Midterm Exam

Econ 8105-8106
Fall 2003
Prof. L. Jones

Please answer both questions.

Each question is *not* weighted equally, so budget your time accordingly.

Problem 1 (65 points)

Consider an economy with two infinitely-lived consumers, indexed by $i = 1, 2$, who receive endowments of a consumption good each period, denoted by e_t^i. There is no production. The endowments are:

$$e_t^1 = \begin{cases} 1 & t = 0, 2, 4, ... \\ 0 & t = 1, 3, 5, ... \end{cases}$$
$$e_t^2 = \begin{cases} 0 & t = 0, 2, 4, ... \\ 1 & t = 1, 3, 5, ... \end{cases}$$

Consumer i has preferences over the consumption good represented by

$$U^i(\underset{\sim}{c}) = \sum_{t=0}^{\infty} \beta_i^t \log c_t^i, \qquad \underset{\sim}{c} = (c_0, c_1, ...)$$

Assume that $\beta_1 > \beta_2$.

(a) (15 points) Define an Arrow-Debreu Equilibrium for this economy.

(b) (25 points) Define a Pareto Optimal allocation for this economy, and prove the First Welfare Theorem (that is, prove that the allocation of any Arrow-Debreu Equilibrium is Pareto Optimal).

Now, consider the problem of a social planner maximizing a weighted sum of the consumers' utilities, i.e. $\lambda_1 U^1(\underset{\sim}{c}) + \lambda_2 U^2(\underset{\sim}{c})$, subject to feasibility constraints.

(c) (5 points) For what λ_1, λ_2 is the solution to this problem a Pareto Optimal allocation?

(d) (15 points) Suppose that the planner weights each consumer equally ($\lambda_1 = \lambda_2$). Solve for the Pareto Optimal allocation. What happens to the consumption of each consumer over time?

(e) (5 points) Are there weights so that the solution to the social planner's problem gives each consumer the same amount of consumption at all dates (i.e. $c_t^1 = c_t^2$ for all t)? Is this Pareto Optimal?

Problem 2 (35 points)

Consider a one-sector Arrow-Debreu economy, with one consumer and one firm that produces both consumption and investment goods. The consumer has an initial endowment $\bar{k}_0$ of capital, and an endowment $\bar{n}_t^i$ of time each period. Suppose that the firm chooses how much of the consumer's intitial capital it wants to buy, then chooses how much of its output of investment good to devote to accumulating its capital stock; that is, the firm's problem is to:

$$\max_{\{c_t, x_t, n_t, y_t, k_t\}_{t=0,1,\ldots}} \sum_{t=0}^{\infty} [p_t(c_t + x_t) - w_t n_t - p_t y_t] - qk_0$$

$$\text{subject to, for all } t:$$

$$c_t + x_t \leq F(k_t, n_t)$$

$$k_{t+1} \leq (1-\delta)k_t + y_t$$

$$c_t, x_t, n_t, y_t, k_t \geq 0$$

Show that, if the solution to this problem is interior, then the problem can be rewritten so that it is as if the firm is paying to rent capital in each period. What would be the rental price of capital?

(Note: you are just asked to consider the firm's problem here; do not concern yourself with the consumer's problem.)