Problem Set #3

Econ 8105-8106
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Question 1

Consider the following social planner's problem:

$$\max_{\{c_t,n_t,l_t,k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t. c_t + k_{t+1} = f(k_t, n_t)$$

$$k_o \; given; n_t + l_t = 1; c_t \geq 0; k_{t+1} \geq 0$$

Show that this problem has a unique solution. If you need some assumptions, state them clearly.

Question 2

Consider the problem of finding a Pareto Optimal allocation in a T-period economy with one consumer and one firm. The consumer has preferences over consumption and leisure represented by the utility function:

$$U(\underline{c}, \underline{l}) = \sum_{t=0}^{T} \beta^t u(c_t),$$

where $0 < \beta < 1$. The firm produces consumption and investment goods according to the technology:

$$c_t + x_t \leq F(k_t, n_t),$$

and capital stock evolves according to:

$$k_{t+1} \leq (1 - \delta)k_t + x_t.$$

Write the first-order necessary conditions for this problem as a second-order difference equation in k (i.e. an equation in k_{t-1}, k_t, and k_{t+1}).

Show that if a sequence of capital stocks satisfies these conditions and $k_{T+1} = 0$, then it is part of a Pareto Optimal allocation (make any additional assumptions you need on u and F).

Suppose now that the time horizon is infinite. What condition do you need to add to the equations found above to show that a sequence of capital stocks satisfying these conditions is part of an optimal allocation? How would you define this additional condition in an Arrow-Debreu equilibrium setting?

Question 3

Complete the "informal constructive proof" of the Second Welfare Theorem presented in class.

Question 4

This question outlines a procedure (usually referred to as "Negishi's method") for using the Second Welfare theorem to find an Arrow-Debreu equilibrium in a specific environment.

Consider an economy with 2 consumers and one firm. The consumers have endowments of hours $\overline{n}_t^1$, $\overline{n}_t^2$, and endowments of capital $\overline{k}^1, \overline{k}^2$. There is no investment and no depreciation, so the capital stocks stay constant over time. The consumers have preferences over consumption and leisure represented by the utility function

$$U^i(\underline{c}^i, \underline{l}^i) = \sum_{t=0}^{\infty} \beta^t \log(c_t^i) \qquad i = 1, 2,$$

where $\beta < 1$. The firm produces the consumption good using labor and capital according to the technology:

$$c_t \leq Ak^{1-\alpha} (n_t)^{\alpha}.$$

(a) Set up the problem of a "social planner" maximizing a weighted sum of the consumers' utilities subject to attaining a feasible allocation. For given weights, solve for the Pareto Optimal allocation.

(b) In the Arrow-Debreu environment of this economy, what would be the prices that implement this optimal allocation as an Arrow-Debreu equilibrium? Find the transfers needed to implement this allocation as an Arrow-Debreu Transfer Equilibrium.

(c) Find the Arrow-Debreu equilibrium of the economy by setting the transfers from (b) equal to zero.

Question 5

Consider the two-sector model presented in class. Define an Arrow-Debreu Equilibrium with transfers (ADTE) in this environment. Show that, if for fixed lump sum taxes and transfers $(T_1, ...T_I)$ an ADTE exists, its allocation is Pareto optimal.

Question 6

Consider an economy with I infinetely-lived consumers, one consumption good produced by one firm and with no capital. State the conditions needed to show that an Arrow Debreu equilibrium in this environment is the same than one in which each consumer's budget constraint is balanced period by period.

Question 7

This question asks you to establish the equivalence of Arrow-Debreu Equilibrium and Sequential Markets Equilibrium in an environment with I consumers, and J firms that produce both consumption and investment goods and sell all their output at price p_t and time horizon is infinite.

(a) Define a Sequential Markets equilibrium in this environment. What should replace $L_T = 0$, which we imposed in the finite horizon case? Why?

(b) In a Sequential Markets environment, consider the constraint $L_t^i \leq A$, for all t, where A is "large enough." Show that this constraint implies that, if $r_t^L > 0$ for all t, then: $\lim_{t\to\infty} \frac{L_t^i}{(1+r_0^L)(1+r_1^L)\cdots(1+r_{t-1}^L)} = 0$,
where r_t^L is the interest paid (or received) on borrowing (or lending) done in period t.

(c) Consider any Sequential Markets Equilibrium. Construct prices for the Arrow-Debreu environment of the economy so that the allocation of the Sequential Markets Equilibrium (without the $L^{'}$s) is the allocation of an Arrow-Debreu equilibrium at those prices.

(d) Consider any Arrow-Debreu equilibrium. Construct prices, interest rates, and borrowing/lending amounts so that the allocation of the Arrow-Debreu equilibrium is a part of the allocation of a Sequential Markets Equilibrium at those prices.

(Note: you may find it easier to start with the model presented in class, in which there are no firms, no capital, and no labor, and each consumer has an endowment of the consumption good in each period.)

Question 8

In this question you are asked to prove some results using the method 1 of aggregation. I.e., assume all firms are identical within and across sectors and the technology is CRS; all households have the same endowment of capital and time; $U^i = U$,for all $i = 1, .., I$.and U is strictly concave. Show that the competitive equilibrium of this economy is the same as one with one firm and one household (state clearly this CE). Show that this CE solves a Planner's problem (state this Planner's problem).