

## Second Midterm Exam

Econ 8105-8106

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Please answer both questions.

Each question is worth 50 points

1. Consider the following single-sector growth problem:

$$\max_{\{c_t, l_t, n_t, k_{t+1}, x_t\}_{t=0,1,\dots}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to:

$$c_t + x_t \leq f(k_t, n_t)$$

$$k_{t+1} \leq k_t(1 - \delta) + x_t$$

$$n_t + l_t \leq \bar{n}$$

$$c_t, k_{t+1}, n_t, l_t \geq 0$$

$k_0$  given.

- (a) Write this problem as a dynamic programming problem in sequence form, that is, as maximizing an infinite discounted sum of a time-stationary return function, with a time-stationary constraint set.  
(Note: there are different ways of doing this; pick one method and be consistent.)
- (b) Write the functional equation (Bellman's equation) for the problem you wrote in (a).
- (c) Provide assumptions on  $u$ ,  $f$ ,  $\beta$ , and  $\delta$  that are sufficient to guarantee that there is a unique bounded and continuous function satisfying the functional equation in (b). Explain briefly why each assumption is needed.
- (d) Now suppose labor is inelastically supplied by the consumer. Write Bellman's equation for this problem and characterize the optimal policy through the first-order and envelope conditions. Define a steady-state for this problem and show that there is a unique positive steady-state. (State any assumptions on  $u$ ,  $f$ ,  $\beta$ , and  $\delta$  that you use.)

2. Consider a one-sector economy with one infinitely-lived representative consumer, one representative firm, and a government. The consumer has preferences represented by utility function:

$$\sum_{t=0}^{\infty} \beta^t [\gamma \log c_t + (1 - \gamma) \log l_t]$$

and the firm has technology constraint:

$$c_t + x_t + g_t \leq F(k_t, n_t)$$

The government finances its purchases of  $g_t$  through a tax on both labor income and capital income at the constant rate  $\tau$ .

- (a) Define a Tax-Distorted Competitive Equilibrium (TDCE) for this economy.
- (b) Characterize the steady-state of the TDCE, and determine how steady-state capital stock, consumption, and labor supply depends on the tax rate  $\tau$ .
- (c) Suppose that instead of income taxes, the government only taxes purchases of consumption and investment goods. Is there a tax system of this sort that will give the same steady-state allocation as in (b)?
- (d) Suppose that instead of purchasing  $g_t$  with the income taxes, the government subsidizes purchases of consumption and investment goods and balances its budget in every period. Is there a way to do this so that the steady-state TDCE allocation is the same as if there were no government at all?