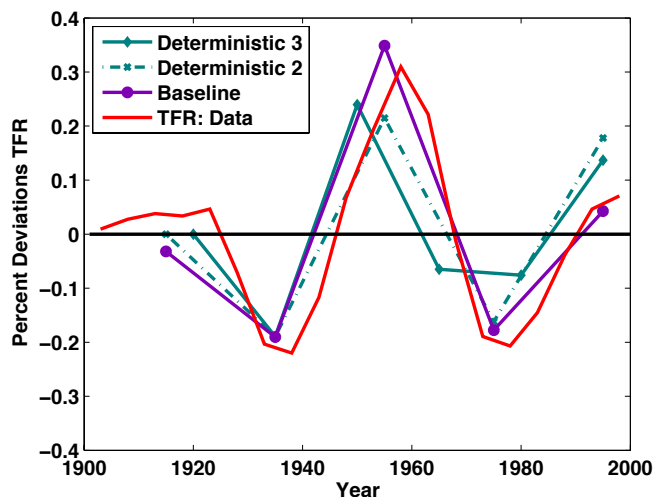
of having two periods working life.

6 Statistical evidence

There is a large statistical literature on the cyclicality of fertility. Sobotka et al. (2010) extensively review this literature and analyze several major recessions in detail including the Great Depression, the recession of the 1970s and economic shocks in Central and Eastern Europe after 1989. In their survey of 26 low-fertility countries from 1980-2008, they find that of 62 recessions (defined as a drop in GDP) 50 were followed by a drop in TFR one year later.

We provide two additional pieces of statistical evidence. First, we analyze data for 17 countries during the Great Depression. We find that countries with a smaller Depression in the 1930s tend to have smaller baby busts and that those with smaller baby busts tend to have smaller baby booms in the 1950s, conditional on the economic circumstances in the 1950s. This last part directly relates to our theory and was not considered in Sobotka et al. (2010). Second, we provide evidence on the recent recession across U.S. states, while Sobotka et al. (2010)'s analysis concentrates on country level data. We find that states with a larger drop in income per capita tend to have a larger reduction in fertility in 2008.

6.1 International evidence on the Great Depression

In this section, we show that countries with a smaller Depression in the 1930s also tend to have smaller baby busts and that those with smaller baby busts tend to have smaller

baby booms in the 1950s, conditional on the economic circumstances in the 1950s. Before we start, several data related issues should be addressed. These relate to data availability, detrending methods and dealing with war periods.

Ideally, we would like to use the same measures of fertility and productivity as we do for the U.S.. However, for the 20th century complete TFR series and consistent TFP measures are only available for few countries (see e.g., Chesnais (1992) for TFR and Cette et al. (2009) for TFP). We therefore use Crude Birth Rates (CBR) mostly from Mitchell (1998) and GDP per capita from Barro and Ursua (2008), instead.[30] This gives us 17 countries for which we have data from 1900 until 1993.[31]

As to detrending, for CBR, we use the same detrending method as we did for TFR in the U.S., namely a Hodrick-Prescott (HP) filter. While for the U.S. a linear detrending method for log-TFP poses few problems, this is not true for all countries we consider. In particular, if one uses linear detrending, countries such as Portugal or Japan show a permanent negative deviation in GDP early in the century and a permanent positive one later on. Besides the linear detrending method, we therefore also use an HP filter in comparison.

Furthermore, the two world wars affected fertility levels in several ways that are unrelated to our mechanism. For example, men being away to fight prevents some women from having children. Typically, there is rapid catching up as soon as the men return, if they do (e.g., see the period 1943-1946 in U.S. data shown in Figure 3). The longer the war involvement and the larger the death rate of soldiers, the more unpredictable this effect on fertility rates becomes. Similar issues emerge for GDP data during war periods. For example, a large part of the increase in output in Germany during the war was not for private consumption but rather war related. We therefore also use dummies for war and immediately after war periods to adjust our measures of deviations in CBR and GDP in comparison to deviations resulting from a simple linear trend and the HP filter.[32]

Given these provisos, we are now ready to analyze sizes of Depressions, baby busts and baby booms. First, we aim to determine the correlation between the size of the Depression in the 1930s and the size of the baby bust at the time across countries. Since the timing of the Depression was different across countries, we divide the 1930s into two periods: 1930 to

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[30]CBR data form 1900 to 1993 are obtained from Mitchell (1998) for all countries, except Germany. German data was combined using Chesnais (1992, Appendix A1) for the 1900 to 1943 period and Statistisches Bundesamt (2007) for 1946 to 1993, linearly interpolating 1944 and 1945. All GDP per capita data comes from Barro and Ursua (2008) and is in millions 1990 International Geary-Khamis dollars.

[31]The main issues are. CBR as opposed to TFR is sensitive to mortality in certain groups of the population such as wars which affect total population but not the number of fertile women. GDP per capita as opposed to TFP or LP is only an indirect measure of the productivity of the labor force and therefore corresponds less well to our model.

[32]For details on detrending procedures, see Supplementary Appendix B.4.

1934 and 1935 to 1939. We then run eight cross-country OLS regressions, one for each time period combined with one of four detrending methods. Let $\widehat{X}_t^i$ denote the percent deviation from trend in variable X in country i in five-year period t.

$$\widehat{CBR}_t^i = \lambda_{1,t} + \lambda_{2,t}\widehat{GDP}_t^i + \varepsilon^i$$

where $t = \{30-34, 35-39\}$. The results for each detrending method are given in Table 6.[33]

Table 6: Cross-Country: Depressions and Busts, 1930s

Dependent Variable	Independent Variable	Detrending Method			
		Linear	HP	Linear No war	HP No war
$\widehat{CBR}_{30-34}^i$	Constant ($\lambda_{1,30-34}$)	-0.0522	-0.0839	-0.0365	-0.0446
	$\widehat{GDP}_{30-34}^i$ ($\lambda_{2,30-34}$)	0.3512	0.4253	0.3933	0.6480
	(p-value)	(0.059)	(0.008)	(0.084)	(0.008)
$\widehat{CBR}_{35-39}^i$	Constant ($\lambda_{1,35-39}$)	-0.0856	-0.1211	-0.0716	-0.0806
	$\widehat{GDP}_{35-39}^i$ ($\lambda_{2,35-39}$)	0.2118	0.5743	0.2482	0.5554
	(p-value)	(0.441)	(0.005)	(0.335)	(0.004)

As can be seen, the coefficient, $\lambda_{2,t}$, is positive for all detrending methods and both time periods. It is larger (with a lower p-value) whenever an HP filter is used rather than linear detrending and whenever the effects of wars are taken into account in detrending. Also, for most detrending methods, $\lambda_{2,t}$ is larger in the early 1930s than in the late 1930s.[34]

Second, we aim to determine whether a larger baby bust in the 1930s implies a larger baby boom in the 1950s, controlling for the GDP deviation in the 1950s. To do this, we run eight cross-country OLS regressions, one for each time period combined with one of four detrending methods.

$$\widehat{CBR}_t^i = \lambda_{3,t} + \lambda_{4,t}\widehat{CBR}_{t-20}^i + \lambda_{5,t}\widehat{GDP}_t^i + \varepsilon^i$$

where $t = \{50-54, 55-59\}$. The results for each detrending method are given in Table 7.[35]

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[33]Tables B.7 to B.8 show the inputs into these regressions, namely percent deviations from trend in CBR and GDP in the 1930s for each country and each detrending method.

[34]Some observations relating to depressions and busts across countries leading to these regression results are described in Supplementary Appendix B.4.1.

[35]Tables B.9 to B.10 Appendix B.4 show the inputs into these regressions, namely percent deviations from trend in CBR in the 1930s and 1950s and in GDP in the 1950s for each country and each detrending method.

Table 7: Cross-Country: Busts and Booms, 1950s vs. 1930s

Dependent Variable	Independent Variable	Detrending Method			
		Linear	HP	Linear No war	HP No war
$\widehat{CBR}_{50-54}^{i}$	Constant ($\lambda_{3,50-54}$)	0.0477	0.0203	0.0853	0.0506
	$\widehat{CBR}_{30-34}^{i}$ ($\lambda_{4,50-54}$)	-0.2795	-0.3680	-0.1527	-0.2792
	(p-value)	(0.229)	(0.058)	(0.634)	(0.372)
	$\widehat{GDP}_{50-54}^{i}$ ($\lambda_{5,50-54}$)	0.2181	0.3316	0.2571	0.2034
	(p-value)	(0.171)	(0.092)	(0.208)	(0.428)
$\widehat{CBR}_{55-59}^{i}$	Constant ($\lambda_{3,55-59}$)	0.0960	0.0296	0.0680	0.0296
	$\widehat{CBR}_{35-39}^{i}$ ($\lambda_{4,55-59}$)	-0.0973	-0.2854	-0.4048	-0.5748
	(p-value)	(0.714)	(0.181)	(0.143)	(0.020)
	$\widehat{GDP}_{55-59}^{i}$ ($\lambda_{5,55-59}$)	0.6052	1.0424	0.4029	0.4887
	(p-value)	(0.012)	(0.004)	(0.045)	(0.046)

As can be seen, the coefficient, $\lambda_{4,t}$, is negative for all detrending methods and both time periods. It is larger in absolute value (with a lower p-value) whenever an HP filter is used rather than linear detrending and, for the late 1950s, whenever the effects of wars are taken into account in detrending. Also, $\lambda_{4,t}$ is larger in the early 1930s than in the late 1930s when the effects of wars are not taken into account in detrending, smaller when they are. Similar to $\lambda_{2,t}$, the coefficient $\lambda_{5,t}$ is positive for all detrending methods and both time periods. However, p-values are larger, especially in the early 1950s.[36]

In sum then, international evidence on the sizes of depressions and baby busts in the 1930s strongly supports our theory while the evidence for the size of the subsequent baby boom, though rightly signed, is somewhat weaker.

6.2 The Great Recession across 25 U.S. states

The most recent recession, starting in 2007, also seems to have had an impact on fertility in the United States. According to the National Center for Health Statistics, the birth rate (the number of births per 1000 women age 15 to 44) fell from 69.5 in 2007 to 66.7 in 2009. This corresponds to a 4 percent drop overall with most of it concentrated in younger age-groups.

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[36]Some observations relating to busts and booms across countries leading to these regression results are given in Supplementary Appendix B.4.1.

Preliminary data for the twelve-months period through June 2010 show a further decrease to a birth rate of 65.5, an additional 3 percent drop.

Data from a Pew Study in April 2010 across 25 U.S. states show that those states that were most affected by the recession between experienced a larger drop in birth rates from 2007 to 2008 (see Figure 8). The correlation coefficient between percent changes in birth rates between 2007 and 2008 and percent changes in income per capita, between 2006 and 2007 and between 2007 and 2008, are 0.56 and 0.52, respectively.

Of course, this recession is still ongoing in the sense that, while GDP per capita may have started to increase again, growth rates have not yet attained their long term levels. Because of this, it is yet to be seen how large the (completed) fertility response to current events will be and whether this generates a baby boom in the 2020s.

Figure 8: Cross-state evidence: Recent Recession
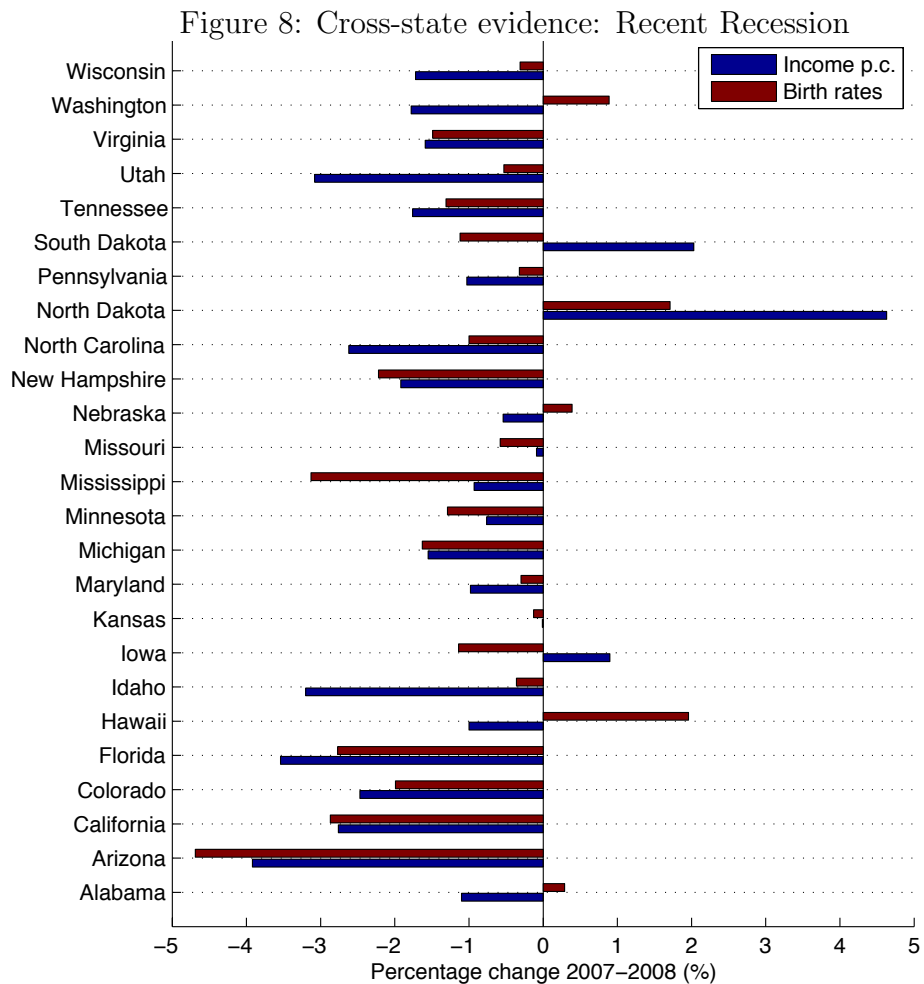
Compared to the U.S., the fertility response to the recent economic downturn among

European countries is less evident if it exists at all. For example, using preliminary data from Eurostat, we find that between 2008 and 2009 many countries only saw their fertility level decrease slightly (and some not at all). The largest drop in fertility was in Spain with 4.6 percent, followed by Portugal with 3.7 percent—two countries whose economy was affected early on in the crisis.

7   Concluding remarks

In this paper, we have studied the properties of models of fertility choice with dynastic altruism in which both aggregate shocks and multiple generations are present. We have shown that in simple versions of this class of models, fertility is procyclical and has dampened oscillations when perturbed away from the steady state. Moreover, these effects are quantitatively significant. In calibrated simulations of the model, we find that in response to a negative shock to income like the Great Depression, the decline in fertility in the model is between 40 and 70 percent of the decline actually seen in the data. Moreover, because of a (mild) productivity boom in the 1950s and early 1960s in conjunction with the oscillatory effects discussed above, the model accounts for between 60 and 90 percent of the baby boom. The key parameter determining the quantitative findings is the nature of the cost of children – goods versus time.

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A Appendix

A.1 Decentralization with transfers

Suppose, every generation can choose their own consumption when age $a = 1, 2, 3$, fertility when age $a = 1$, as well as the following two transfers. Let $tr_t^2(s^t)$ be the transfer people age 2 in period t (i.e. old workers) give to (take from, if negative) their children (i.e. young workers in period t). Let $tr_t^3(s^t)$ be the transfer people age 3 in period t (i.e. retirees) give to (take from, if negative) their children (i.e. old workers in period t). So the budget constraints for $a = 1, .., 3$ are given by

$$c_t^1(s^t) + \theta_t(w_t^1(s^t))n_t(s^t) \leq w_t^1(s^t) + tr_t^2(s^t)$$
$$c_{t+1}^2(s^{t+1}) \leq w_{t+1}^2(s^{t+1}) + tr_{t+1}^3(s^{t+1}) - n_t(s^t)tr_{t+1}^2(s^{t+1})$$
$$c_{t+2}^3(s^{t+2}) \leq -n_t(s^t)tr_{t+2}^3(s^{t+2})$$

Clearly, since $w^3 = 0$ and there is no other income in retirement (e.g. savings, pensions,...), $tr_{t+2}^3(s^{t+2})$ will be negative. $tr_{t+1}^2(s^{t+1})$, on the other hand, may be positive or negative, depending on the relative size of w^1, w^2, s_t and θ or b.

In any period t, the following budget constraints must hold:

$$c_t^1(s^t) + \theta_t(w_t^1(s^t))n_t(s^t) \leq w_t^1(s^t) + tr_t^2(s^t)$$
$$c_t^2(s^t) \leq w_t^2(s^t) + tr_t^3(s^t) - n_{t-1}(s^{t-1})tr_t^2(s^t)$$
$$c_t^3(s^t) \leq -n_{t-2}(s^{t-2})tr_t^3(s^t)$$

Multiplying each budget constraint by N_t^a we get:

$$N_t^1 c_t^1 + N_{t+1}^1 \theta_t(w_t^1) \leq N_t^1 w_t^1 + N_t^1 tr_t^2$$
$$N_t^2 c_t^2 \leq N_t^2 w_t^2 + N_t^2 tr_{t+1}^3 - N_t^1 tr_t^2$$
$$N_t^3 c_t^3 \leq N_t^3 w_t^3 - N_t^2 tr_t^3$$

Substituting out $N_t^2 tr_t^3$ and then $N_t^1 tr_t^2$ in the budget constraint of the retirees feasibility for the dynasty pertains. Thus, feasibility,

$$\sum_{a=1}^3 N_t^a c_t^a + \theta_t(w_t^1)n_t N_t^1 \leq \sum_{a=1}^3 w_t^a N_t^a,$$

holds by construction.[37]

This means that the old face the same budget constraint every period as the planner does. Hence, the solution will be the same.

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[37]It is useful to note the following.

1. We are not restricting transfers in any way but as long as parents care about the utility of their children, they will never take everything from their children.

2. The argument also goes through if the retirees choose two transfers, to their children and to their grandchildren, while old workers don't choose any transfers.

3. "Bequests" are often modeled as a quantity parents choose before dying and children receive (with interest) after parents are dead. The transfers here are "inter-vivos transfers", chosen and received in the same period. In deterministic models, this makes no difference. In stochastic models, it matters whether the transfers are chosen before or after the current shock is observed. In this sense, our transfers cannot be viewed as bequests in the usual sense.

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